LETTER *Special Section on Radio Access Techniques for 3G Evolution*

Performance Improvement by Frequency-Domain Interleaving for OFDM/TDM Using MMSE-FDE in a Wireless Channel

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SUMMARY The use of frequency-domain interleaving on a frame-by-frame basis for orthogonal frequency division multiplexing (OFDM) combined with time division multiplexing (OFDM/TDM) is presented. In conventional OFDM, FDE is not designed to exploit the channel frequency-selectivity and consequently, the frequency diversity gain cannot be obtained. To further improve the bit error rate (BER) performance of conventional OFDM an interleaving technique may be applied, but FDE cannot be fully exploited. In this letter, the OFDM/TDM signal (i.e., several concatenated OFDM signals) frequency components are interleaved at the transmitter and then, minimum mean square error frequency-domain equalization (MMSE-FDE) is applied at the receiver to obtain a larger frequency diversity gain. It is shown that frequency-domain interleaving on a frame-by-frame basis for OFDM/TDM using MMSE-FDE achieves improved BER performance in comparison with conventional OFDM due to enhanced frequency diversity gain.

key words: *OFDM/TDM, MMSE-FDE, frequency-domain interleaving, channel frequency-selectivity*

1. Introduction

Orthogonal frequency division multiplexing (OFDM), which is robust against multipath fading, has a drawback of having a large peak-to-average power ratio (PAPR) [1]. This undesirable feature renders the OFDM particularly sensitive to nonlinear distortions (e.g., high-power amplifier (HPA)). Recently, we proposed OFDM combined with time division multiplexing [2] (in this paper called OFDM/TDM) using minimum mean square error frequency-domain equalization (MMSE-FDE) [3] to improve the transmission performance in terms of the bit error rate (BER) and the PAPR.

Of late, there has been some work on OFDM with interleaving to improve the system performance [4]–[6]. In [4], an interleaved OFDM is used to increase the transmission efficiency; but, since a wireless channel is assumed to be invariant over P consecutive OFDM signals, (i.e., P is a length of the interleaver) the channel frequency-selectivity cannot be exploited and consequently, the system performance cannot be improved. A two-dimensional interleaving was proposed in [5] to improve the BER performance, but the interleaving size must be long if the performance improvement is to be achieved. This increases the latency (i.e., the processing delay) of the system.

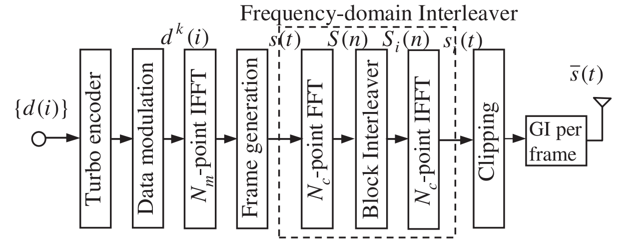
In this letter, frame-by-frame frequency-domain interleaving for OFDM/TDM is applied at the transmitter to obtain a larger frequency diversity gain through MMSE-FDE at the receiver. Unlike conventional OFDM, a combination of frequency-domain interleaving for OFDM/TDM at the transmitter and MMSE-FDE at the receiver can be used to exploit the channel frequency-selectivity and improve the BER performance.

The letter is organized as follows. In Sect. 2, system model is presented. Frequency-domain interleaving on a frame-by-frame basis for OFDM/TDM using MMSE-FDE is presented in Sect. 3, and its performance is evaluated in Sect. 4. Sect. 5 concludes the paper.

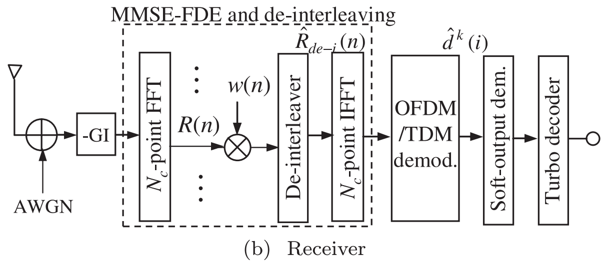
2. System Overview

The OFDM/TDM system model is illustrated in Fig. 1. Throughout this paper, T_c-spaced discrete time representation is used, where T_c represents FFT sampling period.

An information bearing sequence $\mathbf{d}$ of length M is channel coded and punctured, bit interleaved and quadrature phase shift keying (QPSK) modulated to generate N symbol sequence $\mathbf{c}$. The sequence is divided into N/N_c blocks each of having N_c data-modulated symbols, where the mth block is represented as $\{d_m(n);\ n = 0 \sim N_c - 1\}$ for $m = 0 \sim N/N_c - 1$. In this work, we consider a transmission of



(a) Transmitter



(b) Receiver

Fig. 1 OFDM/TDM system model.

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N_c data-modulated symbols without loss of generality and thus, the block index m is omitted in what follows. Each block $\{d(n)\}$ is divided into K subblocks each of having N_m $(=N_c/K)$ data-modulated symbols. The kth subblock is denoted by $\{d^k(i);\ i=0\sim N_m-1\}$, where $d^k(i)=d(kN_m+i)$ for $k=0\sim K-1$. The OFDM/TDM signal is expressed using the equivalent low-pass representation as

$$s(t)=\frac{1}{\sqrt{N_m}}\sum_{i=0}^{N_m-1}d^{\lfloor t/N_m\rfloor}(i)\exp\left\{j2\pi t\frac{i}{N_m}\right\} \tag{1}$$

for $t=0\sim N_c-1$. Frequency-domain frame-by-frame interleaving is applied (see Sect. 3) and then, interleaved signal represented by $\{S_i(n);\ n=0\sim N_c-1\}$ is transformed back to time-domain signal represented by $\{s_i(t);\ t=0\sim N_c–1\}$. Due to frequency-domain interleaving the PAPR of OFDM/TDM will regrow. To limit the PAPR, deliberate amplitude clipping is applied as [1]

$$\bar{s}(t)=\begin{cases}s_i(t), & |s_i(t)|\le\beta\\ \beta\frac{s_i(t)}{|s_i(t)|} & \text{otherwise}\end{cases} \tag{2}$$

for $t=0\sim N_c–1$, where β denotes the predetermined clipping level, which is known at the receiver side. After insertion of the guard interval (GI) the OFDM/TDM signal is multiplied by power coefficient $\sqrt{2E_s/T_c}$ and transmitted over a frequency-selective fading channel, where E_s denotes the data-modulated symbol energy.

At the receiver, N_c-point FFT is applied over the entire OFDM/TDM frame with K concatenated OFDM signals to decompose the received signal into N_c frequency components $\{R(n);\ n=0\sim N_c–1\}$. We note here that $R(n)$ includes a nonlinear degradation term due to deliberate clipping. One-tap MMSE-FDE [7] is applied over the entire OFDM/TDM frame as [3]

$$\hat{R}(n)=R(n)w(n), \tag{3}$$

where $w(n)$ is the MMSE equalization weight, which includes the degradation due to clipping noise, and is given by

$$w(n)=\frac{\left[1-\exp\left(-\beta^2\right)+\frac{\sqrt{\pi}\beta}{2}\mathrm{erfc}\left(\beta\right)\right]H^*(n)}{[1-\exp(-\beta^2)]\,|H(n)|^2+(\frac{E_s}{N_0})^{-1}}, \tag{4}$$

where $\mathrm{erfc}[x]=\frac{2}{\sqrt{\pi}}\int_x^\infty\exp(-t^2)dt$ is the complementary error function. In Eq. (4), $H(n)$ and N_0, respectively, denote the channel gain and the single-sided AWGN power spectrum density. $(\cdot)^*$ denotes complex conjugate operation.

After de-interleaving, the OFDM/TDM signal is transformed back into time-domain by applying N_c-point IFFT to $\{\hat{R}_{de-i}(n);\ n=0\sim N_c-1\}$ (see Sect. III). The time-domain OFDM/TDM signal $\{r(t);\ t=0\sim N_c-1\}$ is obtained as

$$\hat{r}(t)=\sum_{n=0}^{N_c-1}\hat{R}_{de-i}(n)\exp\left\{j2\pi t\frac{n}{N_c}\right\} \tag{5}$$

for $t=0\sim N_c-1$. Thereby, the decision variable for the ith data symbol of the kth OFDM signal can be obtained by applying N_m-point FFT as

$$\hat{d}^k(i)=\frac{1}{\sqrt{N_m}}\sum_{t=kN_m}^{(k+1)N_m-1}\hat{r}(t)\exp\left\{-j2\pi i\frac{t}{N_m}\right\} \tag{6}$$

for $i=0\sim N_m-1$ and $k=0\sim K-1$.

For turbo decoding [8], the log-likelihood ratio (LLR) is required. Let us denote the $2N_c$-bit sequence that constructs the N_c-symbol block $\{d(n);\ n=0\sim N_c-1\}$, by $b=\{b_0(0),b_1(0),b_2(1),b_3(1)\ldots b_{2N_c-2}(N_c-1),b_{2N_c-1}(N_c-1)\}$. Using Eq. (6) and the well-known approximation $\log\sum_n\exp(x_n)\approx\max_n x_n$ [9] the LLR $L[b_j(n)]$ for the jth bit in the nth QPSK symbol $d(n)$ is given by

$$\begin{aligned}L[b_j(n)]\ \approx\ &\min\frac{|\hat{d}(n)-\frac{\alpha}{N_c}\sum_{n=0}^{N_c-1}\hat{H}(n)\hat{s}_0|^2}{2\sigma^2}\\ &-\min\frac{|\hat{d}(n)-\frac{\alpha}{N_c}\sum_{n=0}^{N_c-1}\hat{H}(n)\hat{s}_1|^2}{2\sigma^2},\end{aligned} \tag{7}$$

In the above expression, $\Pr[\cdot|\cdot]$ is the conditional probability and $\{\hat{d}(n)=\hat{d}^k(\lfloor n/N_m\rfloor);\ n=0\sim N_c–1\}$ with $\hat{H}(n)=H(n)w(n)$. $\hat{s}_0$ (or $\hat{s}_1$) is the candidate symbol with $b_j(n)=0$ (or 1) for which the Euclidian distance from $\hat{d}(n)$ is minimum. We assume that residual inter-symbol interference (ISI) after MMSE-FDE and clipping noise are Gaussian random variables and thus, $2\sigma^2$ denotes variance of the sum of residual ISI, the clipping noise and the AWGN after MMSE-FDE given by

$$\begin{aligned}2\sigma^2\ =\ &\frac{2\alpha^2E_s}{T_cN_c}\sum_{n=0}^{N_c-1}\left|\hat{H}(n)-\frac{1}{N_c}\sum_{m=0}^{N_c-1}\hat{H}(m)\right|^2|\Psi(n)|^2\\ &+\frac{2E_sN_m}{T_cN_c}\sum_{n=0}^{N_c-1}\left[1-\exp(-\beta^2)-\alpha^2\right]|\hat{H}(n)|^2|\Psi(n)|^2\\ &+\frac{2N_0}{T_cN_c}\sum_{n=0}^{N_c-1}|w(n)|^2|\Psi(n)|^2,\end{aligned} \tag{8}$$

where $\Psi(n)$ is defined in [3].

The first term in Eq. (8) is the residual ISI power, the second term is the clipping noise power and the third term is the AWGN power. Note that in the case of conventional OFDM ($K=1$) the first term in square brackets will be omited. The LLR values are computed and then, data decoding is performed using the sequence of LLR values as a soft input [8].

3. Frequency-Domain Interleaving for OFDM/TDM Using MMSE-FDE

So far, the use of interleaving and FDE with conventional OFDM has not been fully exploited. This is because FDE is applied to each OFDM signal independently to compensate for the channel distortions. Hence, a larger frequency diversity gain cannot be obtained.

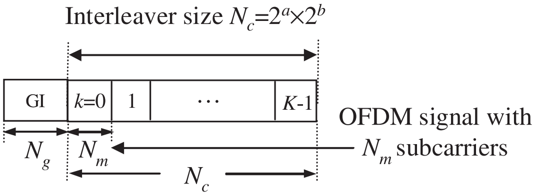

Fig. 2 OFDM/TDM frame structure.

To enhance the frequency diversity effect we introduce a frequency-domain interleaver over the entire OFDM/TDM frame i.e., K concatenated OFDM signals. The combination of frequency domain interleaving and MMSE-FDE at the OFDM/TDM receiver provides improved BER performance due to enhanced frequency diversity gain. At the transmitter (see Fig. 1), after OFDM/TDM modulation, N_c-point FFT is applied over OFDM/TDM frame signal $\{s(t);\ t = 0 \sim N_c - 1\}$ to decompose the signal into N_c frequency components $\{S(n);\ n = 0 \sim N_c - 1\}$ as

$$S(n) = \sum_{t=0}^{N_c-1} s(t) \exp\left\{ - j2\pi n \frac{t}{N_c} \right\}. \tag{9}$$

Here we note a difference between a subcarrier component in the conventional OFDM and the OFDM/TDM frequency component each of carrying a portion of data-modulated symbol. The OFDM/TDM signal frequency components $\{S(n);\ n = 0 \sim N_c - 1\}$ are fed to the interleaver to permute the ordering and obtain the interleaved OFDM/TDM frequency components given by

$$S_i(n) = \pi\{S(n)\} \tag{10}$$

for $n = 0 \sim N_c - 1$, where $\pi\{\cdot\}$ represents the interleaver function. The performed displacement of frequency components is described by an interleaving vector $\pi = \{\pi_0, \pi_1, ..., \pi_{N_c-1}\}$ that defines a mapping from indexes of $\{R(n)\}$ to those of $\{R_i(n)\}$, such that

$$R_i(\pi\{n\}) = R(n) \tag{11}$$

for $n = 0 \sim N_c - 1$.

The size of the interleaver is N_c as shown in Fig. 2. The frequency-domain interleaver formats the OFDM/TDM frequency components in a rectangular array of 2^a rows and 2^b columns. The OFDM/TDM frequency components are write-in row-wise, but they are read-out column-wise. As a result of this reordering of the OFDM/TDM signal frequency components, each data symbol is spread over several frequencies, which is exploited by MMSE-FDE.

After frequency-domain interleaving, the permuted OFDM/TDM frequency components $\{S_i(n);\ n = 0 \sim N_c - 1\}$ are converted by N_c-point IFFT to the time-domain interleaved OFDM/TDM signal $\{s_i(t);\ t = 0 \sim N_c - 1\}$ as

$$s_i(t) = \frac{1}{N_c} \sum_{n=0}^{N_c-1} S_i(n) \exp\left\{ j2\pi t \frac{n}{N_c} \right\}. \tag{12}$$

Then, time-domain interleaved signal is clipped and transmitted over a frequency-selective fading channel.

At the receiver, after MMSE-FDE (i.e., Eq. (3)), the OFDM/TDM frequency components must be permuted back (de-interleaved) before OFDM/TDM demodulation as shown in Fig. 1. The equalized OFDM/TDM signal is fed to block de-interleaver as

$$\hat{R}_{de-i}(n) = \pi^{-1}\{\hat{R}(n)\} \tag{13}$$

with the same rectangular array format as the interleaver at the transmitter side. The equalized OFDM/TDM frequency components are write-in column-wise and they are read-out row-wise. The performed de-interleaving of frequency components is described by an interleaving vector $\pi^{-1} = \{\pi_0^{-1}, \pi_1^{-1}, ..., \pi_{N_c-1}^{-1}\}$ that defines a mapping from indexes of $\{\hat{R}(n)\}$ to those of $\{R_{de-i}(n)\}$, such that

$$R_{de-i}(\pi^{-1}\{n\}) = \hat{R}(n) \tag{14}$$

for $n = 0 \sim N_c - 1$.

Finally, the time-domain OFDM/TDM signal is recovered by applying N_c-point IFFT to $\{\hat{R}_{de-i}(n)\ n = 0 \sim N_c - 1\}$ and then, OFDM demodulation is carried out using N_m-point FFT to obtain decision variables $\{\hat{d}^k(i)\ i = 0 \sim N_m{-}1\}$ required for the LLR computation and turbo decoding.

4. Simulation Results and Discussions

We assume an OFDM/TDM frame size of $N_c = 256$ samples with the GI length of $N_g = 32$ samples and ideal coherent quadrature phase shift keying (QPSK) data modulation/demodulation. The propagation channel is $L = 16$-path block Rayleigh fading channel having uniform power delay profile, where the path gains remain constant over one OFDM/TDM frame length and vary frame-by-frame. We assume that the time delay of the lth path is l samples with $L < N_g$. Perfect knowledge of the channel state information is assumed. A rate 1/3 turbo encoder with constraint length 4 and (13, 15) RCS component encoders is assumed. The parity bit sequences are punctured to obtain coding rate $R = \frac{1}{2}$. Log-MAP decoding with 8 iterations is carried out at the receiver. The information bit sequence length is taken to be $M = 1024$ bits.

To limit the PAPR we have chosen clipping level $\beta = 4$ dB for both OFDM/TDM and conventional OFDM (a lower β will increase the BER, but, as shown in [10], OFDM/TDM using FDE still provides a lower BER over OFDM ($K = 1$)). On the other hand, the frequency diversity gain obtained by interleaving and FDE depends on the channel frequency-selectivity and hence, it is not affected by amplitude clipping at all. Due to this fact the impact of β on the performance is not discussed here.

The BER performance of OFDM/TDM as a function of the average signal energy per bit-to-AWGN power spectrum density ratio E_b/N_0 $(= 0.5 \times R \times (E_s/N_0) \times (1 + N_g/N_c))$ with K as a parameter is illustrated in Fig. 3. The figure shows two

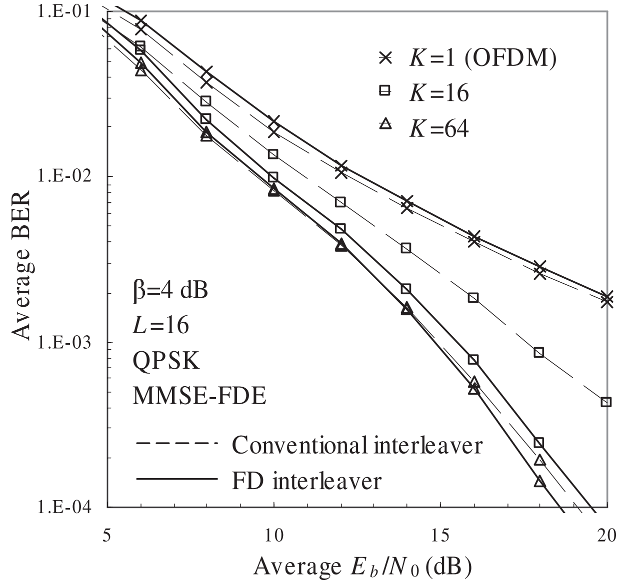

Fig. 3 Average BER vs. E_b/N_0.

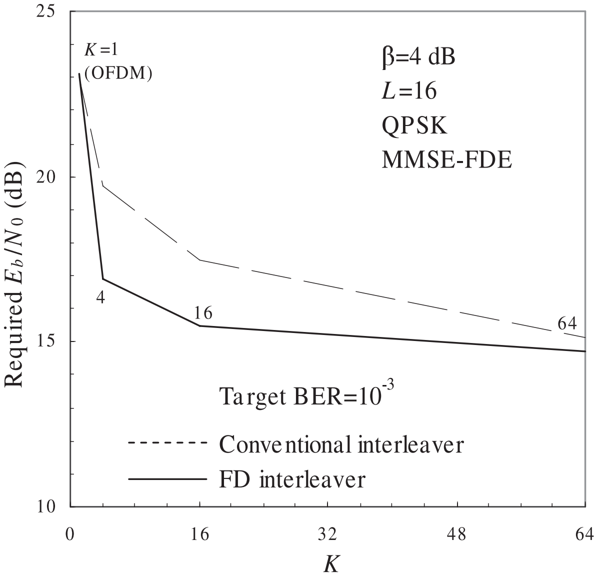

Fig. 4 Required E_b/N_0 vs. K.

cases: (*i*) conventional interleaving where channel interleaving is done immediately after encoding and (*ii*) frequency domain (FD) interleaving as presented in Sect. 3. As can be seen from Fig. 3, for BER = 10^{-2}, OFDM/TDM using MMSE-FDE when K =16 and 64 achieves the E_b/N_0 gain of about 2 and 2.4 dB over the conventional OFDM ($K = 1$), respectively. A reason for this performance improvement is that the transmitted symbol energy is distributed over a K times wider bandwidth in comparison with the conventional OFDM ($K = 1$) and consequently, the interleaving and MMSE-FDE can take advantage of the channel frequency-selectivity to obtain a larger frequency diversity gain. The figure shows that the E_b/N_0 gain further increases for a lower BER. It can be further seen that the proposed FD interleaving with OFDM/TDM using MMSE-FDE improves the BER performance in comparison with the conventional interleaving case.

As shown by Fig. 3, the performance of OFDM ($K = 1$) with conventional interleaving is slightly better than FD interleaving. This is because with conventional interleaving bits belonging to the same data symbol are mapped onto different subcarriers and hence, a higher degree of randomization effect can be obtained resulting in a slightly improved turbo decoding performance (same results are seen in [6], where bit- and symbol-interleaving are equivalent to conventional and frequency domain interleaving, respectively.)

The required E_b/N_0 of OFDM/TDM as a function of the parameter K for a target BER=10^{-3} is illustrated in Fig. 4. As can be seen from Fig. 4, in both cases, OFDM/TDM using MMSE-FDE when K = 4, 16 and 64 reduces the required E_b/N_0 in comparison with the conventional OFDM (K = 1) due to enhanced frequency diversity gain.

5. Conclusion

In this letter, a frame-by-frame frequency-domain interleaving for OFDM/TDM using MMSE-FDE was presented. The OFDM/TDM signal frequency components are interleaved to achieve a larger frequency diversity gain through MMSE-FDE at the receiver. It was shown that the application of frequency-domain interleaving on a frame-by-frame basis for OFDM/TDM using MMSE-FDE can improve the BER performance due to enhanced frequency diversity gain in comparison with the conventional interleaving.

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