Broadband Analog Network Coding

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***Abstract*—In this letter, we present the performance of broadband bi-directional transmission with analog network coding (ANC) in a frequency-selective fading channel. To cope with the channel frequency-selectivity we introduce the use of frequency domain equalization (FDE) with broadband ANC based on orthogonal frequency division multiplexing (OFDM) and single carrier (SC) radio access. We evaluate, by theory and computer simulation, the achievable bit error rate (BER) and ergodic capacity of bi-directional ANC scheme based on OFDM and SC-FDE radio access in a frequency-selective fading channel. Our results show that SC-FDE achieves a better BER performance, but on the other hand, a lower ergodic capacity in comparison with OFDM in a frequency-selective fading channel. Through both analysis and computer simulation, our findings show that a drawback of ANC scheme is its lack of diversity combining at the destination, which causes a slightly lower ergodic capacity in comparison with cooperative relaying irrespective of radio access scheme.**

***Index Terms*—Analog network coding, frequency-selective fading, OFDM, SC, FDE.**

I. Introduction

THE future wireless networks are envisaged to offer a broadband ubiquitous coverage in large areas with high capacity. Network coding had been studied in the context of distributed source coding as a promising technique to improve capacity in wired networks [1]. Although the original investigation of network coding was in the context of wired networks, its potential to improve the performance in wireless networks becomes more significant due to the broadcast nature of the wireless medium [2].

Direct application of network coding at the physical layer (PNC) [3], [4] in a wireless relay network increases the capacity of bi-directional communication. In [3]-[6], the network capacity and the bit error rate (BER) of PNC scheme in a frequency-nonselective fading channel was studied. Henceforth, we refer to these schemes as narrowband PNC. In narowband PNC, the relay uses logical operations to map received signal into a digital bit stream, so that the interference becomes a part of the arithmetic operation in network coding. The narrowband analog network coding (ANC) proposed in [7] is essentially another variation of PNC with a simpler implementation, where the relay amplifies and broadcasts the received signal without any processing [7]. To date, this method has been addressed in the literature mainly within information theory group for bi-directional communication over a narrowband Gaussian symmetric (i.e., link between the users and relay and vice versa is assumed identical) channel. The broadband communication over more realistic wireless multipath (i.e., frequency-selective) channels has not been addressed.

The wireless channel is characterized as a multipath propagation environment giving rise to inter-symbol interference (ISI) that may significantly degrade the system performance [8]. To cope with the channel impairments two broadband transmission techniques are widely used: (i) orthogonal frequency division multiplexing (OFDM) [8] and (ii) single carrier with frequency domain equalization (SC-FDE) [9]. Most of the work on narrowband ANC (and/or PNC) points out its advantage that the network capacity is doubled in comparison with conventional point-to-point communication. However, a more important issue, from the wireless communication perspective, is the achievable performance of ANC scheme in a frequency-selective fading channel. Hence, the use of FDE with ANC scheme in a frequency-selective fading channel based on broadband radio access technologies is required. Henceforth, we refer to this scheme as broadband ANC.

This paper introduces a broadband bi-directional communication with ANC scheme based on OFDM and SC-FDE radio access in a frequency-selective fading channel. The equalization weight based on zero forcing (ZF) and minimum mean square error (MMSE) criteria required for broadband ANC communication based on OFDM and SC-FDE signaling, respectively, are derived. We evaluate the achievable bit error rate (BER) and ergodic channel capacity of broadband ANC in a frequency-selective fading channel by theory and computer simulation. In the case of broadband ANC based on SC-FDE, the BER and ergodic network capacity expressions are derived based on the Gaussian approximation of the residual inter-symbol interference (ISI) after FDE, while the performance of broadband ANC based on OFDM radio access is used as a reference. We, first, compare the achievable BER and then, we compare the ergodic capacities of broadband ANC scheme, conventional relaying [10] and conventional network (i.e., single source and destination terminals without relaying) in a frequency-selective fading channel. Distinct aspect here from related work is that we take into consideration the channel frequency-selectivity and introduce FDE weights to cope the channel distortions from all terminals. Our findings show that a drawback of broadband ANC scheme is its lack of diversity combining at the destination end. A natural consequence is a lower ergodic capacity in comparison with cooperative relaying scheme from the end user's perspective. Hence, an appropriate diversity scheme for broadband ANC should be presented to further improve the capacity similar as cooperative relay networking case. We note here that our analysis is based on the assumption of perfect knowledge of

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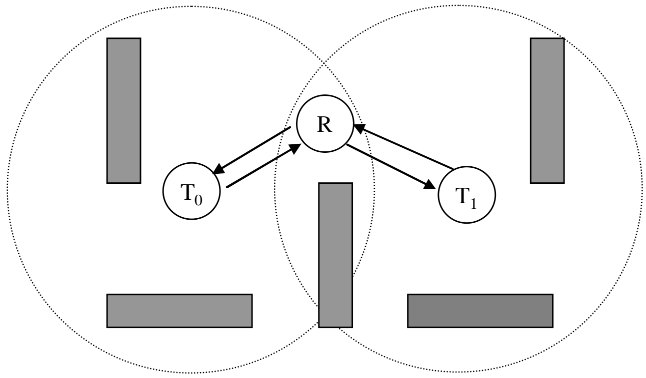


Fig. 1. Network model.

channel state information (CSI) and perfect self-information removal.

The remainder the letter is organized as follows. Section II gives an overview network model. The theoretical performance analysis is presented in Sect. IV. In Sect. V, simulation results and discussions are presented. Section VI concludes the paper.

II. Network Model

The relay network model is illustrated in Fig. 1. We assume that coverage area of terminals T_0 and T_1 include the relay R terminal, while they are out of each other's transmission range. Each terminal is equipped with an omni-directional antenna, and the communication takes place in two orthogonal stages as summarized in Table I. During the first stage, T_0 and T_1 transmit, while during the second stage, the relay broadcasts the received signal to both T_0 and T_1 using amplify-and-forward network protocol [10].

Information bit sequence of length M is channel coded, bit interleaved and mapped into the transmit data symbols, chosen from a complex-valued quadrature phase shift keying (QPSK) modulation scheme. The jth terminal T_j sequence is divided into blocks $\{d_j^m(n);\ n=0 \sim N_c-1\}$ for $m = 0 \sim (M/N_c)-1$, each of having N_c data-modulated symbols with $E[|d_j^m(n)|^2]$=1, where $E[\cdot]$ denotes the ensemble average operation. We consider that M is selected to be a multiple of N_c. Without loss of generality we consider a transmission of N_c data-modulated symbols and thus, the block index m is omitted in what follows.

The SC-FDE transmit signal is given by $\{d_j(n) = s_j(n); n=0\sim N_c-1\}$, while in the case of OFDM $\{d_j(n)\}$ is fed to inverse fast Fourier transform (IFFT) to generate a time domain OFDM signal $\{s_j(t); t=0 \sim N_c-1\}$ [8]. After insertion of N_g-sample guard interval (GI) the signal is multiplied by a power coefficient P_s (=E_s/T_c), where E_s and T_c denote the data-modulated symbol energy and the sampling interval of IFFT. Finally, the signal is transmitted over a frequency-selective fading channel with a discrete-time channel impulse response $h_{m,j}(\tau) = \sum_{l=0}^{L-1} h_{l,m,j}\delta(\tau-\tau_l)$, where L, $h_{l,m,j}$, τ_l and $\delta(t)$ denote the number of paths, the path gain between the jth terminal T_j and relay at stage m, the time delay of the lth path and the delta function. Without loss of generality, we assume $\tau_0 = 0 < \tau_1 < \cdots < \tau_{L-1}$ and that the lth path time delay is $\tau_l = l\Delta$, where Δ (≥ 1) denotes the time delay separation between adjacent paths.

TABLE I
Network protocol.

First stage	Second stage
$T_0,\ T_1\ \rightarrow\ R$	$R\ \rightarrow\ T_0,\ T_1$

Stage I: The signal received at the relay terminal during the first stage may be represented in the frequency domain as

$$R_r(n) = \begin{cases} \sqrt{P_s}S_0(n)H_{0,0}(n)+\sqrt{P_s}S_1(n)H_{0,1}(n)+N_r(n), & \text{SC;} \\ \sqrt{P_s}d_0(n)H_{0,0}(n)+\sqrt{P_s}d_1(n)H_{0,1}(n)+N_r(n), & \text{OFDM,} \end{cases} \tag{1}$$

for $n = 0 \sim N_c-1$, where $S_j(n)$, $H_{0,j}(n)$ and $N_r(n)$ denote the Fourier transforms of the jth terminal T_j transmit signal, the channel gain between T_j and R, and the additive white Gaussian noise (AWGN) at the relay with power spectral density N_0, respectively.

Stage II: The relay terminal amplifies the received signal and broadcasts during the second time slot. The relay terminal normalizes the received signal (1) by a factor of $\beta = \sqrt{E[|R_r(n)|^2]}$ so that the average energy is unity. We assume that the source terminals and the relay transmit with the same power using a half of the total available power.

At the jth terminal T_j, an N_c-point FFT is applied to decompose the received signal into N_c subcarrier (frequency, for SC-FDE) components represented by

$$R_j(n) = \sqrt{P_s}R_r(n)H_{1,j}(n)+N_j(n) \tag{2}$$

for $n=0\sim N_c-1\}$, where $j\in\{0,1\}$. We define $\bar{j}\in\{0,1\}$, where the bar over the expression signifies the unitary complement operation (i.e., 'NOT' operation) that performs logical negation of the value under the bar. The jth terminal T_j removes its self information (i.e., $\{d_j(n)\}$ for OFDM and $\{S_j(n)\}$ for SC-FDE) from the received signal as

$$R_j(n) = \begin{cases} R_j(n) - P_sS_j(n)H_{0,j}(n)H_{1,j}(n), & \text{SC;} \\ R_j(n) - P_sd_j(n)H_{0,j}(n)H_{1,j}(n), & \text{OFDM.} \end{cases} \tag{3}$$

We assume that the destination terminal normalize the received signal (3) by a factor of $\alpha = \sqrt{E[|\tilde{N}_j(n)|^2]/(2N_0/N_cT_c)}$, so that the average energy is unity, where $\tilde{N}_j(n) = \frac{\sqrt{P_s}}{\alpha\beta}H_{1,j}(n)N_r(n)w_j(n)+\frac{1}{\alpha}N_j(n)w_j(n)$ Note that the normalization will not alter the signal-to-interference plus noise power ratio (SINR), but it will assist theoretical derivations in Sect. IV. In the case of SC-FDE transmission N_c-point IFFT is applied to (4) to obtain decision variables for data detection, while for OFDM case (4) denotes the decision variables. Finally, the log-likelihood ratio (LLR) is computed and de-interleaving followed by Viterbi decoding is carried out.

Next we derive the equalization weights that are required for broadband ANC transmission.

III. Frequency Domain Equalization

We introduce one-tap equalization for broadband ANC communication in the frequency domain to combat the channel frequency selectivity. The equalization weights for broadband bi-directional ANC scheme based on OFDM and SC-FDE radio access are derived, where the channel gains from both stages are taken into consideration. To our knowledge, such formulation of FDE weights for broadband ANC has not yet been reported because narrowband ANC [3]-[7] does not consider equalization since the assumption is that the wireless channel is frequency-nonselective and consequently, those schemes cannot be utilized in a frequency-selective fading channel.

The equalized signal at the jth terminal T_j can be represented as

$$\hat{R}_j(n) = R_j(n)w_j(n)$$
$$= \begin{cases} \frac{P_s}{\beta}S_{\bar{j}}(n)\hat{H}_j(n) + \sqrt{P_s}H_{1,j}(n)N_r(n)w_j(n) + \hat{N}_j(n), & \text{SC;} \\ \frac{P_s}{\beta}d_{\bar{j}}(n)\hat{H}_j(n) + \sqrt{P_s}H_{1,j}(n)N_r(n)w_j(n) + \hat{N}_j(n), & \text{OFDM,} \end{cases} \tag{4}$$

for $n = 0 \sim N_c - 1$ with $\hat{H}_j(n) = H_{0,\bar{j}}(n)H_{1,j}(n)w_j(n)$ and $\hat{N}_j(n) = N_j(n)w_j(n)$, where $w_j(n)$ denotes the equalization weight of broadband ANC at the jth terminal T_j.

A. SC-FDE

The equalization weight for SC-FDE signaling is chosen to minimize the mean square error (MSE) between $\hat{R}_j(n)$ and $S_{\bar{j}}(n)$ at the nth frequency as $MSE_j(n) = E[|e_j(n)|^2] = E[|\hat{R}_j(n) - S_{\bar{j}}(n)|^2]$. The MMSE equalization weight $w_j(n)$ for broadband ANC based on SC-FDE radio access at the jth terminal T_j is given by [Appendix A] (5). We note here that the reason why factor 2 is present in the denominator of (5) is because the source terminals and the relay transmit with the half of the total available power.

B. OFDM

The ZF equalization weight $w_j(n)$ for broadband ANC based on OFDM radio access is chosen to satisfy the following condition for all subcarriers $\hat{H}_j(n) = H_{0,\bar{j}}(n)H_{1,j}(n)w_j(n) = 1$. From here, the equalization weight $w_j(n)$ for broadband ANC based on OFDM radio access at the jth terminal T_j can be expressed as

$$w_j(n) = \frac{H^*_{0,\bar{j}}(n)H^*_{1,j}(n)}{|H_{0,\bar{j}}(n)H_{1,j}(n)|^2}. \tag{6}$$

Next, the performance of broadband ANC scheme is theoretically analyzed based on the derived equalization weights in this section.

IV. Performance Analysis

In this section, we derive the expressions for conditional SINR for broadband ANC communication, first, and then, the uncoded bit error rate (BER) and ergodic capacity expressions are presented. Unlike [3]-[7], the performance of broadband ANC is evaluated by theory and numerical evaluation in a frequency-selective fading channel. Broadband ANC based on SC-FDE utilizes the entire available frequency band and consequently, the ISI arises which is compensated at the destination through MMSE equalization. However, MMSE equalization cannot completely eliminate the ISI. Thus, the performance is analyzed based on the Gaussian approximation of the residual ISI at the output of FDE. On the other hand, the analysis on broadband ANC based on OFDM radio access, which is based on general approach of converting the frequency-selective channel into the frequency-nonselective one with application of equalization [8], is used as a reference.

We assume ideal knowledge of CSI with perfect self-information cancelation. The analysis with of imperfect knowledge of CSI and self-information removal is beyond the scope of this study.

A. SINR

The decision variables at the jth terminal T_j after equalization can be expressed as

$$\hat{d}_j(n) = \begin{cases} \frac{P_s}{\alpha\beta}d_{\bar{j}}(n)\tilde{H}_j + I_j(n) + \frac{1}{N_c}\sum_{n=0}^{N_c-1}\tilde{N}_j(n), & \text{SC;} \\ \frac{P_s}{\alpha\beta}d_{\bar{j}}(n)\hat{H}_j(n) + \tilde{N}_j(n), & \text{OFDM,} \end{cases} \tag{7}$$

where $I_j(n)$ denotes the residual ISI after equalization given by (8) with $\tilde{H}_j = \frac{1}{N_c}\sum_{n=0}^{N_c-1} H_{0,\bar{j}}(n)H_{1,j}(n)w_j(n)$. We assume that the residual ISI after FDE, for SC-FDE, can be approximated as a zero-mean complex-valued Gaussian variable and hence, it can be seen from (7) that the OFDM (SC-FDE) equalized (IFFT) output can be seen as a random variable with mean $\frac{P_s}{\alpha\beta}d_{\bar{j}}(n)\hat{H}$ ($\frac{P_s}{\alpha\beta}d_{\bar{j}}(n)\tilde{H}$). The sum of the residual ISI and AWGN can be treated as a new zero-mean complex-valued Gaussian noise with variance $2\sigma^2 = 2\sigma^2_{isi} + 2\sigma^2_n$ given by (9) where $2\sigma^2_{isi}$ and $2\sigma^2_n$ are given in Appendix B. Note that in the case of OFDM, the ISI is eliminated by insertion of the GI longer than the maximum channel time delay. Using (7) and (9), the conditional SINR $\gamma_j[E_s/N_0, \{H_{m,j}(n)\}]$ of broadband ANC at the jth terminal T_j is represented by (10).

B. Uncoded BER

We assume all "1" transmission without loss of generality and QPSK data-modulation. The conditional BER of the jth terminal T_j for the given set of channel gains $\{H_{m,j}(n);\ n = 0 \sim N_c - 1\}$ can be expressed as [11] see (8), where erfc$[\cdot]$ denotes the complementary error function [8]. The average BER at the jth terminal T_j can be numerically evaluated by averaging (11) over all possible realizations of $\{H_{0,j}(n),\ H_{1,j}(n);\ n = 0 \sim N_c - 1\}$ as (12) where $\wp[\{H_{m,j}(n)\}]$ is the joint probability density function of $\{H_{0,j}(n), H_{1,j}(n)\}$.

The evaluation of the theoretical average BER is done by Monte-Carlo numerical computation method as follows. A set of channel gains $\{H_{m,j}(n)\}$ is generated and then, $\{w_j(n)\}$ is computed for particular radio access scheme. The conditional BER as a function of the average signal energy per symbol-to-AWGN power spectrum density ratio E_s/N_0 is computed using (11) for the given set of channel gains $\{H_{m,j}(n)\}$. This is repeated a sufficient number of times to obtain the theoretical average BER given by (12).

C. Ergodic Channel Capacity

The ergodic capacity is the ensamble average of the information rate over the channel distribution. The significance of the ergodic capacity is that for every independent (realization) channel use, we can transmit at the rate defined by ergodic

$$w_j(n)=\frac{H^*_{0,\bar{j}}(n)H^*_{1,j}(n)}{|H_{0,\bar{j}}(n)H_{1,j}(n)|^2+[|H_{0,\bar{j}}(n)|^2+|H_{1,j}(n)|^2+(\frac{E_s}{2N_0})^{-1}](\frac{E_s}{2N_0})^{-1}}. \tag{5}$$

$$I_j(n)=\frac{\sqrt{P_s}}{\alpha\beta}\frac{1}{N_c}\sum_{n=0}^{N_c-1}\hat{H}_j(n)\sum_{t'=0(\neq t)}^{N_c-1}d_{\bar{j}}(t')\exp\left(j2\pi n\frac{t-t'}{N_c}\right) \tag{8}$$

$$2\sigma^2=\begin{cases}\frac{2N_0}{T_cN_c}\frac{1}{N_c}\sum_{n=0}^{N_c-1}|w_j(n)|^2+\frac{\frac{P_s}{N_c}\sum_{n=0}^{N_c-1}|H_{0,\bar{j}}(n)H_{1,j}(n)w_j(n)|^2-|\tilde{H}_j|^2}{|H_{0,\bar{j}}(n)|^2+|H_{1,j}(n)|^2+(\frac{E_s}{2N_0})^{-1}}, & \text{SC;}\\ \frac{2N_0}{T_cN_c}|w_j(n)|^2, & \text{OFDM,}\end{cases} \tag{9}$$

$$\gamma_j\left[\frac{E_s}{N_0},\{H_{m,j}(n)\}\right]=\begin{cases}\frac{|\tilde{H}_j|^2}{(\frac{E_s}{2N_0})^{-1}\frac{1}{N_c}\sum_{n=0}^{N_c-1}[|H_{0,\bar{j}}(n)|^2+|H_{1,j}(n)|^2+(\frac{E_s}{2N_0})^{-1}]|w_j(n)|^2+\frac{1}{N_c}\sum_{n=0}^{N_c-1}|\hat{H}_j|^2-|\tilde{H}_j|^2}, & \text{SC;}\\ \frac{E_s}{2N_0}\frac{|H_{0,\bar{j}}(n)|^2|H_{1,j}(n)|^2}{|H_{0,\bar{j}}(n)|^2+|H_{1,j}(n)|^2+(\frac{E_s}{2N_0})^{-1}}, & \text{OFDM.}\end{cases} \tag{10}$$

capacity with error close to zero assuming that asymptotically optimal codebooks are used.

When the channel is not known at the transmitter the ergodic channel capacity $C_j(E_s/N_0)$ in bps/Hz for the given the average E_s/N_0 of the jth terminal T_j can be computed as [8]

$$\begin{aligned}C_j[E_s/N_0]&=E\left[C_j\left(\frac{E_s}{N_0},\{H_{m,j}(n)\}\right)\right]\\&=\int_0^\infty\cdots\int_0^\infty C_j\left(\frac{E_s}{N_0},\{H_{m,j}(n)\}\right)\\&\quad\times\wp[\{H_{m,j}(n)\}]\prod_n dH_{m,j}(n),\end{aligned} \tag{13}$$

where $C_j(E_s/N_0,\{H_{m,j}(n)\})$ is the conditional channel capacity [8]. A closed-form or convenient expression has not been found for integral in (13) and thus, we resort to the numerical computation approach using

$$\begin{aligned}&C_j\left(\frac{E_s}{N_0}\right)\\&=\frac{1}{N_c}\int_0^\infty\cdots\int_0^\infty\sum_{n=0}^{N_c-1}\log_2\left\{1+\gamma_j\left[\frac{E_s}{N_0},\{H_{m,j}(n)\}\right]\right\}\\&\qquad\times\wp[\{H_{m,j}(n)\}]\prod_n dH_{m,j}(n).\end{aligned} \tag{14}$$

The evaluation of ergodic capacity is done by Monte-Carlo numerical computation method as follows. First, we generate a set of channel gains $\{H_{m,j}(n)\}$ and then, $\{w_j(n)\}$ is computed based on particular radio access scheme. The SINR $\gamma_j(\cdot)$ is computed by (10) to evaluate the ergodic capacity by averaging (14) for the given set of channel gains $\{H_{m,j}(n)\}$ a sufficient number of times.

V. Numerical and Simulation Results

Numerical and computer simulation parameters are shown in Table II. We assume $N_c=256$, GI length of $N_g=32$ samples and ideal coherent QPSK data modulation/demodulation. For forward error control we apply $\{g_1=(111),\ g_2=(101)\}$ convolutional encoder with coding rate 1/2 and constraint length 3. We assume that for each new frame the state of the encoder is initialized before transmission. At the receiver, a hard decision Viterbi decoder is applied. The information bit sequence length is taken to be M=1024 bits. The propagation channel is an $L=16$-path block Rayleigh fading channel. The normalized Doppler frequency $f_DT_s=10^{-4}$, where $1/T_s=1/[T_c(1+N_g/N_c)]$ is the transmission symbol rate is assumed (under this assumption, the path gains can be considered to remain almost constant over one N_c data-modulated symbol block and vary block-by-block). $f_DT_s=10^{-4}$ corresponds to mobile terminal moving speeds of 11 km/h for 5 GHz carrier frequency and transmission data rate of 100 M symbols/sec. We assume that the maximum time delay of the channel L is less than the GI length (i.e., $L<N_g$ with $\Delta=1$) and that

$$\begin{aligned}\wp_{j,b}\left[\frac{E_s}{N_0},\{H_{m,j}(n)\}\right]&=\frac{1}{2}Prob[\Re[\hat{d}_j(n)]<0|\{H_{m,j}(n)\}]+\frac{1}{2}Prob[\Im[\hat{d}_j(n)]<0|\{H_{m,j}(n)\}]\\&=\frac{1}{2}\mathrm{erfc}\left[\sqrt{\frac{1}{4}\gamma_j\left(\frac{E_s}{N_0},\{H_{m,j}(n)\}\right)}\right],\end{aligned} \tag{11}$$

$$\mathcal{P}_{j,b}\left[\frac{E_s}{N_0}\right]=\int\cdots\int\wp_{j,b}\left[\frac{E_s}{N_0},\{H_{m,j}(n)\}\right]\wp[\{H_{m,j}(n)\}]\prod_n dH_{m,j}(n), \tag{12}$$

TABLE II
SIMULATION PARAMETERS.

Transmitter	Data modulation	QPSK
	Block (IFFT) size	N_c=256
	GI	$N_g = 32$
Channel	L-path block Rayleigh fading with $\Delta = 1$	
Receiver	FDE	MMSE, ZF
	Channel Estimation	Ideal

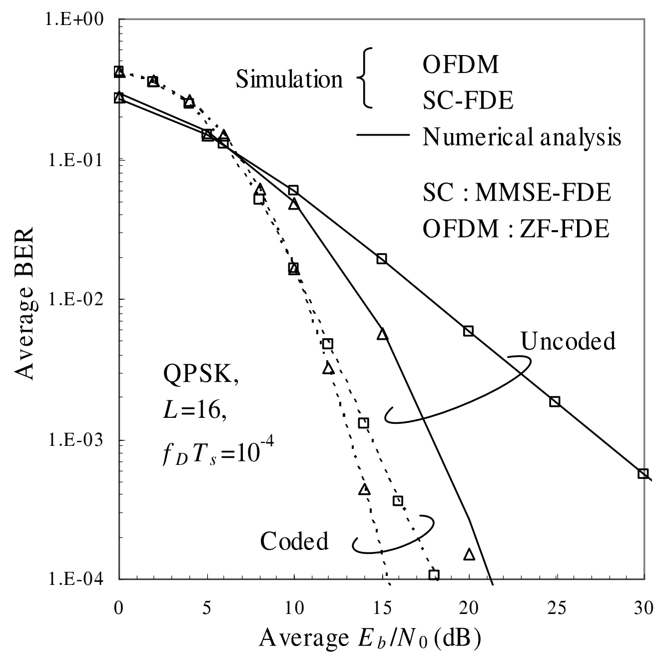
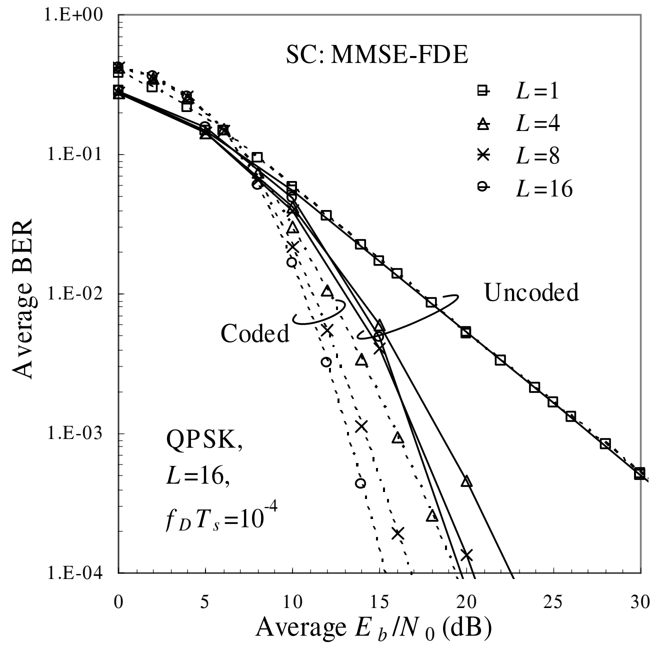


Fig. 2. BER performance in a frequency-selective fading channel.

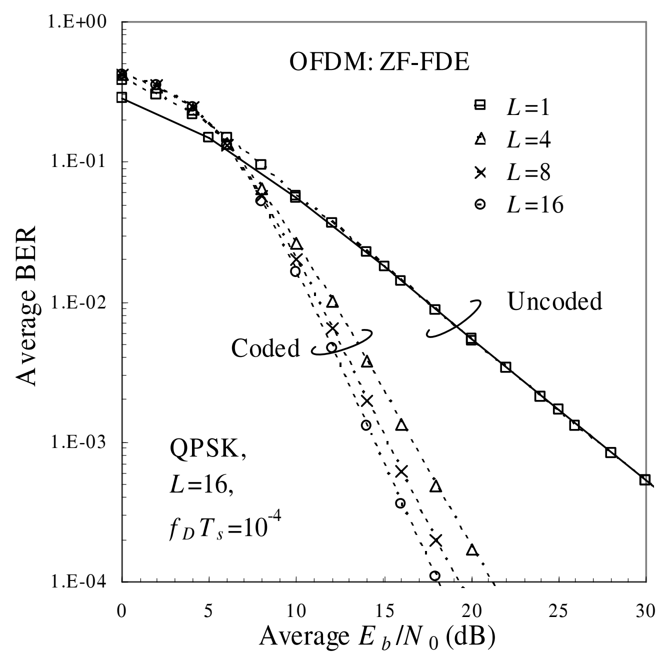
all paths in any channel are independent with each other. We assume the system without knowledge of the channel state information at the transmitters, but the perfect channel state information at the receivers (i.e., relay and destinations). We assume no shadowing loss and pathloss.

A. BER Performance

The uncoded and coded BER performances of broadband ANC is illustrated in Fig. 2 as a function of the average signal energy per bit-to-AWGN power spectrum density ratio E_b/N_0 $(=0.5\times(N_c/M)\times(E_s/N_0)\times(1+N_g/N_c))$. It can be seen from the Fig. 2 that the performance of ANC based on SC with MMSE-FDE transmission improves in comparison with OFDM; for BER=10^{-3}, the E_b/N_0 gain of about 10 dB is obtained. It can be further seen from the figure that the E_b/N_0 gain of broadband ANC based on SC-FDE further increases for a lower BER in comparison with OFDM radio access. Note that in this case, at the destination end, each frequency component of the received signal carries a portion of each data modulated symbol (i.e., each symbol is spread over entire frequency band), which is exploited through MMSE-FDE to obtain frequency-diversity gain and improve the performance. The figure shows a fairly good agreement between computer simulation results and numerical analysis based on the Gaussian approximation of the residual ISI after FDE. We note here that, unlike SC with MMSE-FDE, OFDM system cannot achieve frequency diversity gain unless channel coding is applied.



(a) SC-FDE radio access



(b) OFDM radio access

Fig. 3. Impact of channel frequency-selectivity.

The figure also illustrates that the performance of broadband ANC improves due to the channel coding gain in a frequency-selective fading channel. The performance difference between SC-FDE and OFDM is significantly reduced and it may be further reduced by utilization of the coding techniques such as turbo codes or low density parity check codes. This is left as interesting future work.

B. Impact of Channel Frequency-selectivity

The performance of SC with MMSE-FDE depends on the channel frequency-selectivity and thus, in this subsection we investigate the effect of different propagation scenarios. The channel frequency-selectivity is a function of the number of paths L; as L decreases the channel becomes less frequency-selective and when $L = 1$ it becomes a frequency-nonselective channel (i.e., single-path channel).

Figure 3 illustrates the dependency of the BER performance on the E_b/N_0 with the channel number of paths L as a parameter for both uncoded and coded broadband ANC. It can be seen from the Fig. 3(a) that the performance of broadband ANC based on SC-FDE radio access degrades as L decreases due to less frequency diversity gain. Note that the performance of broadband ACN based on SC-FDE and OFDM radio access for $L = 1$ is the same. As shown in Fig. 3(b), the OFDM system converts the frequency-selective channel into a set of frequency-nonselective channels and consequently, the uncoded performance of broadband ANC based on OFDM access will not be affected by the channel frequency-selectivity at all. However, the channel coded performance of broadband ANC based on OFDM degrades as the number of channel paths L decreases.

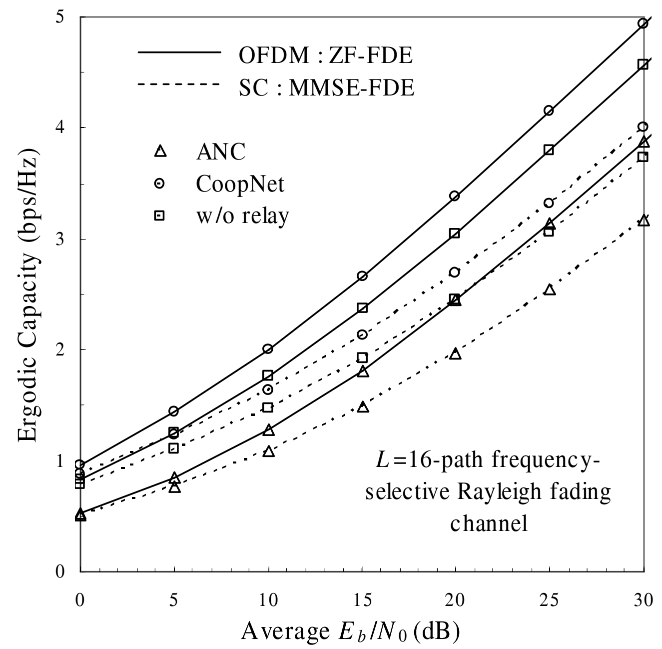


Fig. 4. Ergodic channel capacity.

C. Ergodic Capacity

In this section, the ergodic channel capacity given by (14) is evaluated using Monte Carlo numerical computational method. Figure 4 illustrates the achievable ergodic capacity as a function of the E_b/N_0 with the number of paths $L = 16$ for both SC-FDE and OFDM broadband access between (i) analog network coding (i.e., ANC), (ii) cooperative relay networking (i.e., CoopNet) [10] and (iii) conventional network (i.e., single source and destination terminal without relaying – w/o relay). We note here that cooperative relaying is defined as single source and destination communication assisted by relay using a two-stage process [10]. During the first stage source transmits to both relay and destination, while in the second stage only relay transmits to destination which combines the signals from both stages before data detection. The figure shows that the ergodic capacity of OFDM without relaying is slightly increased in comparison with ANC scheme. This is because in ANC a total available transmit power is divided between the source terminals and the relay, while without relaying the total power is available at the source terminal. It can be also seen from the figure that broadband ANC based on OFDM achieves a higher channel capacity in comparison with SC-FDE radio access under the same channel conditions. The figure shows that the performance with cooperative relay networking is slightly better than broadband ANC due to the gain obtained through diversity combining. We note here that the ergodic capacity of broadband ANC is evaluated from the end user's perspective, however, the capacity on the network level is doubled in comparison with cooperative relaying. Thus, an appropriate diversity scheme for ANC should be employed to further improve the capacity similar as cooperative relay networking case.

VI. Conclusion

In this letter, we presented broadband ANC scheme based on OFDM and SC-FDE radio access in a frequency-selective fading channel. The equalization weights required for broadband ANC communication in a frequency-selective fading channel are derived. The theoretical BER and ergodic capacity analysis for bi-directional ANC scheme using OFDM and SC-FDE radio access in a frequency-selective fading channel was evaluated based on the Gaussian approximation of the residual ISI after MMSE-FDE for SC-FDE, while in case of OFDM the ISI is eliminated by insertion of GI. It was shown that SC-FDE achieves a lower BER, but a worse ergodic capacity in comparison with OFDM under the same channel conditions in a frequency-selective fading channel. Our results show that a drawback of broadband ANC scheme is its lack of diversity combining at the destination, which causes a slightly lower ergodic capacity than cooperative relaying irrespective of transmission scheme. Thus, an appropriate diversity scheme for ANC should be developed to further improve the performance similar as in case of cooperative relay networking. This is left as an interesting future work.

This work is based on the assumption of perfect knowledge of CSI and perfect self-information removal. In particular, the accurate channel estimation for broadband ANC is not an easy task in practice because the users pilot signals during the first stage interfere with each other and the relay cannot accurately estimate their corresponding CSIs. To estimate all CSIs we can allocate four time slots to separate different users' pilot signals, but network throughput is reduced. Moreover, CSIs which are estimated at the relay must be fed back to the user terminals which additionally reduces the throughput. Consequently, a novel CE scheme for broadband ANC should be introduced. This is left as an interesting future study.

Appendix A

The $MSE_j(n)$ term for broadband ANC based on SC-FDE radio access defined in Sect. III can be expressed as

$$MSE_j(n) = \frac{P_s}{\alpha\beta} E[S_{\bar{j}}(n) S_{\bar{j}}^*(n')] |H_{0,\bar{j}}(n) H_{1,j}(n) w_j(n)|^2 + \frac{P_s}{\alpha\beta} E[N_r(n) N_r^*(n')] |H_{0,\bar{j}} w_j(n)|^2 + \frac{1}{\alpha} E[N_j(n) N_j^*(n')] |w_j(n)|^2, \quad \text{(A.1)}$$

where $N_r(n)$ and $N_j(n)$ denote the zero mean and independent identically distributed (i.i.d.) complex Gaussian variable with variance $2N_0/T_c$. Since data symbols are independent (i.e., $E[d(x)d^*(y)] = \delta(x-y)$) we obtain

$$E[S_{\bar{j}}(n) S_{\bar{j}}^*(n')] = \sum_{t=0}^{N_c-1} \sum_{t'=0}^{N_c-1} E[d_{\bar{j}}(t) d_{\bar{j}}^*(t')] \times \exp\left(j2\pi(n-n')\frac{t-t'}{N_c}\right) = \frac{E_s}{T_c} N_c \delta(n-n'). \quad \text{(A.2)}$$

Using (A.1) and (A.2) and after the expectation with respect to the noise components we obtain

$$MSE_j(n) = \frac{E_s}{T_c N_c} \frac{|H_{0,\bar{j}}(n) H_{1,j}(n) w_j(n)|^2}{|H_{0,\bar{j}}(n)|^2 + |H_{1,j}(n)|^2 + (\frac{E_s}{2N_0})^{-1}} + \frac{2N_0}{T_c N_c} |w_j(n)|^2 + \frac{E_s}{T_c N_c} - \frac{2E_s}{T_c N_c} \frac{\Re\left\{E\left[H_{0,\bar{j}}(n) H_{1,j}(n) w_j(n)\right]\right\}}{|H_{0,\bar{j}}(n)|^2 + |H_{1,j}(n)|^2 + (\frac{E_s}{2N_0})^{-1}}, \quad \text{(A.3)}$$

where $\Re\{z\}$ denotes the real part of the complex number z. By solving $\frac{\partial MSE_j(n)}{\partial w_j(n)} = 0$, we obtain the equalization weight given by (5).

APPENDIX B

We first derive the variance of noise and ISI for broadband ANC based on SC-FDE radio access and then, we present the noise variance for OFDM radio access in which case the ISI is eliminated by the insertion of GI.

The variance of ISI for broadband ANC based on SC-FDE can be represented using (8) as

$$2\sigma_{isi}^2 = E[|I(n)|^2] = \frac{P_s}{\alpha^2\beta^2} \frac{1}{N_c^2} \sum_{n=0}^{N_c-1} \sum_{n'=0}^{N_c-1} H_{0,\bar{j}}(n) H_{1,j}(n) H_{0,\bar{j}}^*(n') H_{1,j}^*(n') \times w_j(n) w_j^*(n') \sum_{t'=0(\neq)}^{N_c-1} \sum_{t''=0(\neq)}^{N_c-1} \times E[d_{\bar{j}}(t') d_{\bar{j}}^*(t'')] \exp\left(j2\pi(n-n')\frac{t'-t''}{N_c}\right). \text{(B.1)}$$

Since different data modulate symbols are independent the ISI variance can be written as

$$2\sigma_{isi}^2 = \frac{P_s}{\alpha^2\beta^2} \frac{1}{N_c} \sum_{n=0}^{N_c-1} \sum_{n'=0}^{N_c-1} H_{0,\bar{j}}(n) H_{1,j}(n) H_{0,\bar{j}}^*(n') H_{1,j}^*(n') \times w_j(n) w_j^*(n') [\delta(n-n') - 1] = \frac{P_s}{\alpha^2\beta^2} \left[\frac{1}{N_c} \sum_{n=0}^{N_c-1} |\hat{H}(n)|^2 - |\tilde{H}(n)|^2\right]. \text{(B.2)}$$

The noise variance for broadband ANC based on SC-FDE can be represented as

$$2\sigma_n^2 = E\left[|\frac{1}{N_c} \sum_{n=0}^{N_c-1} \tilde{N}_j(n)|^2\right] = \frac{1}{N_c^2} \sum_{n=0}^{N_c-1} \sum_{n'=0}^{N_c-1} \left[\frac{P_s}{\alpha^2\beta^2} E[N_r(n) N_r^*(n')] \times w_j(n) w_j^*(n') H_{0,\bar{j}}(n) H_{1,j}(n) H_{0,\bar{j}}^*(n') H_{1,j}^*(n') - \frac{1}{\alpha^2} E[N_j(n) N_j^*(n')] w_j(n) w_j^*(n')\right] \exp\left(j2\pi t \frac{n-n'}{N_c}\right). \quad \text{(B.3)}$$

Finally, the noise variance is given by

$$2\sigma_n^2 = \frac{2N_0}{T_c} \frac{1}{N_c} \sum_{n=0}^{N_c-1} |w_j(n)|^2, \quad \text{(B.4)}$$

where the equalization weight $w_j(n)$ is given by (5).

From (7), the noise variance for broadband ANC based on OFDM can be represented as

$$2\sigma_n^2 = E[|\tilde{N}_j(n)|^2] = \frac{P_s}{\alpha^2\beta^2} E[N_r(n) N_r^*(n')] |H_{1,j}(n) w_j(n)|^2 + \frac{1}{\alpha^2} E[N_j(n) N_j^*(n')] |w_j(n)|^2 = |w_j(n)|^2 \frac{2N_0}{T_c N_c}, \quad \text{(B.5)}$$

where the equalization weight $w_j(n)$ is given by (6).

REFERENCES

[1] R. W. Yeung, "Multilevel diversity coding with distortion," *IEEE Trans. Inf. Theory*, vol. IT-41, pp. 412-422, 1995.

[2] R. Ahlswede, N. Cai, S.-Y. R. Li, and R. W. Yeung, "Network information flow," *IEEE Trans. Inf. Theory*, vol. 46, pp. 1204-1216, 2000.

[3] P. Popovski and H. Yomo, "Bi-directional amplification of throughput in a wireless multi-hop network," in *IEEE 63rd Veh. Technol. Conf. (VTC)*, Melbourne, Australia, May 2006.

[4] S. Zhang, S.-C. Liew, and P. Lam, "Hot topic: physical layer network coding," in *Proc. ACM 12th MobiCom 2006*, pp. 358-365, Sep. 2006, Los Angelos, USA.

[5] S. Zhang, S. C. Liew, and L. Lu, "Physical layer network schemes over finite and infinite fields," in *2008 Global Commun. Conf. (Globecom 2008)*, Nov. 2008, New Orleans, USA.

[6] T. Wang and G. B. Giannakis, "Complex field network coding for multiuser cooperative communications," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 3, pp. 561-571, Apr. 2008.

[7] S. Katti, S. S. Gollakota, and D. Katabi, "Embracing wireless interference: analog network coding," *ACM SIGCOM 2007*, Kyoto, Japan, Aug. 2007.

[8] J. G. Proakis, *Digital Communications*, 3rd ed. McGraw-Hill, 1995.

[9] D. Falconer, S. L. Ariyavisitakul, A. Benyamin-Seeyar, and B. Eidson, "Frequency-domain equalization for single-carrier broadband wireless systems," *IEEE Commun. Mag.*, vol. 40, pp.58-66, Apr. 2002.

[10] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 11, pp. 3062-3080, Dec. 2004.