Estimation of CSI for LST BS-CDMA Systems using Special Sequences with Pre-Defined Spectral Nulls

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***Abstract*—Layered space-time block spread CDMA (LST BS-CDMA) has been previously proposed as an alternative to conventional CDMA systems for high-rate mobile communications. This paper proposes two approaches to obtain channel state information (CSI) for LST BS-CDMA system. The basic approach suffers from high computational complexity and poor performance in systems with high order of transmitter diversity due to inter-layer interference (ILI). The second approach reduces computation complexity and eliminates ILI, making it suitable for systems with higher order of transmitter diversity. The detailed simulation studies show that the highly accurate CSI obtained using the improved technique leads to considerable superior performance.**

***Index Terms*—Channel estimation, single-carrier, frequency-domain equalization, block spread, layered space-time.**

I. INTRODUCTION

IN multicarrier (MC) systems based on orthogonal frequency division multiplexing (OFDM) realization, the entire channel is divided into parallel narrowband subchannels. Each subcarrier is designed to be narrow enough such that flat-fading condition is ensured during the worst frequency-selective channels that the system is designed for. This allows each subcarrier to be equalized using a simple one-tap multiplication. However, OFDM-based MC systems are prone to two major issues, namely the presence of high peak-to-average power ratio (PAPR) and sensitivity to carrier frequency offset (CFO) [1]. Moreover, error performance of OFDM-based systems are dominated by those subchannels in deep fades [2]. Recently, there has been a growing interest in SC system employing cyclic prefix (CP) that allows frequency-domain equalization (FDE) to be used [3]. SC-FDE systems avoid the PAPR and CFO issues while retaining the advantages of MC system. The distribution of bit energy over all subcarriers during equalization make SC-FDE system robust against channel with deep fades.

In order to introduce frequency diversity to each symbol, MC-CDMA proposed in [2] exploits the multiple available subcarriers to carry the different chips belonging to a single symbol. Alternatively, CP-assisted CDMA with SC-FDE has been proposed in [4]. However, the conventional CDMA processing in these schemes exhibit poor uplink performance due to code orthogonality destruction. Moreover, the despreading operation that comes after the completion of signal recovery processing means that the system complexity is proportional to the processing gain. Block spread CDMA (BS-CDMA) is proposed in [5] to overcome such limitations of conventional CDMA system. In high-rate systems, duration of a single chip is significantly smaller compared to channel coherence time (CCT). The small temporal channel variation allows re-ordering of processing sequence at the BS-CDMA receiver that leads to symbol-level processing. Compared to conventional CDMA schemes, BS-CDMA system is able to preserve code orthogonality during asynchronous reception that yields multi-user interference free (MUI-free) performance under certain mobility condition and offers considerable processing reduction at the receiver.

Creation of multiple-input-multiple-output (MIMO) channels by employing multiple transmit and receive antennnas promises tremendous enhancement in system performance and capacity. Increased data rate can be delivered by different data from different antenna simultaneously [6]. Combining such MIMO architecture with BS-CDMA, LST BS-CDMA scheme proposed in [7] enhances transmission rate and has been shown to be MUI-free under low mobility condition, making it an attractive choice for uplink.

The use of additional antenna to transmit additional data results in the superposition of signals from the different transmit antenna that gives rise to inter-layer interference (ILI) if not mitigated properly. Therefore, the availability of accurate channel parameter estimates is critical to facilitate practical deployment of LST BS-CDMA system. In the context of MIMO-OFDM, the issue of channel estimate is thoroughly addressed in [8]. The initial method proposed encounters practical limitation as large matrix inversion is required. When knowledge of channel delay spread (CDS) is made available to the receiver, a method based on tap-selection approach is proposed in [8]. This paper addresses the challenges involved in performing channel estimation in specific context of LST BS-CDMA system. Using the minimum mean-square-error (MSE) approach, the channel estimates is the solution to the cost function that minimizes MSE from all subcarriers. Theoretical expression for the MSE estimator is derived and design of the optimum pilot sequence is presented. Alternatively, the use of special pilot sequence with pre-defined nulls is proposed in this paper. By assigning mutually exclusive set of indices to each transmit antenna, ILI is avoided and hence improved estimation accuracy is achieved. In addition, the computation complexity involved is reduced and becomes independent to the degree of transmitter diversity employed.

The rest of the paper is organized as follows. Section II

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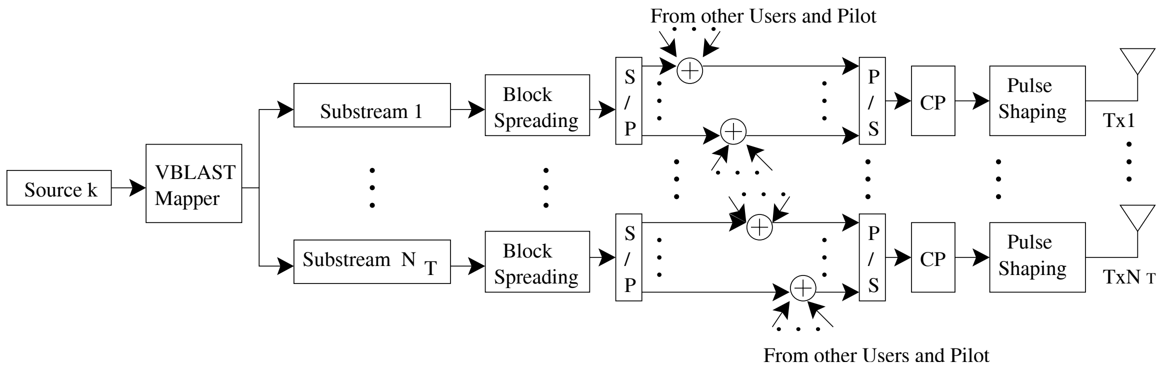

Fig. 1. LST BS-CDMA transmitter architecture.

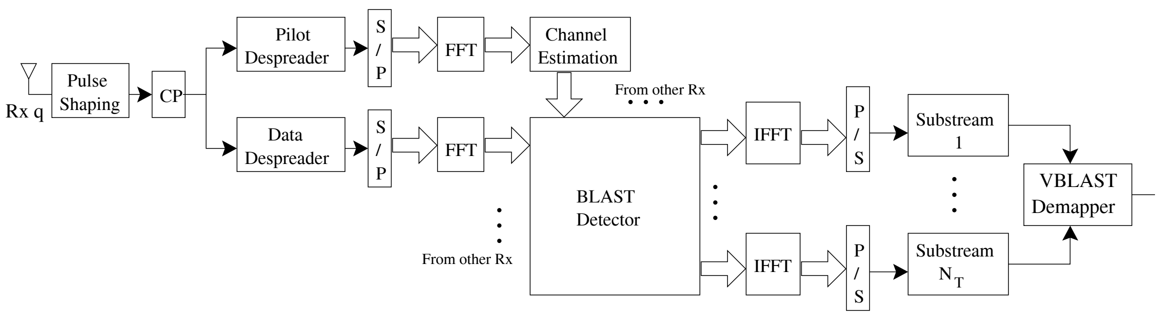

Fig. 2. LST BS-CDMA receiver architecture.

describes the LST BS-CDMA architecture. Section III describes the procedures to construct sequences with pre-defined non-null tones. Section IV and V describes a basic channel estimation technique and its performance. Section VI describes an improved estimation technique. Section VII discusses the details of simulation studies. Section VIII concludes the paper.

Notation: Bold lower case denotes column vector; bold upper case denotes matrix; $[.]^T$ and $[.]^+$ denote transpose and matrix psuedo-inverse; $\circledast$ denotes circular convolution; $\mathbf{I}_m$ denotes identity matrix of size $m \times m$; $[\mathbf{a}]_i$ denotes i^{th} row entry of column vector $\mathbf{a}$; $[\mathbf{A}]_{ij}$ denotes i^{th} row and j^{th} column of matrix $\mathbf{A}$;

II. LST BS-CDMA: Baseband Model

The generalized baseband transmitter structure is depicted in Fig. 1 [7], where only the k^{th} user is shown out of K users. In LST BS-CDMA, the stream of information bit, $d_k(i)$, is grouped into chunks of $N_T M \times 1$ block, $\mathbf{d}_k = \left[d_k((i-1)N_T M+1), \cdots, d_k(iN_T M)\right]$. Each block is split into N_T parallel blocks of size $M \times 1$, $\mathbf{d}_{k,p}$ ($p \in [1, N_T]$), where each will be transmitted independently using different antenna. Here, N_T represents the number of antenna used at the transmitter. The block spreading operation takes in the $M \times 1$ block, $\mathbf{d}_{k,p}$ to produce a $G \times M$ matrix, $\mathbf{X}_p = \left[c_k^{(1)}\mathbf{d}_{k,p} \cdots \quad c_k^{(G)}\mathbf{d}_{k,p}\right]^T$. The chips are then read row-wise where each row of chip makes up a single chip-block, $\mathbf{s}_{k,p}^{(g)} = c_k^{(g)}\mathbf{d}_{k,p}$. During the g^{th} (out of G) interval, the signal resulting from the summation of K users' and the pilot chip-block is given as $\mathbf{x}_p^{(g)} = \sum_{k=1}^{K} \mathbf{s}_{k,p}^{(g)} + \mathbf{t}_p^{(g)}$, where $\mathbf{t}_p^{(g)} = c_t^{(g)}\mathbf{t}_p$, $\mathbf{t}_p = \left[t_p(0) \cdots t_p(M-1)\right]^T$ is the $M \times 1$ pilot chips to be transmitted from antenna p. $c_k^{(g)}$ and $c_t^{(g)}$ are the g^{th} chip of the spread code assigned to k^{th} user and pilot respectively. Before transmission, CP insertion is performed.

The generalized baseband receiver structure is depicted in Fig. 2. After CP removal and serial-to-parallel (S/P) conversion, the signals received during the g^{th} interval is given as, $\mathbf{r}_q^{(g)} = \sum_{k=1}^{K}\sum_{p=1}^{N_T} \mathbf{h}_{qp}^{(g)} \circledast \mathbf{s}_{k,p}^{(g)} + \sum_{p=1}^{N_T} \mathbf{h}_{qp}^{(g)} \circledast \mathbf{t}_p^{(g)} + \mathbf{n}_q^{(g)}$ where $\mathbf{h}_{qp}^{(g)}$ is the channel between the q^{th} receive and p^{th} transmit antenna during the g^{th} chip-block interval and $\mathbf{n}_q^{(g)}$ is the additive white Gaussian noise (AWGN) component. The block despreading operation takes in the G received chip-blocks to re-construct the $G \times M$ chip matrix. Despreading is then performed by multiplying each row of the chip matrix with the corresponding chip and then summing all the rows together. If given the assumption that the channel remains largely invariant throughout the G intervals, $\mathbf{h}_{qp} \approx \mathbf{h}_{qp}^{(1)} \approx \cdots \approx \mathbf{h}_{qp}^{(G)}$. Without loss of generality, assume that signal detection is carried out for user 1. The result of the despreading operation is given as $\mathbf{y}_q = \sum_{p=1}^{N_T} \mathbf{h}_{qp} \circledast \mathbf{d}_{1,p} + \sum_{g=1}^{G} c_1^{(g)} \mathbf{n}_q^{(g)}$. The output of the FFT operation is $\bar{\mathbf{y}}_q = \sum_{p=1}^{N_T} \bar{\mathbf{H}}_{qp}\bar{\mathbf{d}}_{1,p} + \bar{\mathbf{n}}_q$, where $\bar{\mathbf{H}}_{qp} = diag\left\{\bar{h}_{qp}(0), \cdots, \bar{h}_{qp}(M-1)\right\}$ is the diagonal matrix with each diagonal entry corresponding to a channel frequency tone between the q^{th} receive and p^{th} transmit antenna, $\bar{\mathbf{d}}_{1,p} = \left[\bar{d}_{1,p}(0) \cdots \bar{d}_{1,p}(M-1)\right]^T$ is the frequency response of $\mathbf{d}_{1,p}$ and $\bar{\mathbf{n}}_q = [\bar{n}_q(0) \cdots \bar{n}_q(M-1)]^T$ is the frequency response of $\tilde{\mathbf{n}}_q = \sum_{g=1}^{G} c_1^{(g)} \mathbf{n}_q^{(g)}$. The m^{th} output tones from N_R receive antennas can be arranged to form $N_R \times 1$ matrix, $\bar{\mathbf{y}}(m) = \bar{\mathbf{H}}(m)\bar{\mathbf{d}}_1(m) + \bar{\mathbf{n}}(m)$, where $\left[\bar{\mathbf{H}}(m)\right]_{qp} = \bar{h}_{qp}(m)$, $\left[\bar{\mathbf{d}}_1(m)\right]_p = \bar{d}_{1,p}(m)$ and $\left[\bar{\mathbf{n}}(m)\right]_q = \bar{n}_q(m)$. Equalization is then performed on per-tone basis [7], followed by FFT and demodulation operations to recover the transmitted signals.

III. SPECIAL SEQUENCE WITH PRE-DEFINED FREQUENCY TONE NULLING

Designing pilot sequences that have nulling of frequency component at pre-defined indices exhibit desirable features for a SC-FDE based system. A special class of repeated finite-length sequence [9] is considered in this context. The detailed procedures to construct such sequence are laid out below.

A. Repeated Finite-Length Sequence

For consistency, note that the symbol representation will follow that of Section II. M corresponds to the block size, N_T represents the number of transmit antennas and M is assumed to be divisible by N_T.

1. Given an arbitrary sequence of length-$(\frac{M}{N_T})$
   $\mathbf{t} = \left[t(0) \cdots t(\frac{M}{N_T}-1)\right]^T$
2. Repeat or append $\mathbf{t}$ with itself N_T times to generate a sequence of length M
   $\hat{\mathbf{t}} = \left[\mathbf{t} \cdots \mathbf{t}\right]^T$

It was shown in [9] that the m^{th} frequency tone, $\bar{t}(m)$ of $\hat{\mathbf{t}}$ is given as

$$\bar{t}(m) = \begin{cases} N_T \displaystyle\sum_{l=0}^{\frac{M}{N_T}-1} t(l) e^{\frac{-j2\pi ml}{M}} & \text{for } m = 0, N_T, \cdots \\ 0 & \text{Otherwise} \end{cases} \tag{1}$$

B. Sequence with Pre-Defined Null Tones

Earlier, non-null tones of a repeated finite-length sequence was shown to exist only at index kN_T (for $k = 0 \ to \ \frac{M}{N_T}-1$). However, in order to facilitate channel estimation in SC-FDE based system, it is necessary to generate such sequence with non-null components at different index set. This can be achieved by rotating the phase of the sequence generated using the above procedure. Now, let $\Omega_p = \{kN_T + p - 1\} \ for \ k = 0 \ to \ \frac{M}{N_T} - 1$ and $p = 0 \ to \ N_T - 1$. A repeated finite-length sequence $\hat{\mathbf{t}}_p$ will have its non-null tones at index $m \in \Omega_p$ when it is phase-rotated by $\theta_p = \frac{2\pi(p-1)}{M}$. The m^{th} element of $\tilde{\mathbf{t}}_p$, the phase-rotated copy of $\hat{\mathbf{t}}_p$ is given by $\left[\tilde{\mathbf{t}}_p\right]_m = e^{\frac{j2\pi(p-1)(m-1)}{M}} \left[\hat{\mathbf{t}}\right]_m$. As a result, it can be shown that the m^{th} frequency tone of $\tilde{\mathbf{t}}_p$ is given by [10]

$$\bar{t}_p(m) = \begin{cases} N_T \displaystyle\sum_{l=0}^{\frac{M}{N_T}-1} t_p(l) e^{-\frac{j2\pi(m-p-1)l}{M}} \\ \qquad \text{for } m = p-1, N_T+p-1, \cdots \\ 0 \qquad\qquad \text{Otherwise} \end{cases} \tag{2}$$

The ability to generate such sequence can be utilized to coordinate the multiple antennas at the transmitter in order to avoid ILI. This sequence will be used for the improved algorithm presented at the later part of the paper.

IV. CHANNEL ESTIMATION

In this paper, a dedicated spread code is assigned for pilot and the pilot chips are transmitted simultaneously with users' data. The result of performing despreading on the received signal, $\mathbf{r}_q^{(g)}$ (using pilot-assigned spread code), followed by FFT operation can be written as, $\bar{\mathbf{z}}_q = \sum_{p=1}^{N_T} \mathbf{F}_M \mathbf{T}_p \mathbf{F}_M^{\mathcal{H}} \mathbf{F}_M \mathbf{h}_{qp} + \bar{\mathbf{n}}_q$, where $\mathbf{F}_M$ is a M-point FFT matrix with $[\mathbf{F}_M]_{ab} = \frac{1}{\sqrt{M}} e^{-\frac{j2\pi ab}{M}}$ and $\mathbf{T}_p$ is a circular Toeplitz matrix with the first column as $\tilde{\mathbf{t}}_p$. According to [11], circular Toeplitz matrix is diagonalizable by pre- and post-multiplying with FFT and IFFT matrix. The output of the FFT operation can simplified into $\bar{\mathbf{z}}_q = \sum_{p=1}^{N_T} \bar{\mathbf{T}}_p \mathbf{F}_{M|L} \mathbf{h}_{qp|L} + \bar{\mathbf{n}}_q$, where $\bar{\mathbf{T}}_p$ is a diagonal matrix with its diagonal elements as tones of the pilot symbols transmitted from the p^{th} antenna, $\mathbf{F}_{M|L}$ is now a $M \times L$ matrix containing only the first L columns of $\mathbf{F}_M$ and $\mathbf{h}_{qp|L}$ is the channel parameters with all the trailing $(M-L)$ zeros omitted. L is the length of CP. In the following, two approaches that minimize the MSE across all subcarriers are described.

A. Basic Approach

The channel esitmation problem is one that minimizes the following cost function:

$$\xi = \| \bar{\mathbf{z}}_q - \sum_{p=1}^{N_T} \bar{\mathbf{T}}_p \mathbf{F}_{M|L} \tilde{\mathbf{h}}_{qp|L} \|^2 \tag{3}$$

Differentiating the cost function w.r.t. $\tilde{\mathbf{h}}_{qp|L}$

$$\frac{\delta \xi}{\delta \tilde{\mathbf{h}}_{qp|L}} = -2 \left(\mathbf{F}_{M|L}^{\mathcal{H}} \bar{\mathbf{T}}_p^{\mathcal{H}}\right) \left(\bar{\mathbf{z}}_q - \sum_{p=1}^{N_T} \bar{\mathbf{T}}_p \mathbf{F}_{M|L} \tilde{\mathbf{h}}_{qp|L}\right) \tag{4}$$

and the estimate of channel parameters is obtained based on the following:

$$\begin{aligned} \left(\frac{\delta\xi}{\delta\tilde{\mathbf{h}}_{q1|L}} \quad \cdots \quad \frac{\delta\xi}{\delta\tilde{\mathbf{h}}_{qN_T|L}}\right)^T &= \mathbf{0}^T \\ \Rightarrow \mathbf{u}_q - \mathbf{Q}\tilde{\mathbf{h}}_q &= \mathbf{0} \\ \tilde{\mathbf{h}}_q &= \mathbf{Q}^{-1}\mathbf{u}_q \end{aligned} \tag{5}$$

where $\mathbf{u}_q$ and $\mathbf{Q}$ is defined respectively as

$$\mathbf{u}_q = \begin{pmatrix} \mathbf{u}_{q1} \\ \vdots \\ \mathbf{u}_{qN_T} \end{pmatrix} \text{ and } \quad \mathbf{Q} = \begin{pmatrix} \mathbf{Q}_{11} & \cdots & \mathbf{Q}_{1N_T} \\ \vdots & \ddots & \vdots \\ \mathbf{Q}_{N_T1} & \cdots & \mathbf{Q}_{N_TN_T} \end{pmatrix} \tag{6}$$

and $\mathbf{u}_{qp} = \mathbf{F}_{M|L}^{\mathcal{H}} \bar{\mathbf{T}}_p^{\mathcal{H}} \bar{\mathbf{z}}_{\mathbf{q}}$, $\mathbf{Q}_{p_1p_2} = \mathbf{F}_{M|L}^{\mathcal{H}} \bar{\mathbf{T}}_{p_1}^{\mathcal{H}} \bar{\mathbf{T}}_{p_2} \mathbf{F}_{M|L}$ and $\tilde{\mathbf{h}}_q = \left[\tilde{\mathbf{h}}_{q1}^T \cdots \tilde{\mathbf{h}}_{qN_T}^T\right]^T$ is the channel estimate that minimizes the MSE. The need to perform matrix inversion of size $N_TL \times N_TL$ limits the practicality of this technique.

B. Tap Selection-Based Channel Estimator

In typical practical environment, $\mathbf{h}_{qp|L}$ contains mostly null. When the channel delay profile and location of non-null coefficients are known, the size of matrix inversion can be reduced considerably. Let the number of taps with significant gain coefficient be L_0. The cost function in Eqn. (3) can be re-expressed as:

$$\xi = \| \bar{\mathbf{z}}_q - \sum_{p=1}^{N_T} \bar{\mathbf{T}}_p \hat{\mathbf{F}}_M \hat{\mathbf{h}}_{qp} \|^2 \tag{7}$$

where $\hat{\mathbf{h}}_{qp} = \left(\hat{h}_{qp}(l_0)\cdots\hat{h}_{qp}(l_{L_0-1})\right)^T$ is the channel vector consisting only L_0 significant taps, $\hat{\mathbf{F}}_M$ is of size $M \times L$ and $\left[\hat{\mathbf{F}}_M\right]_{ab} = e^{-\frac{j2\pi a l_b}{M}}$. Following similar steps described earlier, the new estimated channel parameter that minimizes MSE is given by:

$$\begin{pmatrix}\hat{\mathbf{h}}_{q1}\\ \vdots \\ \hat{\mathbf{h}}_{qN_T}\end{pmatrix} = \begin{pmatrix}\hat{\mathbf{Q}}_{11} & \cdots & \hat{\mathbf{Q}}_{1N_T}\\ \vdots & \ddots & \vdots\\ \hat{\mathbf{Q}}_{N_T1} & \cdots & \hat{\mathbf{Q}}_{N_TN_T}\end{pmatrix}^{-1}\begin{pmatrix}\hat{\mathbf{u}}_{q1}\\ \vdots \\ \hat{\mathbf{u}}_{qN_T}\end{pmatrix} \tag{8}$$

where $\hat{\mathbf{u}}_{qp} = \hat{\mathbf{F}}_M^{\mathcal{H}}\bar{\mathbf{T}}_p^{\mathcal{H}}\bar{\mathbf{z}}_q$ and $\hat{\mathbf{Q}}_{p_1p_2} = \hat{\mathbf{F}}_M^{\mathcal{H}}\bar{\mathbf{T}}_{p_1}^{\mathcal{H}}\bar{\mathbf{T}}_{p_2}\hat{\mathbf{F}}_M$. The size of the matrix to be inverted is now considerably reduced to $N_TL_0 \times N_TL_0$.

V. Performance Analysis

A. Error Bound Evaluation

Since the received signal is made up of superposition between signals transmitted from N_T antennae, the correlation vector $\mathbf{u}_{qp}$ can be decomposed into:

$$\begin{aligned}\mathbf{u}_{qp} &= \mathbf{F}_{M|L}^{\mathcal{H}}\bar{\mathbf{T}}_p^{\mathcal{H}}\left(\sum_{p_1=1}^{N_T}\bar{\mathbf{T}}_{p_1}\mathbf{F}_{M|L}\mathbf{h}_{qp_1|L} + \mathbf{n}_q\right)\\ &= \left(\mathbf{Q}_{p1}\cdots\mathbf{Q}_{pN_T}\right)\left(\mathbf{h}_{q1|L}^T\cdots\mathbf{h}_{qN_T|L}\right)^T + \mathcal{N}_p\end{aligned} \tag{9}$$

where $\mathcal{N}_p = \mathbf{F}_{M|L}^{\mathcal{H}}\bar{\mathbf{T}}_p^{\mathcal{H}}\bar{\mathbf{n}}_q$ is the noise term. With this and by letting $\mathcal{N} = \left(\mathcal{N}_1^T\cdots\mathcal{N}_{N_T}^T\right)^T$, $\mathbf{u}_q$ and $\tilde{\mathbf{h}}_q$ can be expressed as:

$$\begin{aligned}\mathbf{u}_q &= \left(\mathbf{u}_{q1}^T\cdots\mathbf{u}_{qN_T}^T\right)^T\\ &= \mathbf{Q}\mathbf{h}_q + \mathcal{N}\end{aligned} \tag{10}$$

$$\tilde{\mathbf{h}}_q = \mathbf{Q}^{-1}\mathbf{u}_q \quad = \mathbf{h}_q + \mathbf{Q}^{-1}\mathcal{N} \tag{11}$$

The mean of the channel estimate is $E\left\{\tilde{\mathbf{h}}_q\right\} = E\left\{\mathbf{h}_q\right\} + E\left\{\mathbf{Q}^{-1}\mathcal{N}\right\} = E\left\{\mathbf{h}_q\right\}$. Similarly to [8], the MSE of the channel estimate is given as:

$$MSE = \frac{1}{N_TL}E\left\{\parallel \tilde{\mathbf{h}}_q - \mathbf{h}_q \parallel^2\right\} \quad = \frac{\sigma^2}{N_TL}Tr\left\{\mathbf{Q}^{-1}\right\} \tag{12}$$

where $Tr\{.\}$ corresponds to the Trace of matrix $\{.\}$ and σ^2 is the variance of the AWGN component.

B. Optimum Pilot Design

For the general case of $N_T > 2$, derivation of $\mathbf{Q}^{-1}$ is non-trivial. However, for $N_T = 2$, if the pilot tones are assumed to be constant, i.e. $|\bar{t}_p(l)|^2 = \gamma$ and thus $\mathbf{Q}_{pp} = \gamma\mathbf{I}_{L\times L}$, $\mathbf{Q}^{-1}$ can easily be derived as:

$$\mathbf{Q}^{-1} = \frac{1}{M\gamma}\begin{pmatrix}\mathbf{I}_{L\times L} + \frac{\mathbf{Q}_{12}}{M\gamma}\left(\mathbf{I}_{L\times L} - \frac{\mathbf{Q}_{12}^{\mathcal{H}}\mathbf{Q}_{12}}{M^2\gamma^2}\right)^{-1}\frac{\mathbf{Q}_{12}^{\mathcal{H}}}{M\gamma} \\ -\left(\mathbf{I}_{L\times L} - \frac{\mathbf{Q}_{12}^{\mathcal{H}}\mathbf{Q}_{12}}{M^2\gamma^2}\right)^{-1}\frac{\mathbf{Q}_{12}^{\mathcal{H}}}{M\gamma} \\ \frac{\mathbf{Q}_{12}}{M\gamma}\left(\mathbf{I}_{L\times L} - \frac{\mathbf{Q}_{12}^{\mathcal{H}}\mathbf{Q}_{12}}{M^2\gamma^2}\right)^{-1} \\ \left(\mathbf{I}_{L\times L} - \frac{\mathbf{Q}_{12}^{\mathcal{H}}\mathbf{Q}_{12}}{M^2\gamma^2}\right)^{-1}\end{pmatrix} \tag{13}$$

Let $\left\{\lambda_l^2\right\}$ $for\ l = 0\ to\ L-1$ be the set of eigenvalues for $\mathbf{Q}_{12}^{\mathcal{H}}\mathbf{Q}_{12}$. The MSE is then found to be

$$MSE = \frac{\sigma^2}{2M\gamma L}\left(L + \sum_{l=0}^{L-1}\frac{M^2\gamma^2 + \lambda_l^2}{M^2\gamma^2 - \lambda_l^2}\right) \tag{14}$$

It can easily be deduced that minimum MSE occurs when $\lambda_l = 0\ for\ l = 0\ to\ L-1$ or equivalently

$$MSE \geq \frac{\sigma^2}{2M\gamma L}(2L) = \frac{\sigma^2}{M\gamma} \tag{15}$$

when pilots with such property is used, they are said to be optimum because since $\mathbf{Q}_{12}^{\mathcal{H}}\mathbf{Q}_{12} = \mathbf{0}$, ILI is totally eliminated from the system. In addition, matrix inversion consists only of trivial division as $\mathbf{Q}$ is now a diagonal matrix, $\mathbf{Q} = diag\left\{\mathbf{Q}_{11}\mathbf{Q}_{22}\right\}$. One possible pilot design is to let $\bar{t}_2(m) = (-1)^m\bar{t}_1(m)$. To construct such pilots, the phase-rotated repeated finite-length sequence presented earlier can be used by following the procedures given below:

1. Generate two arbitrary sequences of length $(\frac{M}{2})$ and let the two sequences be denoted $\mathbf{t}_1^1$ and $\mathbf{t}_1^2$ respectively.
2. Repeat or append $\mathbf{t}_1^1$ with itself to generate $\hat{\mathbf{t}}_1^1$ of length M, $\hat{\mathbf{t}}_1^1 = \left((\mathbf{t}_1^1)^T(\mathbf{t}_1^1)^T\right)^T$
3. Perform step 2 on $\mathbf{t}_1^2$ to generate $\hat{\mathbf{t}}_1^2$, $\hat{\mathbf{t}}_1^2 = \left((\mathbf{t}_1^2)^T(\mathbf{t}_1^2)^T\right)^T$
4. Phase-rotate $\hat{\mathbf{t}}_1^1$ and $\hat{\mathbf{t}}_1^2$ by $\theta_0 = \frac{2\pi(0)}{M}$ and $\theta_1 = \frac{2\pi(1)}{M}$ to generate $\tilde{\mathbf{t}}_1^1$ and $\tilde{\mathbf{t}}_1^2$ respectively.
5. Let $\tilde{\mathbf{t}}_2^1 = \tilde{\mathbf{t}}_1^1$ and $\tilde{\mathbf{t}}_2^2 = -\tilde{\mathbf{t}}_1^2$.
6. Pilot symbols to be transmitted from antenna 1 and 2 are $\tilde{\mathbf{t}}_1 = \tilde{\mathbf{t}}_1^1 + \tilde{\mathbf{t}}_1^2$ and $\tilde{\mathbf{t}}_2 = \tilde{\mathbf{t}}_2^1 + \tilde{\mathbf{t}}_2^2$

Instead of an arbitrary sequence, Chu's sequence [12] can be used to generate constant amplitude tone $\left|\bar{t}_p(m)\right|^2 = \gamma\ for\ m = 0\ to\ M-1$. At this point, it is easy to verify that $\bar{t}_2(m) = (-1)^m\bar{t}_1(m)$. The error performance and pilot design presented here is only for $N_T = 2$. For $N_T > 2$, the optimum pilot design can be obtained by first deriving $\mathbf{Q}^{-1}$ and then minimizing the ILI components.

VI. Improved Channel Estimation

Due to the high computational complexity involved, practical suitability of the basic approach is limited. While the tap selection method reduces the complexity significantly, it requires knowledge of channel delay spread (CDS). In addition, both approaches suffer from ILI that grows proportionally with the order of transmit diversity used. A novel channel estimation model that eliminates ILI and reduces complexity is proposed next.

A. Mathematical Model

In the methods described earlier, ILI occurs because all antennas are transmitting simultaneously at all tones. If the transmit antennas can be made to 'cooperate' such that each antenna only transmit at pre-assigned subcarriers (and null on others) and they do not overlap onto one another, ILI can be avoided. Now, let the pre-defined non-null tone index for the p^{th} antenna be $\Omega_p = \{kN_T + p\}$ for $k = 0\ to\ \frac{M}{N_T} - 1$ and $\tilde{\mathbf{t}}_p$ be the pilot sequence generated using the above procedure. After despreading and FFT operation, the received signal is

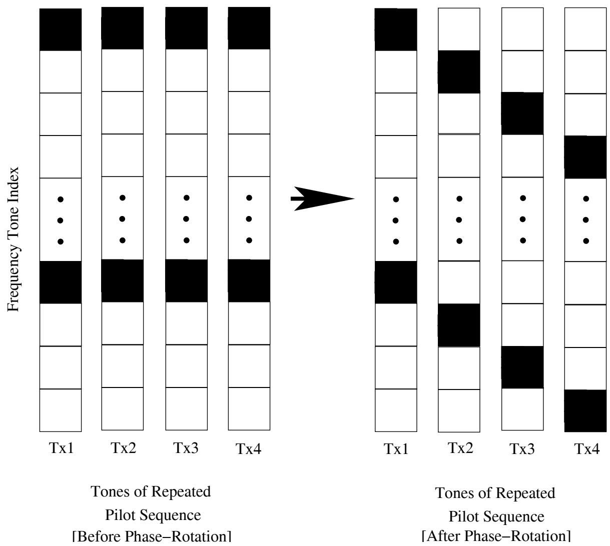

Fig. 3. Pilot tones arrangement.

"clear" from ILI and the received signal at subcarrier $k \in \Omega_p$ can be written as:

$$\bar{\mathbf{z}}_{q|\Omega_p} = \bar{\mathbf{T}}_{p|\Omega_p}\mathbf{F}_{M|L|\Omega_p}\mathbf{h}_{qp|L} \tag{16}$$

where $\bar{\mathbf{z}}_{q|\Omega_p}$ and $\mathbf{F}_{M|L|\Omega_p}$ consists of rows (row index $\in \Omega_p$) from $\bar{\mathbf{z}}_q$ and $\mathbf{F}_{M|L}$ respectively. $\bar{\mathbf{T}}_{p|\Omega_p} = diag\{\bar{t}_p(kN_T + p)\}\ \ for\ k = 0\ to\ \frac{M}{N_T} - 1$. This allows channel estimation to be carried out individually for each antenna without ILI. The cost function for the p^{th} antenna is then defined as:

$$\xi_p = \parallel \bar{\mathbf{z}}_{q|\Omega_p} - \bar{\mathbf{T}}_{p|\Omega_p}\mathbf{F}_{M|L|\Omega_p}\mathbf{h}_{qp|L} \parallel^2 \tag{17}$$

After differentiating the cost function, estimate of the channel parameter is obtained as:

$$\begin{aligned}\mathbf{u}_{qp|\Omega_p} - \mathbf{Q}_{pp|\Omega_p}\mathbf{h}_{qp|L} &= \mathbf{0}\\ \mathbf{h}_{qp|L} &= \mathbf{Q}^{-1}_{pp|\Omega_p}\mathbf{u}_{qp|\Omega_p}\end{aligned} \tag{18}$$

where $\mathbf{Q}_{pp|\Omega_p} = \mathbf{F}^{\mathcal{H}}_{M|L|\Omega_p}\bar{\mathbf{T}}^{\mathcal{H}}_{p|\Omega_p}\bar{\mathbf{T}}_{p|\Omega_p}\mathbf{F}_{M|L|\Omega_p}$ and $\mathbf{u}_{qp|\Omega_p} = \mathbf{F}^{\mathcal{H}}_{M|L|\Omega_p}\bar{\mathbf{T}}^{\mathcal{H}}_{p|\Omega_p}\bar{\mathbf{z}}_{q|\Omega_p}$. It can be seen that the size of matrix to be inverted is $L \times L$ or independent of the number of transmit antenna. If knowledge of CDS is available, the simplified tap-selection approach can be used to reduce computational complexity further.

B. Pilot Design

In the improved method, each antenna transmits a pilot sequence with non-null components at pre-defined indices, unique to itself. The index set assigned to p^{th} transmit antenna is $\Omega_p = \{p, N_T + p, \cdots\}$. The pilot sequences are repeated finite-length sequences constructed from the steps described in Section III-A. Before transmission, pilot sequence to be transmitted from the p^{th} antenna is phase-rotated by $\theta_{p-1},\ p = 1\ to\ N_T$ as described in Section III-B. For illustration purpose, the pilot tones transmitted from $N_T (N_T = 4)$ antennae are given as shown in Figure 3.

C. Performance Analysis

The estimator proposed in Section IV minimizes MSE across all subcarriers w.r.t. all N_T channels. In the improved method, pilot tones from different transmit antennae do not superpose or interfere with one another. The size of matrix inversion is therefore reduced by a factor of N_T and no ILI is present. For the general case of $N_T \geq 1$, the MSE of the improved estimator is given as:

$$\begin{aligned}MSE &= \frac{1}{L}E\left\{\parallel \tilde{\mathbf{h}}_{qp|L} - \mathbf{h}_{qp|L} \parallel^2\right\}\\ &= \frac{1}{L}E\left\{(\mathbf{Q}^{-1}_{pp}\mathcal{N}_p)^{\mathcal{H}}(\mathbf{Q}^{-1}_{pp}\mathcal{N}_p)\right\}\\ &= \frac{\sigma^2}{L}Tr\left\{\mathbf{Q}^{-1}_{pp}\right\}\\ &= \frac{\sigma^2}{M\gamma}\end{aligned} \tag{19}$$

Now, assume that length-($\frac{M}{N_T}$) pilot sequence for each antenna is generated based on Chu's sequence with constant amplitued γ_0. It can easily be derived from Eqn (2) that $\bar{t}_p(m) = \gamma = N_T\gamma_0$ for $m \in \Omega_p$. Thus, estimation accuracy improves when the transmitter diversity order increases.

VII. Simulation Studies

An LST BS-CDMA system using carrier frequency of 2Ghz is simulated. Transmit diversity order of 2 and 4 are used to investigate the performance degradation due to ILI. Processing gain of 32 is used with one of the spreading codes reserved for pilot. The number of users is set as 16. The simulated model assumes transmission bandwidth of 4Mhz. Three different block sizes are considered, namely 64, 128 and 256. The length of CP, L_{CP}, is set as one-quarter of the block size. The total duration to transmit G chip-blocks is 0.64, 1.28 and 2.56 msecs respectively. BPSK modulation is used. Doppler frequency of 40 and 200Hz that corresponds to pedestrian and high mobility users are used. The communication link assumes L_0 independent Rayleigh distributed paths [13] and L_0 is set as one-quarter of L_{CP}. For system with transmitter diversity order of 2 and 4, each user enjoys transmission rate of 200kbps and 400kbps respectively. Overall, the system delivers transmission efficiency of 1.55 and 3.1 bps/Hz respectively.

Figure 4 compares the MSE performance of the basic estimator employing tap-selection agains the improved estimator for block size of 128 and Doppler frequency of 40Hz. For 2×2 system, the tap-selection technique performs worse than the improved technique when it employs random pilot. Meanwhile, when optimum pilot is used instead, the tap selection technique performs better than the improved method. This is in line with earlier hypothesis that says the use of optimum pilot for the tap-selection technique eliminates ILI. For 4×4 system, the performance of tap-selection technique performs considerably poorer than the improved technique as non-optimum pilot is used.

Figure 5 shows the MSE performance of the improved channel estimator under varying block sizes. As shown in the Figure, estimation accuracy improvement is observed as the block size is increased. However, the gradient of the MSE decreases when $M = 256$. One reason for such behaviour

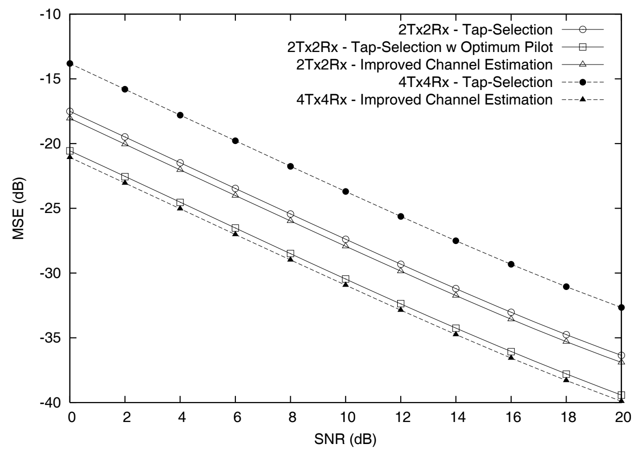

Fig. 4. MSE comparison across different methods.

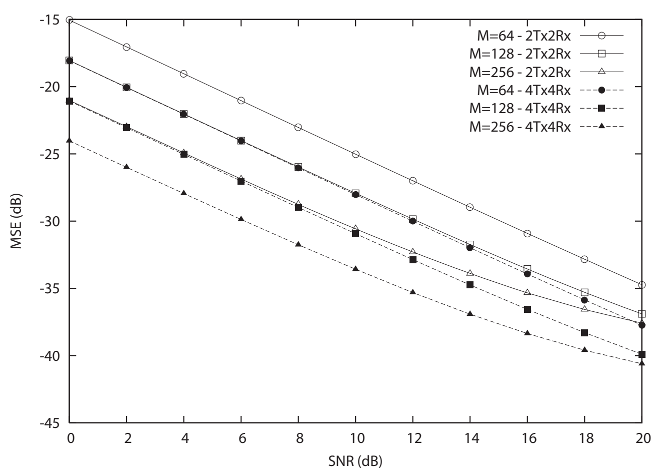

Fig. 5. Performance of improved estimator, $F_d = 40Hz$.

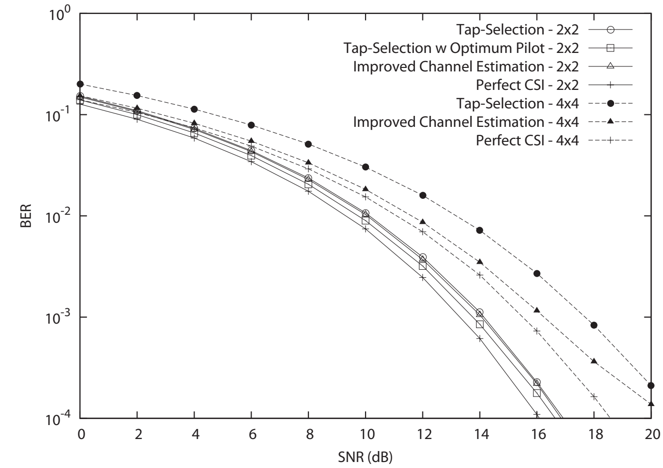

Fig. 6. Performance of LST BS-CDMA with Doppler frequency, $F_d = 40Hz$ and block size, $M = 128$.

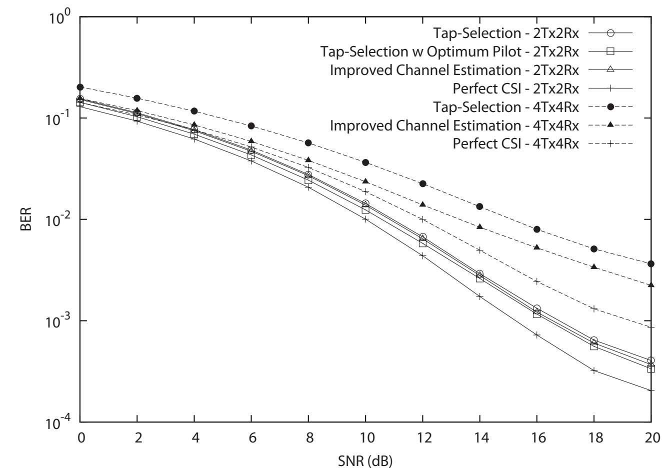

Fig. 7. Performance of LST BS-CDMA with Doppler frequency, $F_d = 200Hz$ and block size, $M = 64$.

is that the MSE is approaching its lower bound and there will come to a point when any increase in block size yields no significant improvement. Another reason is the larger amount of variation experienced with larger block size causes degradation to the estimation quality.

Figure 6 and 7 shows the BER performance of LST BS-CDMA system with Doppler frequency of 40 and 200Hz respectively. The result in Figure 6 is obtained with block size chosen as 128 and MMSE equalizer is used. From Figure 6, it can be seen that the system performance with basic estimator is about 0.4dB worse than that with the improved estimator for 2×2 antenna system. For 4×4 system, the basic estimator and improved estimator performs about 1.5dB and 0.3dB worse than that when perfect CSI is available. In Figure 7, smaller block size of 64 is used to counter the more rapid variation during high Doppler scenario. For 2×2 case, the BER performance of all the estimators are close to one another and they are about 0.6dB worse than the ideal case at BER level of 10%. For 4×4 case, the basic and improved estimator performs about 2.2dB and 1dB worse than the ideal case respectively.

VIII. Conclusion

This paper proposes two approaches for obtaining channel estimates for LST BS-CDMA system. In the first approach, all antennae transmit pilot sequences with non-null component at all tones and therefore vulnerable to ILI. Channel can be estimated by minimizing MSE w.r.t. all tones. Assuming the knowledge of channel delay profile, significant reduction in computation complexity can be achieved by using the tap-selection based approach. The second approach assign different tone indices to different antenna. This results in computational complexity reduction by a factor of the transmitter diversity order, N_T, and ILI is totally eliminated. Impact of imperfect CSI on the performance of LST BS-CDMA system was also investigated. For LST BS-CDMA system with higher transmitter diversity order, less accurate channel estimation is obtained using the basic estimator, due to ILI. Hence, the improved estimator is more suitable for systems with higher order of transmitter diversity.

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