

PAPER

# Joint Water Filling-MRT Downlink Transmit Diversity for a Broadband Single-Carrier Distributed Antenna Network

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**SUMMARY** In this paper, joint water filling and maximal ratio transmission (joint WF-MRT) downlink transmit diversity for a single-carrier distributed antenna network (SC DAN) is proposed. The joint WF-MRT transmit weight allocates the transmit power in both transmit antenna dimension and frequency dimension, i.e., the power allocation is done both across frequencies based on WF theorem and across transmit antennas based on MRT strategy. The cumulative distribution function (CDF) of the channel capacity achievable by joint WF-MRT transmit diversity is evaluated by Monte-Carlo numerical computation method. The channel capacities achievable with joint WF-MRT, MRT, and WF transmit weight (WF transmit weight is done across transmit antennas and frequencies based on WF theorem) are compared. It is shown that the joint WF-MRT transmit weight provides the highest channel capacity among three transmit weights. **key words:** distributed antenna network, transmit diversity, channel capacity, frequency-selective channel

## 1. Introduction

In broadband wireless systems, the received signal suffers from frequency-selective fading in addition to the shadowing and path losses [1]. The distributed antenna network (DAN) or the distributed antenna system (DAS) [2]–[7] is a promising wireless network to mitigate the negative impacts of fading, shadowing loss, and path loss. In DAN, many antennas connected by optical fiber cable to a base station (BS) are spatially distributed so that with a high probability, some antennas can always be visible from a mobile station (MS). There are two ways to utilize DAN: transmit/receive diversity [6]–[11] and spatial multiplexing [12]–[15]. In this paper, we consider the single-carrier (SC) DAN downlink transmit diversity.

Recently, we evaluated the channel capacity distribution of the DAN downlink in a frequency-nonselective channel [7]. In [7], the maximal ratio transmit (MRT) diversity [10] is considered. The MRT diversity is the optimal diversity in a frequency-nonselective channel, but is not necessarily optimal in a frequency-selective channel. In this paper, we propose a new joint water filling and MRT (joint WF-MRT) downlink transmit diversity.

The well known advantages of SC transmission over multi-carrier (MC) transmission (e.g., orthogonal frequency division multiplexing (OFDM)) is its low peak-to-average power ratio (PAPR) property. However, an introduction of

joint WF-MRT increases the PAPR of SC signals. In this paper, therefore, we investigate the PAPR of SC using WF-MRT to compare with that of OFDM.

The remainder of this paper is organized as follows. In Sect. 2, we present the SC DAN downlink transmit diversity model. In Sect. 3, we develop a downlink capacity expression and then, by using the Lagrange multiplier method, derive the joint WF-MRT transmit weight that maximizes the channel capacity for the given channel realization. In Sect. 4, we evaluate, by the Monte Carlo numerical computation method, the cumulative distribution function (CDF) of the SC DAN downlink channel capacity.

## 2. SC DAN Downlink Transmission

### 2.1 SC DAN Model

We consider an SC DAN in which transmit antennas are uniformly distributed over a service area as shown in Fig. 1. The SC DAN downlink transmit diversity model is illustrated in Fig. 2. Distributed antennas are ranked in the ascending order of the path loss plus shadowing loss and  $N_t$  antennas which have the smallest path loss plus shadowing loss are selected for transmit diversity. A single user is assumed in this paper.

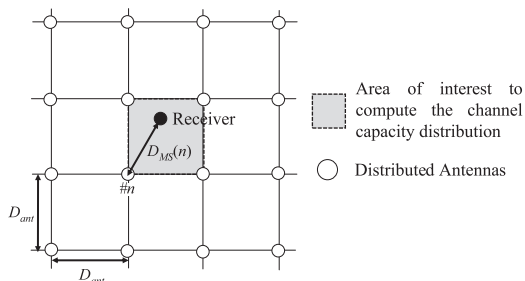


Fig. 1 Transmit antenna distribution.

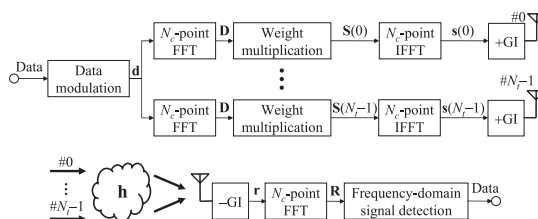


Fig. 2 SC DAN downlink transmit diversity model.

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## 2.2 Transmit Signal

An information bit sequence is transformed into the data-modulated symbol sequence, which is then divided into a sequence of blocks of  $N_c$  symbols each, where  $N_c$  is the size of fast Fourier transform (FFT). In this paper, without loss of generality, a transmission of the single block, i.e.,  $\{d(t); t = 0 \sim N_c - 1\}$ , from  $N_t$  distributed antennas is considered. The symbol block to be transmitted is represented by  $\mathbf{d} = [d(0), \dots, d(N_c - 1)]^T$ .

An  $N_c$ -point FFT is applied to transform  $\mathbf{d}$  into the frequency-domain signal vector  $\mathbf{D} = [D(0), \dots, D(k), \dots, D(N_c - 1)]^T$  as

$$\mathbf{D} = \mathbf{F}\mathbf{d}, \quad (1)$$

where

$$\mathbf{F} = \frac{1}{\sqrt{N_c}} \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & e^{-j2\pi \frac{1 \times 1}{N_c}} & \dots & e^{-j2\pi \frac{1 \times (N_c - 1)}{N_c}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j2\pi \frac{(N_c - 1) \times 1}{N_c}} & \dots & e^{-j2\pi \frac{(N_c - 1) \times (N_c - 1)}{N_c}} \end{bmatrix} \quad (2)$$

is the FFT matrix of size  $N_c \times N_c$ . At the  $n$ th transmit antenna ( $n = 0 \sim N_t - 1$ ), an  $N_c \times N_c$  diagonal transmit weight matrix  $\mathbf{W}(n) = \text{diag}[W(n, 0), \dots, W(n, N_c - 1)]^T$  is multiplied to  $\mathbf{D}$  to obtain  $\mathbf{S}(n) = [S(n, 0), \dots, S(n, N_c - 1)]^T$  as

$$\mathbf{S}(n) = \mathbf{W}(n)\mathbf{D}, \quad n = 0 \sim N_t - 1. \quad (3)$$

An  $N_c$ -point inverse FFT (IFFT) is applied to transform into the time-domain transmit signal block  $\mathbf{s}(n) = [s(n, 0), \dots, s(n, N_c - 1)]^T$  as

$$\mathbf{s}(n) = \mathbf{F}^H \mathbf{S}(n), \quad (4)$$

where  $(\cdot)^H$  is the Hermitian transpose operation. The last  $N_g$  samples in the block are copied as a cyclic prefix (CP) and inserted into the guard interval (GI) placed at the beginning of each block before transmission.

## 2.3 Channel Model

The broadband channel is characterized by the distant dependent path loss, log-normally distributed shadowing loss, and frequency-selective fading. The received power  $P_r(n)$  for an MS whose distance from the  $n$ th distributed antenna is  $D_{MS}(n)$  can be modeled as [1]

$$P_r(n) = (P_t(n) \cdot D_{ant}^{-\alpha}) \cdot \hat{D}(n)^{-\alpha} \cdot 10^{-\frac{\eta(n)}{10}}, \quad (5)$$

where  $P_t(n)$  is the transmit power,  $\hat{D}(n) = D_{MS}(n)/D_{ant}$  is the normalized distance,  $\alpha$  is the path loss exponent, and  $\eta(n)$  is the shadowing loss (dB) characterized by a Gaussian variable with zero-mean and standard variation  $\sigma$ . Assuming that the frequency-selective channel is composed of  $L$  distinct paths with different time delays, the channel impulse response  $h(n, \tau)$  between the  $n$ th distributed antenna and the

MS receive antenna is given by

$$h(n, \tau) = \sqrt{\Omega(n)} \sum_{l=0}^{L-1} h_l(n) \delta(\tau - \tau_l), \quad (6)$$

where  $h_l(n)$  and  $\tau_l$  are respectively the complex-valued path gain and the time delay of the  $l$ th path between the  $n$ th transmit antenna and the MS with  $h_l(n)$  being the complex-valued path gain and  $E[\sum_{l=0}^{L-1} |h_l(n)|^2] = 1$  and

$$\Omega(n) = \hat{D}(n)^{-\alpha} \cdot 10^{-\frac{\eta(n)}{10}}. \quad (7)$$

## 2.4 Received Signal

The GI-removed received signal block  $\mathbf{r} = [r(0), \dots, r(N_c - 1)]^T$  can be expressed using the matrix form as

$$\mathbf{r} = \sqrt{\frac{2E_s}{T_s}} \mathbf{h}\mathbf{s} + \mathbf{n}, \quad (8)$$

where  $E_s = (P_t(n) \cdot R^{-\alpha})T_s$  with  $T_s$  being the data symbol length,  $\mathbf{s} = [\mathbf{s}(0)^T, \dots, \mathbf{s}(N_t - 1)^T]^T$ ,  $\mathbf{h}$  is the  $N_c \times N_t N_c$  channel impulse response matrix given as

$$\mathbf{h} = [\mathbf{h}(0), \dots, \mathbf{h}(N_t - 1)], \quad (9)$$

with

$$\mathbf{h}(n) = \begin{bmatrix} h_0(n) & & & h_{L-1}(n) \\ \vdots & h_0(n) & & \ddots \\ & \vdots & h_0(n) & \mathbf{0} & h_{L-1}(n) \\ h_{L-1}(n) & & \vdots & \ddots & \\ & h_{L-1}(n) & & h_0(n) & \\ & & h_{L-1}(n) & \vdots & \ddots \\ \mathbf{0} & & & \ddots & h_0(n) \end{bmatrix}, \quad (10)$$

and  $\mathbf{n} = [n(0), \dots, n(N_c - 1)]^T$  is the noise vector. The  $t$ th element,  $n(t)$ , of  $\mathbf{n}$  is the zero-mean additive white Gaussian noise (AWGN) having variance  $2N_0/T_s$  with  $N_0$  being the one-sided noise power spectrum density.

The received signal block  $\mathbf{r}$  is transformed by an  $N_c$ -point FFT into the frequency-domain signal  $\mathbf{R} = [R(0), \dots, R(N_c - 1)]^T$  as

$$\mathbf{R} = \mathbf{F}\mathbf{r} = \sqrt{\frac{2E_s}{T_s}} \mathbf{H}\mathbf{W}\mathbf{F}\mathbf{d} + \mathbf{N}, \quad (11)$$

where  $\mathbf{H} = [\mathbf{H}(0), \dots, \mathbf{H}(N_t - 1)]^T$ ,  $\mathbf{W} = [\mathbf{W}(0), \dots, \mathbf{W}(N_t - 1)]^T$ , and  $\mathbf{N} = \mathbf{F}\mathbf{n} = [N(0), \dots, N(N_c - 1)]^T$ .  $\mathbf{H}(n)$  is given as

$$\begin{aligned} \mathbf{F}\mathbf{h}(n)\mathbf{F}^H &= \mathbf{H}(n) \\ &= \text{diag}[H(n, 0), H(n, 1), \dots, H(n, N_c - 1)], \end{aligned} \quad (12)$$

with

$$H(n, k) = \sum_{l=0}^{L-1} h_l(n) \exp(-j2\pi k\tau_l/N_c), \quad k=0 \sim N_c - 1. \quad (13)$$

### 3. Joint WF-MRT Transmit Weight

#### 3.1 Maximization Problem of SC DAN Downlink Transmit Diversity

According to Ref. [14], the channel capacity  $C(\mathbf{W})$  of DAN downlink transmit diversity is given by

$$C(\mathbf{W}) = \frac{1}{N_c} \log_2 \left[ \det \left( \mathbf{I} + \frac{E_s}{N_0} \tilde{\mathbf{H}} \tilde{\mathbf{H}}^H \right) \right] \quad (14)$$

for SC and MC, where  $\tilde{\mathbf{H}} = \mathbf{H}\mathbf{W}\mathbf{F}$  for SC DAN and  $\tilde{\mathbf{H}} = \mathbf{H}\mathbf{W}$  for MC DAN. Since  $\mathbf{F}\mathbf{F}^H = \mathbf{I}$ , the channel capacity  $C(\mathbf{W})$  of SC DAN is the same as that of MC DAN and is given by

$$C(\mathbf{W}) = \frac{1}{N_c} \sum_{k=0}^{N_c-1} \log_2 \left( 1 + \frac{E_s}{N_0} \left| \sum_{n=0}^{N_t-1} H(n, k) W(n, k) \right|^2 \right), \quad (15)$$

where  $E[\mathbf{d}\mathbf{d}^H] = \mathbf{I}$ . We want to derive the set of transmit weights that maximizes the channel capacity for the given channel realization  $\mathbf{H}$ . The maximization problem can be written as

$$\begin{aligned} \max. C(\mathbf{W}) &= \frac{1}{N_c} \sum_{k=0}^{N_c-1} \log_2 \left( 1 + \frac{E_s}{N_0} \left| \sum_{n=0}^{N_t-1} H(n, k) W(n, k) \right|^2 \right) \\ \text{s.t.} \quad &\begin{cases} \sum_{n=0}^{N_t-1} \sum_{k=0}^{N_c-1} |W(n, k)|^2 = N_c \\ 0 \leq |W(n, k)|^2 \text{ for } n=0 \sim N_t-1 \text{ and } k=0 \sim N_c-1 \end{cases} \end{aligned} \quad (16)$$

However, it is quite difficult to solve Eq. (16). In this paper, therefore we derive the transmit weight that can maximize the upper bound of Eq. (15). Using Cauchy-Schwarz inequality [18], Eq. (15) can be upper bounded as

$$\begin{aligned} \max. C(\mathbf{W}) &\leq \frac{1}{N_c} \sum_{k=0}^{N_c-1} \log_2 \left( 1 + \frac{E_s}{N_0} \sum_{n=0}^{N_t-1} |H(n, k)|^2 \sum_{n=0}^{N_t-1} |W(n, k)|^2 \right). \end{aligned} \quad (17)$$

In Eq. (17), the equality holds if and only if

$$\frac{W(0, k)}{H^*(0, k)} = \dots = \frac{W(n, k)}{H^*(n, k)} = \dots = \frac{W(N_t - 1, k)}{H^*(N_t - 1, k)}. \quad (18)$$

#### 3.2 Derivation of $\mathbf{W}$

To derive the optimal  $\mathbf{W}$ , first, the permuted index  $k' = (0 \sim N_c - 1)$  is introduced so that  $\sum_{n=0}^{N_t-1} |H(n, 0)|^2 \geq \dots \geq \sum_{n=0}^{N_t-1} |H(n, k')|^2 \geq \dots \geq \sum_{n=0}^{N_t-1} |H(n, N_c - 1)|^2$ . The optimality condition given by Eq. (17) can now be rewritten as

$$\max. C_{\text{upper}}(\mathbf{W})$$

$$\begin{aligned} &= \frac{1}{N_c} \sum_{k'=0}^{N_c-1} \log_2 \left( 1 + \frac{E_s}{N_0} \sum_{n=0}^{N_t-1} |H(n, k')|^2 \sum_{n=0}^{N_t-1} |W(n, k')|^2 \right) \\ \text{s.t.} \quad &\begin{cases} \sum_{n=0}^{N_t-1} \sum_{k'=0}^{N_c-1} |W(n, k')|^2 = N_c \\ 0 \leq |W(n, k')|^2 \text{ for } n=0 \sim N_t-1 \text{ and } k'=0 \sim N_c-1 \end{cases} \end{aligned} \quad (19)$$

We assume that  $\sum_{n=0}^{N_t-1} |W(n, k')|^2$  has  $m$  non-zero elements and  $(N_c - m)$  zero elements ( $0 \leq m \leq N_c - 1$ ), where  $m$  is determined so that  $C_{\text{upper}}(\mathbf{W})$  is maximized. Since  $C_{\text{upper}}(\mathbf{W})$  is a monotonic increasing function of  $\sum_{n=0}^{N_t-1} |H(n, k')|^2$ ,  $k' = (0 \sim N_c - 1)$ ,  $\sum_{n=0}^{N_t-1} |W(n, k')|^2 \neq 0$  for  $k' = (0 \sim m - 1)$  and  $\sum_{n=0}^{N_t-1} |W(n, k')|^2 = 0$  for  $k' = (m \sim N_c - 1)$ . Eq. (19) can be rewritten as

$$\begin{aligned} \max. C_{\text{upper}}(\mathbf{W}) &= \frac{1}{N_c} \sum_{k'=0}^{N_c-1} \log_2 \left( 1 + \frac{E_s}{N_0} \sum_{n=0}^{N_t-1} |H(n, k')|^2 \sum_{n=0}^{N_t-1} |W(n, k')|^2 \right) \\ \text{s.t.} \quad &\begin{cases} g_1(\mathbf{W}) = \sum_{k'=0}^{m-1} \sum_{n=0}^{N_t-1} |W(n, k')|^2 - N_c = 0 \\ g_2(\mathbf{W}) = \sum_{k'=m}^{N_c-1} \sum_{n=0}^{N_t-1} |W(n, k')|^2 - 0 = 0 \\ f(\mathbf{W}(n)) = - \sum_{n=0}^{N_t-1} |W(n, k')|^2 \leq 0, \\ k' = 0 \sim N_c - 1 \end{cases} \end{aligned} \quad (20)$$

The optimality problem of Eq. (20) can be solved using the Lagrange multiplier method. The Lagrangian function  $J$  can be expressed as [19]

$$\begin{aligned} J &= C_{\text{upper}}(\mathbf{W}) + \kappa \cdot g_1(\mathbf{W}) + \mu \cdot g_2(\mathbf{W}) \\ &\quad + \sum_{k'=0}^{N_c-1} \sum_{n=0}^{N_t-1} \Psi(n, k') \cdot f(\mathbf{W}(n)), \end{aligned} \quad (21)$$

where  $\kappa$ ,  $\mu$ , and  $\{\Psi(n, k'); n = (0 \sim N_t - 1), k' = (0 \sim N_c - 1)\}$  are the Lagrange multipliers. The optimal solution satisfies [20], [21]

$$\left\{ \begin{array}{l} \frac{\partial J}{\partial \sum_{n=0}^{N_t-1} |W(n, k')|^2} = 0 \\ \sum_{k'=0}^{m-1} \sum_{n=0}^{N_t-1} |W(n, k')|^2 - N_c = 0 \\ \sum_{k'=m}^{N_c-1} \sum_{n=0}^{N_t-1} |W(n, k')|^2 - 0 = 0, \quad k' = 0 \sim N_c - 1. \\ \sum_{n=0}^{N_t-1} |W(n, k')|^2 \geq 0 \\ \Psi(n, k') \geq 0 \\ \Psi(n, k') \sum_{n=0}^{N_t-1} |W(n, k')|^2 = 0 \end{array} \right. \quad (22)$$

Using Eqs. (21)–(22), we obtain

$$\sum_{n=0}^{N_t-1} |W(n, k')|^2 = \max \left\{ \varphi_{\text{jointWF-MRT}} - \frac{\left(\frac{E_s}{N_0}\right)^{-1}}{\sum_{n'=0}^{N_t-1} |H(n', k')|^2}, 0 \right\}, \quad (23)$$

where

$$\varphi_{\text{jointWF-MRT}} = \frac{1}{m} \left\{ N_c + \sum_{k'=0}^{m-1} \frac{\left(\frac{E_s}{N_0}\right)^{-1}}{\sum_{n'=0}^{N_t-1} |H(n', k')|^2} \right\}. \quad (24)$$

From Eqs. (18) and Eq. (23) and since  $k'$  is the permuted index,  $W_{\text{jointWF-MRT}}(n, k)$  can be derived as

$$\begin{aligned} & W_{\text{jointWF-MRT}}(n, k) \\ &= \frac{H^*(n, k)}{\sqrt{\sum_{n'=0}^{N_t-1} |H(n', k)|^2}} \left[ \max \left\{ \varphi_{\text{jointWF-MRT}} - \frac{\left(\frac{E_s}{N_0}\right)^{-1}}{\sum_{n'=0}^{N_t-1} |H(n', k)|^2}, 0 \right\} \right]^{\frac{1}{2}}. \end{aligned} \quad (25)$$

In this paper, we call the transmit weight of Eq. (25) as the joint WF-MRT transmit weight.

### 3.3 MRT and WF Transmit Weights

MRT transmit weight which maximizes the received signal-to-noise power ratio (SNR) is given by [10]

$$W_{\text{MRT}}(n, k) = \frac{H^*(n, k)}{\sqrt{\frac{1}{N_c} \sum_{n'=0}^{N_t-1} \sum_{k'=0}^{N_c-1} |H(n', k')|^2}}. \quad (26)$$

Another transmit weight can also be derived based on the water filling theorem [16]. This transmit weight is called WF transmit weight in this paper. The WF transmit weight is derived by applying the water filling theorem to an array of  $N_t N_c$  channel gains,  $[H(0, 0), \dots, H(0, N_c - 1), H(1, 0), \dots, H(1, N_c - 1), \dots, H(N_t - 1, 0), \dots, H(N_t - 1, N_c - 1)]$ . The WF transmit weight  $W_{\text{WF}}(n, k)$  is given by

$$\begin{aligned} & W_{\text{WF}}(n, k) \\ &= \frac{H^*(n, k)}{|H(n, k)|} \left[ \max \left\{ \varphi_{\text{WF}} - \frac{\left(\frac{E_s}{N_0}\right)^{-1}}{|H(n, k)|^2}, 0 \right\} \right]^{\frac{1}{2}}, \end{aligned} \quad (27)$$

where  $\varphi_{\text{WF}}$  is chosen so that  $\sum_{n=0}^{N_t-1} \sum_{k=0}^{N_c-1} |W_{\text{WF}}(n, k)|^2 = N_c$ .

### 3.4 Discussion

When  $N_t = 1$ , Eq. (25) reduces to

$$\begin{aligned} & W_{\text{jointWF-MRT}}(0, k) \\ &= \frac{H^*(0, k)}{|H(0, k)|} \left[ \max \left\{ \varphi_{\text{jointWF-MRT}} - \frac{\left(\frac{E_s}{N_0}\right)^{-1}}{|H(0, k)|^2}, 0 \right\} \right]^{\frac{1}{2}}, \end{aligned} \quad (28)$$

which is identical to the WF transmit weight  $W_{\text{WF}}(n, k)$  given by Eq. (27). This suggests that the transmit power allocation is done across  $N_c$  frequencies based on the WF theorem. On the other hand, when  $N_t > 1$  in a frequency-nonselctive channel (i.e.,  $L = 1$ ),  $H(n, k) = H(n)$  for  $k = (0 \sim N_c - 1)$  and therefore, Eq. (25) reduces to

$$W_{\text{jointWF-MRT}}(n, k) = \frac{H^*(n)}{\sqrt{\sum_{n'=0}^{N_t-1} |H(n')|^2}}, \quad (29)$$

which is identical to the MRT transmit weight  $W_{\text{MRT}}(n, k)$  given by Eq. (26) with  $H(n, k) = H(n)$ . This suggests that the transmit power allocation is done across  $N_t$  transmit antennas based on the MRT strategy.

As a consequence, the joint WF-MRT transmit weight allocates the transmit power in both transmit antenna dimension and frequency dimension; power allocation both across  $N_c$  frequencies based on the WF theorem across and across  $N_t$  transmit antennas based on the MRT strategy.

## 4. Numerical Evaluation

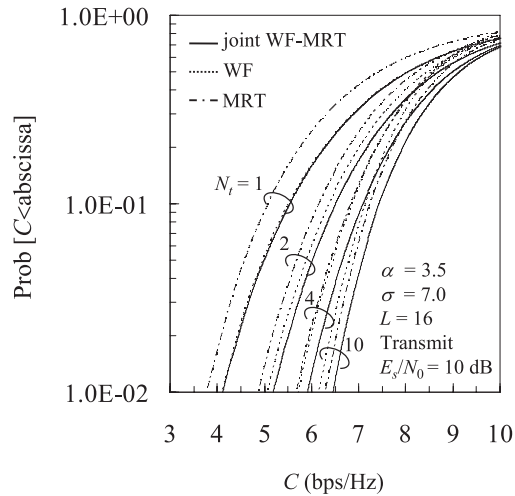
The numerical evaluation condition is summarized in Table 1. The distribution of channel capacity is evaluated by Monte-Carlo numerical computation method. The channel is assumed to be a frequency-selective block Rayleigh fading channel having a symbol-spaced  $L = 16$ -path uniform power delay profile. Ideal channel estimation is assumed. The user location is uniformly distributed over the area of interest.

### 4.1 Comparison of Transmit Weight

Figure 3 shows the cumulative distribution function (CDF)

**Table 1** Numerical evaluation condition.

Power delay profile	Uniform
No. of paths	$L = 16$
Time delay	$\tau_l = l, l = (0 \sim L - 1)$
No. of selected antennas	$N_t = 1, 2, 3, \dots, 10$
Path loss exponent	$\alpha = 3.5$
Shadowing loss standard deviation	$\sigma = 7.0$ (dB)
FFT size	$N_c = 256$
Normalized transmit $E_s/N_0$	$E_s/N_0 = 0, 5, 10$ (dB)
Channel estimation	Ideal

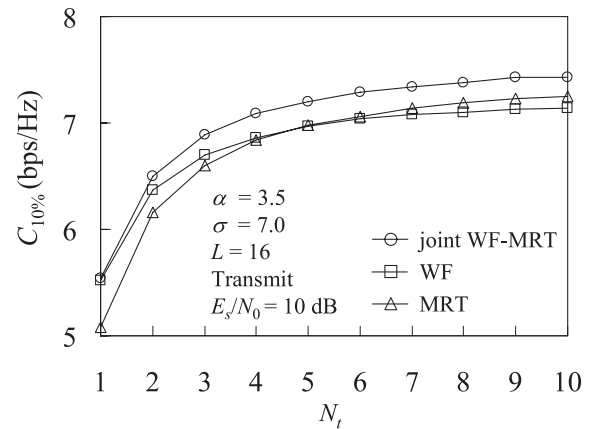


**Fig. 3** CDF of channel capacity.

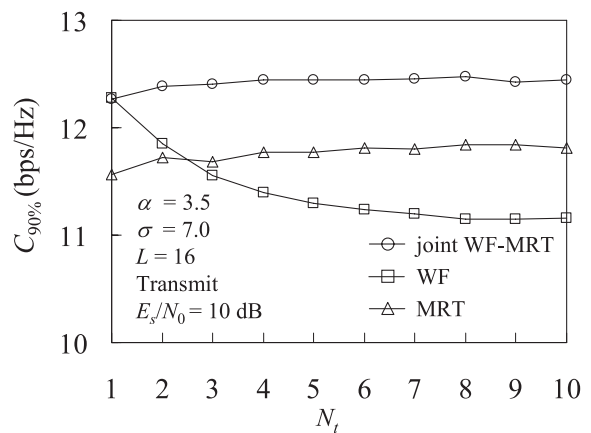
of channel capacity with  $N_t$  as a parameter when the transmit  $E_s/N_0 = 10$  dB for joint WF-MRT transmit weight, WF transmit weight, and MRT transmit weight. From Fig. 3, we obtained the 10% channel capacity  $C_{10\%}$  (below which the channel capacity falls with 10% probability), the 90% channel capacity  $C_{90\%}$  (which the channel capacity exceeds with 10% probability), and the ergodic capacity  $\bar{C}$ . They are plotted in Fig. 4 as a function of  $N_t$ .

It can be seen from Fig. 4 that the joint WF-MRT transmit weight provides the highest channel capacity among three transmit weights. As  $N_t$  increases,  $C_{10\%}$  and  $\bar{C}$  increase (see Figs. 4(a) and (c)); however,  $C_{90\%}$  is almost constant except for the WF transmit weight (see Fig. 4(b)). The reason for this is discussed below. Due to the increased diversity order, the probability that the capacity drops is reduced as  $N_t$  increases. This contributes to the increase in the value of  $C_{10\%}$ . This happens if the MS is far from all antennas. On the other hand, if the MS is close to one of distributed antennas, higher capacity can be obtained. How the transmit diversity can increase the capacity in this case can be discussed using  $C_{90\%}$ . Since most of transmit power is allocated to an antenna to which the MS is close, the diversity gain does not increase that much even if  $N_t$  is increased.

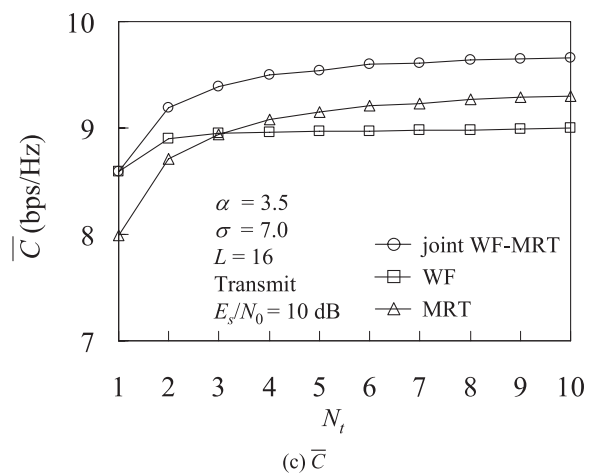
When  $N_t = 1$ , the WF transmit weight becomes identical with the joint WF-MRT transmit weight which is the optimum weight. However, when  $N_t > 1$ , the WF transmit weight is not optimal anymore and therefore, the achievable channel capacity only slightly increases as  $N_t$  increases. On



(a)  $C_{10\%}$

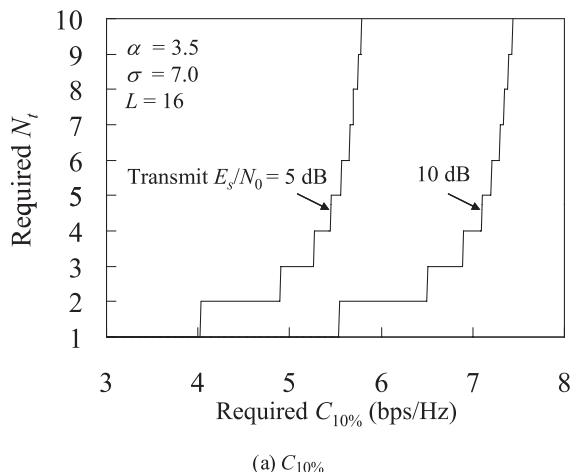
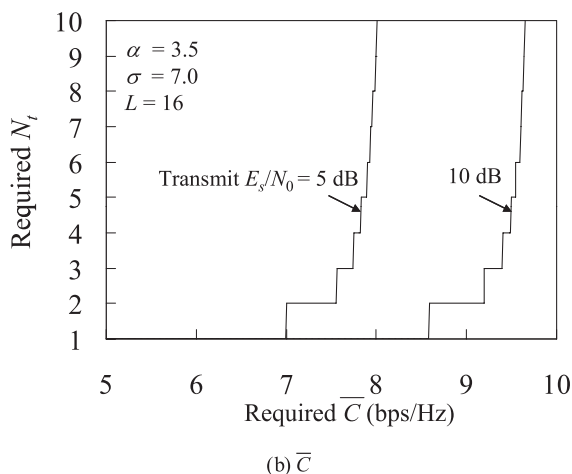


(b)  $C_{90\%}$



**Fig. 4** Channel capacity as a function of  $N_t$ .

the other hand, although the MRT transmit weight is not optimal as well, it can achieve increasing antenna diversity gain as  $N_t$  increases. This is a reason why the WF transmit weight provides larger channel capacity than the MRT

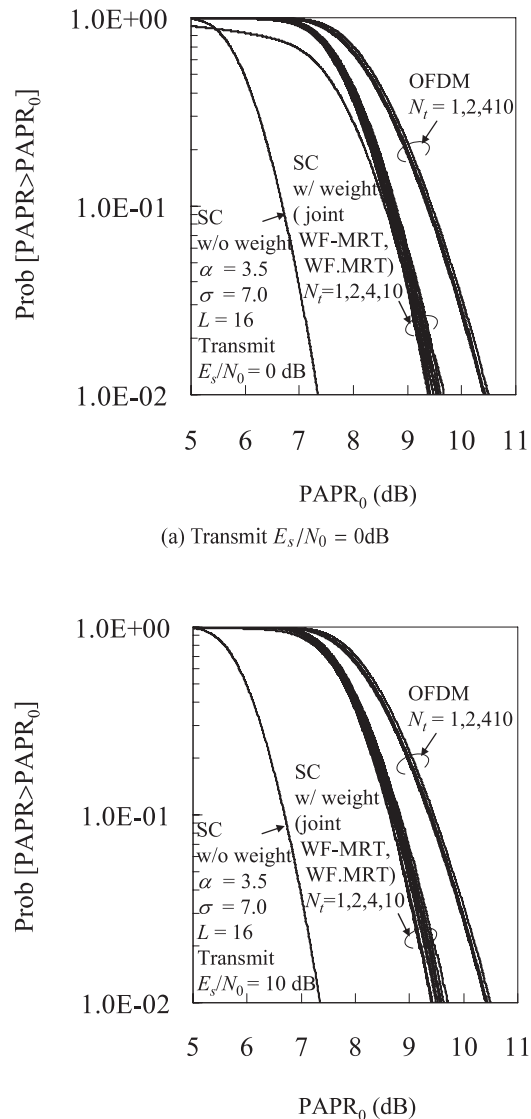
(a)  $C_{10\%}$ (b)  $\bar{C}$ **Fig. 5** Required  $N_t$  for achieving the required channel capacity.

transmit weight when  $N_t = 1$ , but the latter provides larger channel capacity for a large  $N_t$ .

Figure 5 plots the value of  $N_t$  to achieve the required channel capacity ( $C_{10\%}$  and  $\bar{C}$ ) with the transmit  $E_s/N_0$  as a parameter for joint WF-MRT transmit weight. From Fig. 5(a), if we want to achieve the required  $C_{10\%} = 4.0$  bps/Hz,  $N_t = 2$  is needed when the transmit  $E_s/N_0 = 5$  dB, while  $N_t = 1$  is sufficient when the transmit  $E_s/N_0 = 10$  dB. In other words, as  $N_t$  increases from 1 to 2, the transmit  $E_s/N_0$  can be decreased by 5 dB. On the other hand, to achieve the required  $C_{10\%} = 6.0$  bps/Hz when the transmit  $E_s/N_0 = 5$  dB, more than 10 antennas are needed. This is because the distributed antennas excluding the 4 closest antennas are far away from MS (see Fig. 1) and their path losses are very large, therefore their contributions to the diversity gain are negligibly small.

## 4.2 PAPR

PAPR is defined as [12]

(a) Transmit  $E_s/N_0 = 0$  dB(b) Transmit  $E_s/N_0 = 10$  dB**Fig. 6** CCDF of PAPR.

$$\text{PAPR} = \frac{\max \{|s(n, t)|^2\}}{E \left[ |s(n, t)|^2 \right]},$$

$$n = 0, \dots, N_t - 1, t = 0, 1/8, \dots, N_c - 1. \quad (30)$$

In this paper, we assume QPSK modulation. The PAPR was measured generating 8 times oversampled transmit signal waveforms. Fig. 6 plots the complementary CDF (CCDF) of PAPR when transmit  $E_s/N_0 = 0$  and 10 dB. It can be seen from figure that with joint WF-MRT, WF, and MRT transmit weights, the PAPR level of SC signals is increased, but it is still lower than that of OFDM signals. The PAPR level of SC signal at CCDF =  $10^{-2}$  is lower than OFDM signals by about 0.9 dB. Furthermore, it can be seen from Fig. 6 that the same PAPR distribution can be observed irrespective of the transmit weight and transmit  $E_s/N_0$ . The reason for this is left as a future study topic.

## 5. Conclusion

In this paper, we proposed the joint WF-MRT transmit diversity for the SC DAN downlink in a frequency-selective channel. The joint WF-MRT transmit weight allocates the transmit power in both transmit antenna dimension and frequency dimension. We evaluated the CDF of channel capacity by the Monte-Carlo numerical computation method and compared the channel capacities achievable by the joint WF-MRT, WF, and MRT transmit weights. It was shown that the joint WF-MRT transmit weight can achieve the highest channel capacity among the three transmit weights. The use of  $N_t = 4$  antennas is sufficient when transmit antennas are uniformly distributed. Although the PAPR level (at CCDF =  $10^{-2}$ ) of joint WF-MRT transmit weight is about 2.2 dB higher than without transmit weight, it is still lower than OFDM by about 0.9 dB. Furthermore, it was found that joint WF-MRT, WF, and MRT transmit weights provide almost the same PAPR distribution. The reason for this is left as a future study topic.

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## Appendix: Derivation of Eq. (25)

From Eq. (18),

$$W(n, k) = A \cdot H^*(n, k), n = 0 \sim N_t - 1, \quad (\text{A} \cdot 1)$$

where A is constant. Substituting Eq. (A·1) into Eq. (23) gives

$$A^2 \cdot \sum_{n=0}^{N_t-1} |H(n, k')|^2 = \max \left\{ \varphi_{\text{jointWF-MRT}} \frac{\left(\frac{E_s}{N_0}\right)^{-1}}{\sum_{n'=0}^{N_t-1} |H(n', k')|^2}, 0 \right\}. \quad (\text{A} \cdot 2)$$

From Eqs. (A·1) and (A·2), Eq. (25) is derived.



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