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Joint transmit/receive one-tap minimum mean square error frequency-domain equalisation for broadband multicode direct-sequence code division multiple access

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Abstract: Multicode direct-sequence code division multiple access (DS-CDMA) can flexibly support multimedia services with various data rates simply by changing the code multiplexing order. The use of simple one-tap frequency-domain equalisation (FDE) at a receiver is known to improve the bit error rate (BER) performance of multicode DS-CDMA in a severe frequency-selective fading environment. However, the BER performance improvement is limited due to the presence of the residual inter-chip interference (ICI). The authors propose a joint transmit/receive minimum mean square error (MMSE) FDE, which carries out one-tap transmit FDE and one-tap receive FDE jointly based on the MMSE criterion. The authors theoretically derive a suboptimal set of transmit and receive FDE weights and investigate the BER performance improvement by computer simulation in a freqeuncy-selective Rayleigh fading channel. The proposed scheme improves the received signal-to-interference plus noise ratio after despreading, and consequently, the BER performance can be significantly improved compared to the conventioal receive MMSE-FDE by (a) making the variations in the equivalent channel gain shallower to reduce the residual ICI for large code multiplexing order U and (b) allocating the transmit power to the frequencies having good condition to improve the received signal-to-noise ratio for a small U. This is conformed by the computer simualtion.

1 Introduction

Multicode direct-sequence code division multiple access (DS-CDMA) can flexibly support multimedia services with various data rates simply by changing the code multiplexing order. In the third generation (3G) mobile communication systems, multicode DS-CDMA using coherent rake combining is adopted to support data services of up to a few Mbps [1]. The use of coherent rake combining can take advantage of the channel frequency selectivity and achieves the frequency diversity gain (or path diversity gain), which contributes to improve the achievable bit error rate (BER) performance. However, this is true only for the case where a moderate number of propagation paths exist.

Broadband packet data services of transmission rates close to 1 Gbps are demanded in the next generation wireless communication systems [2]. Broadband wireless channel comprises many propagation paths having different time delays and becomes a severe frequency-selective fading channel [3]. For a broadband packet access, the chip rate needs to be significantly increased. As the chip rate increases, the number of resolvable propagation paths tends to increase and the inter-chip interference (ICI) gets stronger, thereby severely degrading the achievable BER performance.

Recently, multi-carrier (MC) signal transmission (i.e. orthogonal frequency division multiplexing (OFDM) and MC-CDMA) has been attracting attention [4–7]. In

OFDM, data symbols are transmitted in parallel using a number of narrowband orthogonal subcarriers. In MC-CDMA which is a combination of OFDM and frequency-domain spreading, simple one-tap frequency-domain equalisation (FDE) based on the minimum mean square error (MMSE) criterion is used before despreading. One-tap MMSE-FDE exploits the channel frequency selectivity and achieves a good BER performance.

One-tap MMSE-FDE is also applicable to DS-CDMA [8–15]. Coherent rake combining can be replaced by MMSE-FDE to achieve significantly improved BER performance in a severe frequency-selective fading channel. When using one-tap MMSE-FDE, the received time-domain chip block is transformed into the frequency-domain signal block by using fast Fourier transform (FFT) and one-tap MMSE-FDE is carried out. After MMSE-FDE, inverse FFT (IFFT) is applied to transform the frequency-domain signal block into the time-domain chip block. One-tap MMSE-FDE can reduce the ICI and can significantly improve the BER performance compared to coherent rake combining. Instead of using the receive MMSE-FDE, the transmit MMSE-FDE (or pre-MMSE-FDE) [16–20] can be used to improve the BER performance of DS-CDMA. However, with the single use of the transmit FDE or the receive FDE, the BER performance improvement is limited due to the presence of residual ICI after despreading [9–12, 17]. In [21], how the residual ICI after MMSE-FDE limits the BER performance improvement was discussed for multicode DS-CDMA.

One approach for reducing the residual ICI is the introduction of an iterative ICI cancellation to the receive MMSE-FDE [22–24]. In each iteration stage, the residual ICI replica is generated using the log-likelihood ratio obtained from the decision results of the previous stage and is subtracted from the received frequency-domain signal before re-performing MMSE-FDE. However, the use of iterative ICI cancellation technique increases the computational complexity in the receiver.

In a communication system using time division duplex, the channel state information (CSI) is available at the transmitter and therefore, the transmit FDE can be employed to further improve the BER performance. Provided that CSI is available at both the transmitter and the receiver, one-tap transmit FDE and one-tap receive FDE can be carried out jointly based on the MMSE criterion. Since the transmit/receive FDE weights interact with each other, it is quite difficult, if not impossible, to derive the optimal set of FDE weights. In [25], iterative joint optimisation of transmit/receive frequency-domain equalisation using the least mean square algorithm was proposed for a non-spread single-carrier signal transmission (equivalent to DS-CDMA with spreading factor, SF = 1). However, no theoretical analysis was given to find an optimal set of transmit and receive FDE weights.

In this paper, we propose a joint transmit/receive MMSE-FDE which can suppress the residual ICI and improve the BER performance of multicode DS-CDMA compared to the conventional receive MMSE-FDE. We theoretically derive a suboptimal set of transmit and receive FDE weights. First, we derive the receive MMSE-FDE weight by viewing a concatenation of the transmit FDE and the propagation channel as an equivalent channel. Second, we derive the transmit MMSE-FDE weight that minimises the total mean square error (MSE) over the received signal block when using the above receive MMSE-FDE. Using the Gaussian approximation of the residual ICI, we derive the conditional BER expression when using the proposed joint transmit/receive MMSE-FDE for the given channel realisation. The achievable average BER performance is evaluated by Monte-Carlo numerical computation method using the derived conditional BER expression and is confirmed by computer simulation. Turbo-coded BER performance when using the proposed joint transmit/receive MMSE-FDE is also evaluated by computer simulation.

The rest of the paper is organised as follows. Section 2 presents the signal representation of multicode DS-CDMA using the proposed joint transmit/receive MMSE-FDE. A suboptimal set of transmit/receive MMSE weights is derived in Section 3. Section 4 derives the conditional BER expression. The numerical and simulation results are discussed in Section 5. Section 6 concludes this paper.

2 Signal representation

2.1 System model

Fig. 1 illustrates the transmission system model of multicode DS-CDMA using the proposed joint transmit/receive MMSE-FDE. Throughout the paper, chip-spaced discrete-time signal representation is used. It is assumed that the perfect CSI is available at both the transmitter and receiver.

2.2 Transmit signal

At the transmitter, an information bit sequence is transformed into a data-modulated symbol sequence, which is serial-to-parallel (S/P) converted into U parallel streams $\{d_u(i);\ i = \ldots, -1, 0, 1, \ldots\}$, $u = 0 \sim U-1$. Then, each stream is spread by using an orthogonal spreading code with spreading factor (SF), $\{c_u(t);\ t = 0 \sim \mathrm{SF}-1\}$, $u = 0 \sim U-1$. The resulting U chip sequences are combined to form the multicode chip sequence, which is further multiplied by a scramble sequence $\{c_{\mathrm{scr}}(t);\ t = \ldots, -1, 0, 1, \ldots\}$ to obtain the multicode DS-CDMA chip block. The resulting chip block can be expressed using the vector form as $\boldsymbol{x} = [x(0), \ldots, x(t), \ldots, x(N_c-1)]^{\mathrm{T}}$ with

$$x(t) = \sum_{u=0}^{U-1} d_u\left(\left\lfloor \frac{t}{\mathrm{SF}} \right\rfloor\right) c_u(t \bmod \mathrm{SF})\, c_{\mathrm{scr}}(t) \qquad (1)$$

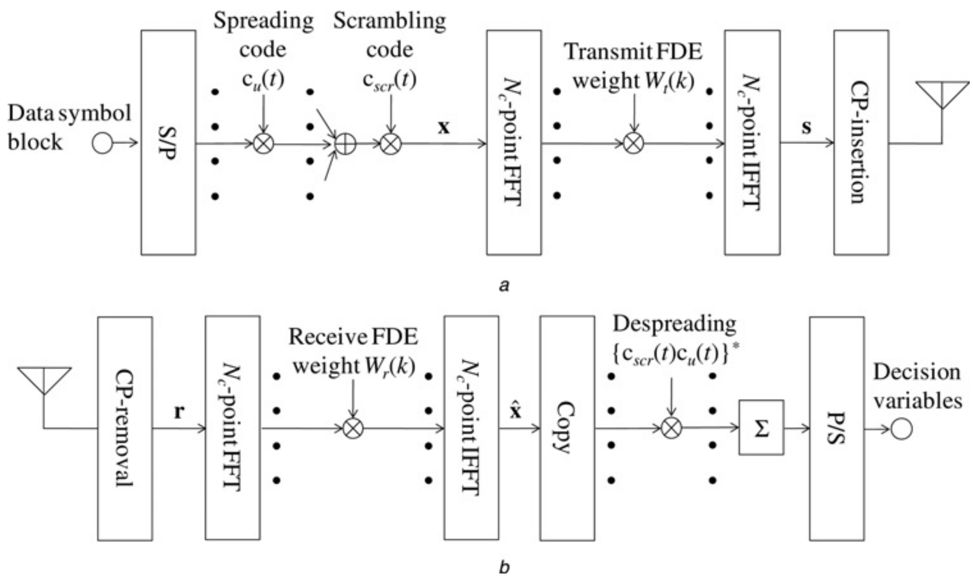

Figure 1 *Transmission system model of multicode DS-CDMA using proposed joint transmit/receive MMSE-FDE*
a Transmitter
b Receiver

The chip vector $\boldsymbol{x}$ is transformed by using an N_c-point FFT into the frequency-domain signal vector $\boldsymbol{X} = [X(0), \ldots, X(k), \ldots, X(N_c - 1)]^T$ as

$$\boldsymbol{X} = \boldsymbol{F}\boldsymbol{x} \tag{2}$$

where $\boldsymbol{F}$ is an $N_c \times N_c$ FFT matrix given as

$$\boldsymbol{F} = \frac{1}{\sqrt{N_c}}\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{-j2\pi((1\times 1)/N_c)} & \cdots & e^{-j2\pi((1\times(N_c-1))/N_c)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j2\pi(((N_c-1)\times 1)/N_c)} & \cdots & e^{-j2\pi(((N_c-1)\times(N_c-1))/N_c)} \end{bmatrix} \tag{3}$$

with $\boldsymbol{F}^H\boldsymbol{F} = \boldsymbol{F}\boldsymbol{F}^H = \boldsymbol{I}((\cdot)^H$ represents the Hermitian transpose and $\boldsymbol{I}$ denotes an $N_c \times N_c$ unit diagonal matrix).

One-tap transmit FDE weight $\boldsymbol{W}_t(k)$ is multiplied to $\boldsymbol{X}(k)$, $k = 0 \sim N_c - 1$. The frequency-domain signal vector $\boldsymbol{S} = [S(0), \ldots, S(k), \ldots, S(N_c - 1)]^T$ after performing the transmit FDE is given as

$$\boldsymbol{S} = C\boldsymbol{W}_t\boldsymbol{X} \tag{4}$$

where $\boldsymbol{W}_t = \text{diag}\{W_t(0), \ldots, W_t(k), \ldots, W_t(N_c - 1)\}$ is the transmit weight matrix and C is the transmit power normalisation factor, which is introduced to keep the average transmit power intact, and is given as

$$C = \sqrt{\frac{N_c}{\text{tr}(\boldsymbol{W}_t\boldsymbol{W}_t^H)}} \tag{5}$$

An N_c-point IFFT is applied to $\boldsymbol{S}$ to obtain the transmit time-domain signal vector $\boldsymbol{s} = [s(0), \ldots, s(t), \ldots, s(N_c - 1)]^T = \boldsymbol{F}^H\boldsymbol{S}$. After the insertion of N_g-sample cyclic prefix (CP) into the guard interval (GI), the signal block is transmitted over a frequency-selective channel.

2.3 Received signal

The propagation channel is assumed to be a chip-spaced L-path frequency-selective block fading channel. The complex-valued path gain and the delay time of the lth path are denoted by h_l and τ_l, $l = 0 \sim L - 1$, respectively. The CP length is assumed to be the same or longer than the maximum channel delay time, $\tau_L - 1$. The received signal block $\boldsymbol{r} = [r(0), \ldots, r(t), \ldots, r(N_c - 1)]^T$ after the CP removal can be expressed as

$$\boldsymbol{r} = \sqrt{\frac{2E_c}{T_c}}\boldsymbol{h}\boldsymbol{s} + \boldsymbol{n} \tag{6}$$

where E_c and T_c are the average transmit chip energy and chip duration, respectively, $\boldsymbol{h}$ is an $N_c \times N_c$ circulant channel matrix given by

$$\boldsymbol{h} = \begin{bmatrix} h_0 & & & h_{L-1} & \cdots & h_1 \\ h_1 & \ddots & & & \ddots & \vdots \\ \vdots & & h_0 & 0 & & h_{L-1} \\ h_{L-1} & & h_1 & \ddots & & \\ & \ddots & \vdots & & \ddots & \\ 0 & & h_{L-1} & \cdots & \cdots & h_0 \end{bmatrix} \tag{7}$$

and $\boldsymbol{n} = [n(0), \ldots, n(t), \ldots, n(N_c - 1)]^T$ is the noise vector with $n(t)$ being an independent zero-mean additive white Gaussian noise having variance $2N_0/T_c$. (N_0 is the one-sided noise power spectrum density.)

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N_c-point FFT is carried out on $\boldsymbol{r}$ to obtain the frequency-domain received signal vector $\boldsymbol{R} = [R(0), \ldots, R(k), \ldots, R(N_c - 1)]^T$ as

$$\boldsymbol{R} = \boldsymbol{F}\boldsymbol{r} = \sqrt{\frac{2E_c}{T_c}}\boldsymbol{C}\boldsymbol{H}\boldsymbol{W}_t\boldsymbol{X} + \boldsymbol{N} \tag{8}$$

where $\boldsymbol{H} = \boldsymbol{F}\boldsymbol{h}\boldsymbol{F}^H$ and $\boldsymbol{N} = \boldsymbol{F}\boldsymbol{n}$. Owing to the circulant property of $\boldsymbol{h}$, the $N_c \times N_c$ channel matrix $\boldsymbol{H}$ is diagonal [26]. The kth diagonal element of $\boldsymbol{H}$ is given by

$$H(k) = \sum_{l=0}^{L-1} h_l \exp\left(-j2\pi k\frac{\tau_l}{N_c}\right) \tag{9}$$

One-tap receive FDE weight $W_r(k)$ is multiplied by $R(k)$, $k = 0 \sim N_c - 1$. The frequency-domain signal vector $\hat{\boldsymbol{X}} = [\hat{X}(0), \ldots, \hat{X}(t), \ldots, \hat{X}(N_c - 1)]^T$ after receive FDE is given as $\hat{\boldsymbol{X}} = \boldsymbol{W}_r\boldsymbol{R}$, where $\boldsymbol{W}_r = \mathrm{diag}\{W_r(0), \ldots, W_r(k), \ldots, W_r(N_c - 1)\}$ is the receive weight matrix. Then, an N_c-point IFFT is carried out on $\hat{\boldsymbol{X}}$ to obtain the equalised DS-CDMA chip vector $\hat{\boldsymbol{x}} = [\hat{x}(0), \ldots, \hat{x}(t), \ldots, \hat{x}(N_c - 1)]^T$ as

$$\hat{\boldsymbol{x}} = \boldsymbol{F}^H\hat{\boldsymbol{X}} = \sqrt{\frac{2E_c}{T_c}}\boldsymbol{C}\boldsymbol{F}^H\boldsymbol{W}_r\boldsymbol{H}\boldsymbol{W}_t\boldsymbol{X} + \boldsymbol{F}^H\boldsymbol{W}_r\boldsymbol{N} \tag{10}$$

Finally, despreading is done on $\hat{\boldsymbol{x}}$ to obtain the decision variable $\hat{d}_u(i)$ to estimate the transmitted symbol $d_u(i)$ as

$$\hat{d}_u(i) = \frac{1}{\mathrm{SF}}\sum_{t=i\mathrm{SF}}^{(i+1)\mathrm{SF}-1} \hat{x}(t)c_{\mathrm{scr}}^*(t)c_u^*(t \bmod \mathrm{SF}) \tag{11}$$

3 Derivation of suboptimal set of transmit/receive FDE weights

3.1 Mean square error

We introduce the relative equalisation error to derive the transmit MMSE-FDE weight [17, 18]. The error vector $\boldsymbol{e} = [e(0), \ldots, e(t), \ldots, e(N_c - 1)]^T$ between the transmit chip vector $\boldsymbol{x}$ and the equalised transmit chip vector $\hat{\boldsymbol{x}}$ normalised by the transmit signal amplitude is defined as

$$\boldsymbol{e} = \frac{\hat{\boldsymbol{x}} - \sqrt{(2E_c/T_c)}\boldsymbol{C}\boldsymbol{x}}{\sqrt{(2E_c/T_c)(1/N_c)\mathrm{tr}[E(\boldsymbol{x}\boldsymbol{x}^H)]}\boldsymbol{C}} \tag{12}$$

Using (10) and (12), we can show that the total MSE is given as

$$\begin{aligned} e(\boldsymbol{W}_t, \boldsymbol{W}_r) &= \mathrm{tr}[E(\boldsymbol{e}\boldsymbol{e}^H)] \\ &= N_c\mathrm{tr}[(\boldsymbol{W}_r\boldsymbol{H}\boldsymbol{W}_t - \boldsymbol{I})(\boldsymbol{W}_r\boldsymbol{H}\boldsymbol{W}_t - \boldsymbol{I})^H] \\ &\quad + \left(\frac{U}{\mathrm{SF}}\frac{E_s}{N_0}\right)^{-1}\mathrm{tr}[\boldsymbol{W}_t\boldsymbol{W}_t^H]\mathrm{tr}[\boldsymbol{W}_r\boldsymbol{W}_r^H] \end{aligned} \tag{13}$$

where E_s is the average transmit symbol energy. If we set $\boldsymbol{W}_r = \boldsymbol{I}$ (or $\boldsymbol{W}_t = \boldsymbol{I}$), we obtain the conventional transmit MMSE-FDE weight (or receive MMSE-FDE weight) by solving $\partial e(\boldsymbol{W}_t, \boldsymbol{I})/\partial\boldsymbol{W}_t = 0$ (or $\partial e(\boldsymbol{I}, \boldsymbol{W}_r)/\partial\boldsymbol{W}_r = 0$) as

$$\{\boldsymbol{W}_t, \boldsymbol{W}_r\} = \begin{cases} \left\{\left[\boldsymbol{H}^H\boldsymbol{H} + \left(\frac{U}{\mathrm{SF}}\frac{E_s}{N_0}\right)^{-1}\boldsymbol{I}\right]^{-1}\boldsymbol{H}^H, \ \boldsymbol{I}\right\} & \text{transmit MMSE} - \text{FDE} \\ \left\{\boldsymbol{I}, \ \left[\boldsymbol{H}^H\boldsymbol{H} + \left(\frac{U}{\mathrm{SF}}\frac{E_s}{N_0}\right)^{-1}\boldsymbol{I}\right]^{-1}\boldsymbol{H}^H\right\} & \text{receive MMSE} - \text{FDE} \end{cases} \tag{14}$$

For the joint transmit/receive MMSE-FDE, $\boldsymbol{W}_t$ and $\boldsymbol{W}_r$ interact with each other and hence, it is quite difficult, if not impossible, to derive the optimal set of MMSE weights. Therefore we derive a suboptimal set of FDE weights. First, we derive the receive MMSE-FDE weight according to the Wiener filter theory [27] by viewing a concatenation of the transmit FDE and the propagation channel as an equivalent channel. Second, we derive the transmit MMSE-FDE weight that minimises the total MSE over the received signal block when using the above receive MMSE-FDE. In the next subsection, we will derive a suboptimal set of MMSE weights.

3.2 Transmit/receive MMSE weight matrices

3.2.1 Receive MMSE weight matrix: A concatenation of the transmit one-tap FDE $\boldsymbol{W}_t$ and the propagation channel $\boldsymbol{H}$ is viewed as an equivalent channel $\boldsymbol{H}\boldsymbol{W}_t$. The MMSE solution to the receive FDE weight matrix $\boldsymbol{W}_r$ can be derived using (13) as

$$\boldsymbol{W}_r = \left[\boldsymbol{W}_t^H\boldsymbol{H}^H\boldsymbol{H}\boldsymbol{W}_t + \frac{1}{N_c}\left(\frac{U}{\mathrm{SF}}\frac{E_s}{N_0}\right)^{-1} \times \mathrm{tr}[\boldsymbol{W}_t\boldsymbol{W}_t^H]\cdot\boldsymbol{I}\right]^{-1}\boldsymbol{W}_t^H\boldsymbol{H}^H \tag{15}$$

3.2.2 Optimality condition for deriving transmit MMSE weight matrix: Substituting (15) into (13) gives

$$\begin{aligned} e(\boldsymbol{W}_t) = \frac{1}{N_c}\left(\frac{U}{\mathrm{SF}}\frac{E_s}{N_0}\right)^{-1}\mathrm{tr}[\boldsymbol{W}_t\boldsymbol{W}_t^H] &\times \mathrm{tr}\left[\left(\boldsymbol{W}_t\boldsymbol{W}_t^H\boldsymbol{H}^H\boldsymbol{H} + \frac{1}{N_c}\left(\frac{U}{\mathrm{SF}}\frac{E_s}{N_0}\right)^{-1}\right.\right. \\ &\left.\left.\times\,\mathrm{tr}[\boldsymbol{W}_t\boldsymbol{W}_t^H]\cdot\boldsymbol{I}\right)^{-1}\right] \end{aligned} \tag{16}$$

The desired transmit FDE weight matrix $\boldsymbol{W}_t$ is the one which minimises the above $e(\boldsymbol{W}_t)$. It can be understood that the phase of each diagonal element of $\boldsymbol{W}_t$ can be arbitrary since $\boldsymbol{H}^H\boldsymbol{H}$ is a real-valued diagonal matrix and $e(\boldsymbol{W}_t)$ is a function of

$\{|W_t(k)|^2;\ k = 0 \sim N_c - 1\}$. Hence, a real-valued diagonal matrix is sufficient for the transmit FDE matrix.

Since $e(\boldsymbol{W}_t)$ is a convex function of the transmit FDE weight, the global optimum solution that minimises $e(\boldsymbol{W}_t)$ of (16) under the transmit power constraint can be derived. Let us introduce a power constraint $\mathrm{tr}[\boldsymbol{W}_t\boldsymbol{W}_t^H] = N_c$ with $|W_t(k)|^2 \geq 0$ for $k = 0 \sim N_c - 1$. The optimality condition can be expressed as follows

$$\min e(\boldsymbol{W}_t)$$
$$\text{s.t. } \mathrm{tr}[\boldsymbol{W}_t\boldsymbol{W}_t^H] = N_c \text{ and } 0 \leq |W_t(k)|^2 \quad \text{for } k = 0 \sim N_c - 1 \tag{17}$$

which can be solved by the non-linear programming and the solution satisfies Karush–Kuhn–Tucker (KKT) condition [28–30]. To derive the optimal $\boldsymbol{W}_t$, first, diagonal elements of $\boldsymbol{H}$ are permutated in descending order of $|H(k)|$, $k = 0 \sim N_c - 1$, and the permutated diagonal matrix $\boldsymbol{G}$ is introduced as $\boldsymbol{G} = \mathrm{diag}\{G(0), \ldots, G(q), \ldots, G(N_c - 1)\}$ (i.e. $|G(0)| \geq |G(1)| \geq \ldots \geq |G(N_c - 1)|$ and $G(q) = H(q')$, $q = 0 \sim N_c - 1$, $q' = 0 \sim N_c - 1$). The same ordering permutation as $\boldsymbol{H} \rightarrow \boldsymbol{G}$ is applied to the diagonal elements of $\boldsymbol{W}_t\boldsymbol{W}_t^H$, where we introduce the permutated diagonal matrix as $\boldsymbol{P} = \mathrm{diag}\{P(0), \ldots, P(q), \ldots, P(N_c - 1)\}$. The optimality condition given by (17) can be rewritten, by replacing $(1/N_c)((U/\mathrm{SF})(E_s/N_0))^{-1}$ with Ω, as

$$\min e(\boldsymbol{P}) = \Omega\mathrm{tr}[\boldsymbol{P}] \times \mathrm{tr}[(\boldsymbol{P}\boldsymbol{G}\boldsymbol{G}^H + \Omega\,\mathrm{tr}[\boldsymbol{P}]\cdot\boldsymbol{I})^{-1}]$$
$$= \sum_{q=0}^{N_c-1} \frac{\Omega\sum_{q'=0}^{N_c-1} P(q')}{P(q)|G(q)|^2 + \Omega\sum_{q'=0}^{N_c-1} P(q')}$$
$$\text{s.t. } \sum_{q=0}^{N_c-1} P(q) = N_c \text{ and } \{P(q);\ q = 0 \sim N_c - 1\} \geq 0 \tag{18}$$

3.2.3 Derivation of transmit MMSE weight matrix: Below, the optimum solution for the condition (18) is represented by $\boldsymbol{P}_0 = \mathrm{diag}\{P_0(0), \ldots, P_0(q), \ldots, P_0(N_c - 1)\}$. We assume that $\boldsymbol{P}_0$ has m non-zero diagonal elements and $(N_c - m)$ zero diagonal elements $(0 < m \leq N_c)$. Since $e(\boldsymbol{P}_0)$ is a monotonically decreasing function of $|G(q)|^2$, $q = 0 \sim N_c - 1$, it can be said that $P_0(q) \neq 0$ for $q = 0 \sim m - 1$ and $P_0(q) = 0$ for $q = m \sim N_c - 1$. The optimality condition (18) can be rewritten as

$$\min e(\boldsymbol{P}) = \sum_{q=0}^{N_c-1} \frac{\Omega\sum_{q'=0}^{N_c-1} P(q')}{P(q)|G(q)|^2 + \Omega\sum_{q'=0}^{N_c-1} P(q')}$$
$$\text{s.t.} \begin{cases} g_1(\boldsymbol{P}) = \sum_{q=0}^{m-1} P(q) - N_c = 0 \\ g_2(\boldsymbol{P}) = \sum_{q=m}^{N_c-1} P(q) - 0 = 0 \\ f(P(q)) = -P(q) \leq 0, \quad q = 0 \sim N_c - 1 \end{cases} \tag{19}$$

Lagrangian function J can be expressed as

$$J = e(\boldsymbol{P}) + \kappa \cdot g_1(\boldsymbol{P}) + \mu \cdot g_2(\boldsymbol{P}) + \sum_{q=0}^{N_c-1} \psi_q \cdot f(P(q))$$
$$= \sum_{q=0}^{N_c-1} \frac{\Omega\sum_{q'=0}^{N_c-1} P(q')}{P(q)|G(q)|^2 + \Omega\sum_{q'=0}^{N_c-1} P(q')} + \kappa \cdot \left\{\sum_{q=0}^{m-1} P(q) - N_c\right\} + \mu \cdot \left\{\sum_{q=m}^{N_c-1} P(q) - 0\right\} + \sum_{q=0}^{N_c-1} \psi_q \cdot \{-P(q) - 0\} \tag{20}$$

where κ, μ and $\{\psi_q;\ q = 0 \sim N_c - 1\}$ are the Lagrangian multipliers. $\boldsymbol{P}_0$ must satisfy the KKT condition [28, 29]. We obtain

$$\left.\frac{\partial J}{\partial P(q)}\right|_{P_0(q)} = 0, \quad q = 0 \sim N_c - 1 \tag{21}$$
$$\sum_{q=0}^{m-1} P_0(q) - N_c = 0 \tag{22}$$
$$\sum_{q=m}^{N_c-1} P_0(q) = 0 \tag{23}$$
$$P_0(q) \geq 0, \quad q = 0 \sim N_c - 1 \tag{24}$$
$$\psi_q \geq 0, \quad q = 0 \sim N_c - 1 \tag{25}$$

and

$$\psi_q P_0(q) = 0, \quad q = 0 \sim N_c - 1 \tag{26}$$

Using (19)–(26), we obtain

$$P_0(q) = \theta_{m-1}\frac{\sqrt{\Omega}}{|G(q)|} - \frac{\Omega N_c}{|G(q)|^2}, \quad q = 0 \sim m - 1 \tag{27}$$

and $P_0(q) = 0$ for $q = m \sim N_c - 1$, where

$$\theta_{m-1} = N_c\left(1 + \sum_{q=0}^{m-1}\frac{\Omega}{|G(q)|^2}\right)\left(\sqrt{\Omega}\sum_{q=0}^{m-1}\frac{1}{|G(q)|}\right)^{-1} \tag{28}$$

To obtain $\boldsymbol{P}_0$, the value of m must be found. The necessary condition for the value of m is given in Appendix 1. A simple algorithm to find the value of m is presented in Appendix 2.

After applying the converse permutation to $\boldsymbol{P}_0$, we obtain

$$\boldsymbol{W}_t\boldsymbol{W}_t^H = \mathrm{diag}\{|W_t(0)|^2, \ldots, |W_t(k)|^2, \ldots, |W_t(N_c - 1)|^2\} \tag{29}$$

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with

$$|W_t(k)|^2 = \max\left\{\left(\frac{\theta_{m-1}\sqrt{\Omega}}{|H(k)|} - \frac{N_c\Omega}{|H(k)|^2}\right), 0\right\} \quad (30)$$

where $\mathrm{tr}[\boldsymbol{W}_t\boldsymbol{W}_t^H] = N_c$. $\boldsymbol{W}_t$ is the diagonal matrix, and therefore the proposed joint transmit/receive MMSE-FDE still retains low complexity property.

4 Conditional BER analysis

We derive the conditional signal-to-noise power ratio of the multicode DS-CDMA for the given channel realisation. When using the proposed joint transmit/receive MMSE-FDE, the equalised chip vector $\hat{\boldsymbol{x}}$ of (10) is given as

$$\hat{\boldsymbol{x}} = \sqrt{\frac{2E_c}{T_c}}\boldsymbol{F}^H\hat{\boldsymbol{H}}\boldsymbol{F}\boldsymbol{x} + \boldsymbol{F}^H\boldsymbol{W}_r\boldsymbol{N} \quad (31)$$

where $\hat{\boldsymbol{H}} = \boldsymbol{W}_r\boldsymbol{H}\boldsymbol{W}_t$ is an $N_c \times N_c$ diagonal matrix whose (k, k)th element is given by $\hat{H}(k) = W_r(k)H(k)W_t(k)$. The diagonal and off-diagonal elements of $\boldsymbol{F}^H\hat{\boldsymbol{H}}\boldsymbol{F}$ give the desired and residual ICI components, respectively. The tth element $\hat{x}(t)$ of $\hat{\boldsymbol{x}}$ is given as

$$\hat{x}(t) = \sqrt{\frac{2E_c}{T_c}}\left(\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{H}(k)\right)x(t) + \sqrt{\frac{2E_c}{T_c}}\frac{1}{N_c} \times \sum_{k=0}^{N_c-1}\hat{H}(k)\sum_{\substack{\tau=0\\ \tau\neq t}}^{N_c-1}x(\tau)\exp\left(j2\pi k\frac{t-\tau}{N_c}\right) + n(t) \quad (32)$$

Using (11) and (32), the decision variable $\hat{d}_u(i)$ for the ith transmit symbol $d_u(i)$ can be expressed as

$$\hat{d}_u(i) = \sqrt{\frac{2E_c}{T_c}}\left(\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{H}(k)\right)d_u(i) + \mu_{ICI} + \mu_n \quad (33)$$

where μ_{ICI} and μ_n are, respectively, the residual ICI and noise, and are given as

$$\begin{cases}\mu_{ICI} = \dfrac{1}{SF}\sqrt{\dfrac{2E_c}{T_c}}\displaystyle\sum_{t=iSF}^{(i+1)SF-1}\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{H}(k)\sum_{\substack{\tau=0\\ \tau\neq t}}^{N_c-1}x(\tau) \times \exp\left(j2\pi k\frac{t-\tau}{N_c}\right)\\ \mu_n = \dfrac{1}{SF}\displaystyle\sum_{t=iSF}^{(i+1)SF-1}\frac{1}{N_c}\sum_{k=0}^{N_c-1}W_r(k)\sum_{\tau=0}^{N_c-1}n(\tau)\exp\left(j2\pi k\frac{t-\tau}{N_c}\right)\end{cases} \quad (34)$$

It can be understood from (33) and (34) that $\hat{d}_u(i)$ is a random variable with mean $\sqrt{2E_c/T_c}(1/N_c)\sum_{k=0}^{N_c-1}\hat{H}(k)\, d_u(i)$. As the scramble sequence is used to make the multicode DS-CDMA chip signal white-noise like, μ_{ICI} can be approximated as a zero-mean complex-valued Gaussian variable. The sum of μ_{ICI} and μ_n can be treated as a new zero-mean complex-valued Gaussian variable μ. The variance of μ is given by

$$\begin{aligned}2\sigma_\mu^2 &= E[|\mu_{ICI}|^2] + E[|\mu_n|^2]\\ &= \frac{1}{SF}\frac{2N_0}{T_c}\left[\frac{1}{N_c}\sum_{k=0}^{N_c-1}|W_r(k)|^2 + \left(\frac{U}{SF}\frac{E_s}{N_0}\right)\right.\\ &\quad\left.\times\left\{\frac{1}{N_c}\sum_{k=0}^{N_c-1}|\hat{H}(k)|^2 - \left|\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{H}(k)\right|^2\right\}\right]\end{aligned} \quad (35)$$

Using the Gaussian approximation of the residual ICI, the conditional BER for the given channel realisation of $\boldsymbol{H}$ can be given as [31]

$$p_b\left(\frac{E_s}{N_0}, \boldsymbol{H}\right) = \begin{cases}\dfrac{1}{2}\mathrm{erfc}\left[\sqrt{\dfrac{1}{4}\gamma(E_s/N_0, \boldsymbol{H})}\right] & \text{for QPSK}\\ \dfrac{3}{8}\mathrm{erfc}\left[\sqrt{\dfrac{1}{20}\gamma(E_s/N_0, \boldsymbol{H})}\right] + \dfrac{1}{4}\mathrm{erfc}\left[\sqrt{\dfrac{9}{20}\gamma(E_s/N_0, \boldsymbol{H})}\right] & \\ -\dfrac{1}{8}\mathrm{erfc}\left[\sqrt{\dfrac{5}{4}\gamma(E_s/N_0, \boldsymbol{H})}\right] & \text{for 16QAM}\end{cases} \quad (36)$$

where $\mathrm{erfc}[x] = (2/\sqrt{\pi})\int_x^\infty \exp(-t^2)dt$ is the complementary error function and $\gamma(E_s/N_0, \boldsymbol{H})$ denotes the conditional signal-to-interference plus noise ratio (SINR) given as (see (37))

The achievable average BER can be numerically evaluated by averaging (36) over all possible realisations of $\boldsymbol{H}$.

5 Numerical and simulation results

5.1 Numerical and simulation condition

Numerical and simulation condition is shown in Table 1. A block transmission using $N_c = 256$ and $N_g = 32$ is considered. The spreading factor is set to SF = 256 and

$$\gamma\left(\frac{E_s}{N_0}, \boldsymbol{H}\right) = \frac{2(E_s/N_0)\left|(1/N_c)\sum_{k=0}^{N_c-1}\hat{H}(k)\right|^2}{(1/N_c)\sum_{k=0}^{N_c-1}|W_r(k)|^2 + ((U/SF)(E_s/N_0))\left\{(1/N_c)\sum_{k=0}^{N_c-1}|\hat{H}(k)|^2 - \left|(1/N_c)\sum_{k=0}^{N_c-1}\hat{H}(k)\right|^2\right\}} \quad (37)$$

the code multiplexing order U is varied from 1 to 256. An $L = 16$-path frequency-selective block Rayleigh fading channel having uniform power delay profile ($E[|h_l|^2] = 1/L$) and perfect CSI are assumed.

5.2 Equivalent channel

Fig. 2 shows a one-shot observation of the actual frequency-selective channel. The amplitude of the channel gain $H(k)$ of an $L = 16$-path frequency-selective channel is plotted. Below, the equivalent channel gain $W_r(k)H(k)W_t(k)$ of the proposed joint transmit/receive MMSE-FDE will be discussed. Fig. 3*a* shows a one-shot observation of the equivalent channel seen after the receive MMSE-FDE (i.e. $|W_r(k)H(k)W_t(k)|$) of the proposed joint transmit/receive MMSE-FDE when the transmit bit energy-to-noise power spectrum density ratio E_b/N_0 ($=0.5(\text{SF} \times E_c/N_0)(1 + N_g/N_c)^{-1}$) $= 6$ dB and $U = 256$. For comparison, the equivalent channel seen after the conventional receive MMSE-FDE is also plotted. The channel variation still remains and thus, the ICI still remains after despreading (this remaining ICI is called the residual ICI in this paper). The residual ICI limits the BER performance improvement.

When the proposed joint transmit/receive MMSE-FDE is used, the transmit FDE acts differently depending on the amplitude level of the channel gain. When the channel gain amplitude $|H(k)|$ at the kth frequency is large enough, the kth element of (29) can be approximated as $|W_t(k)|^2 \propto |H(k)|^{-1}$, and therefore the joint use of the transmit FDE and the receive FDE can suppress the amplitude variations in the equivalent channel gain $W_r(k)H(k)W_t(k)$ compared to the conventional receive MMSE-FDE. On the other hand, when the channel gain amplitude $|H(k)|$ is quite small, the squared value of the transmit FDE weight is forced to 0, and therefore we have $W_r(k)H(k)W_t(k) = 0$. It can be clearly seen from Fig. 3 that as the code multiplexing order U gets smaller, the proposed scheme tends to allocate the transmit signal power to the frequencies having good condition (note that the conventional receive MMSE-FDE allocates the transmit power always equally over all frequencies).

5.3 Statistical properties of residual ICI and received SNR

The probability density functions of $\text{Re}[\mu_{\text{ICI}}]$ and $\text{Re}[\hat{d}_u(i)]$ of (33) are plotted with U as a parameter in Fig. 4 for QPSK and $E_b/N_0 = 6$ dB. It should be noted that because of the symmetric distribution of the residual ICI, only $\text{Re}[\mu_{\text{ICI}}] \geq 0$ is shown in Fig. 4*a*. Without loss of generality, we assume $d_u(i) = (1/\sqrt{2}) + j(1/\sqrt{2})$ for all i in a block. It can be seen from Fig. 4*a* that, compared to the receive MMSE-FDE, the proposed joint transmit/receive MMSE-FDE suppresses more the residual ICI after despreading when $U = 256$, but increases it when $U = 1$ and 16. This is because, as discussed in Section 5.2, the transmit power is allocated to the frequencies having good condition and improve the received SNR when U is small. The transmit signal spectrum is significantly distorted by the transmit FDE and hence, the ICI is increased. This increased ICI can be suppressed by the receive FDE and the despreading process. However, the ICI still remains after the despreading as understood from the residual SINR given by (37); the second term of denominator is the residual ICI. However, negative effect of the residual ICI on the received SINR can be offset by the increased received SNR. The SNR increase can be understood from Fig. 4*b*; as U gets smaller, $\text{Re}[\hat{d}_u(i)]$ is shifted towards right as U becomes smaller. In summary, the proposed scheme improves the received SINR and consequently, the BER performance compared to the conventional receive MMSE-FDE by (a) suppressing the variations in the equivalent channel gain to reduce more the residual ICI for large U and (b) allocating the transmit power to the frequencies having good condition to improve the received SNR for small U.

Table 1 Numerical and simulation condition

DS-CDMA	data modulation	QPSK, 16QAM
	FFT block size (chips)	$N_c = 256$
	GI length (chips)	$N_g = 32$
	spreading factor	SF = 256
	code multiplexing order	$U = 1$–256
channel model	frequency-selective block Rayleigh fading	
	number of paths	$L = 16$
	delay time of the lth path	$\tau_l = l$
	power delay profile	uniform
	channel estimation	ideal

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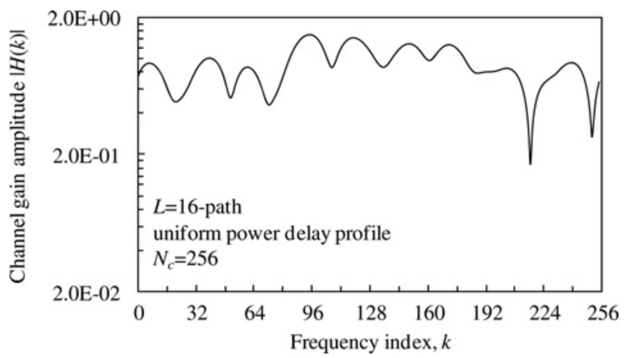

Figure 2 *One-shot observation of actual frequency-selective channel*

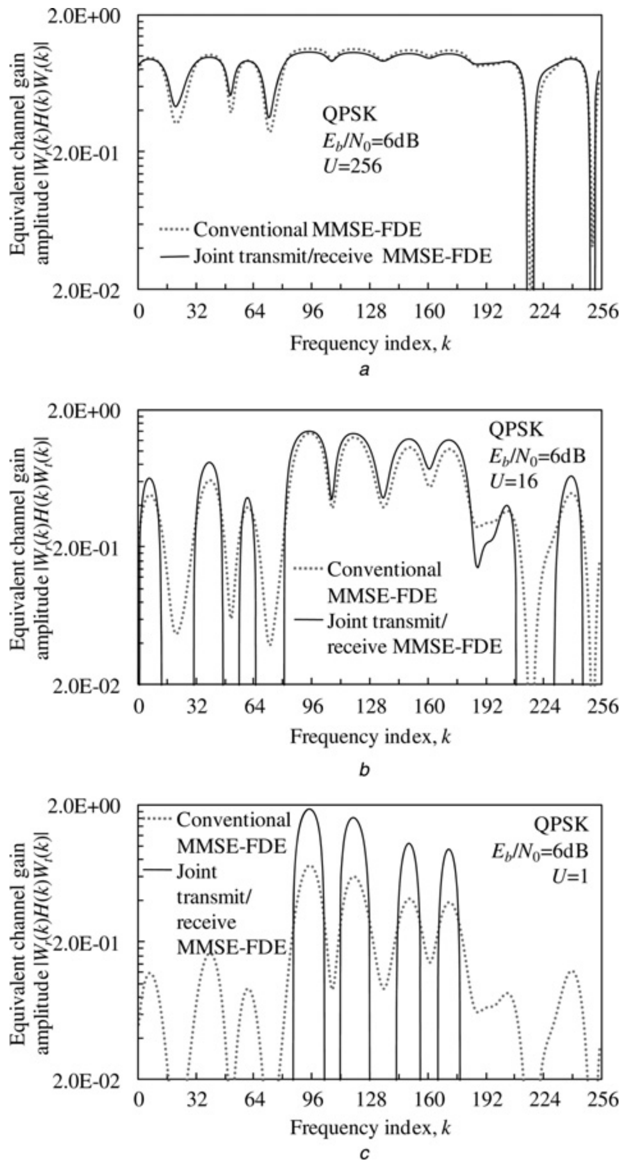

Figure 3 *Equivalent channel seen after the joint transmit/receive MMSE-FDE*

a $U = 256$
b $U = 16$
c $U = 1$

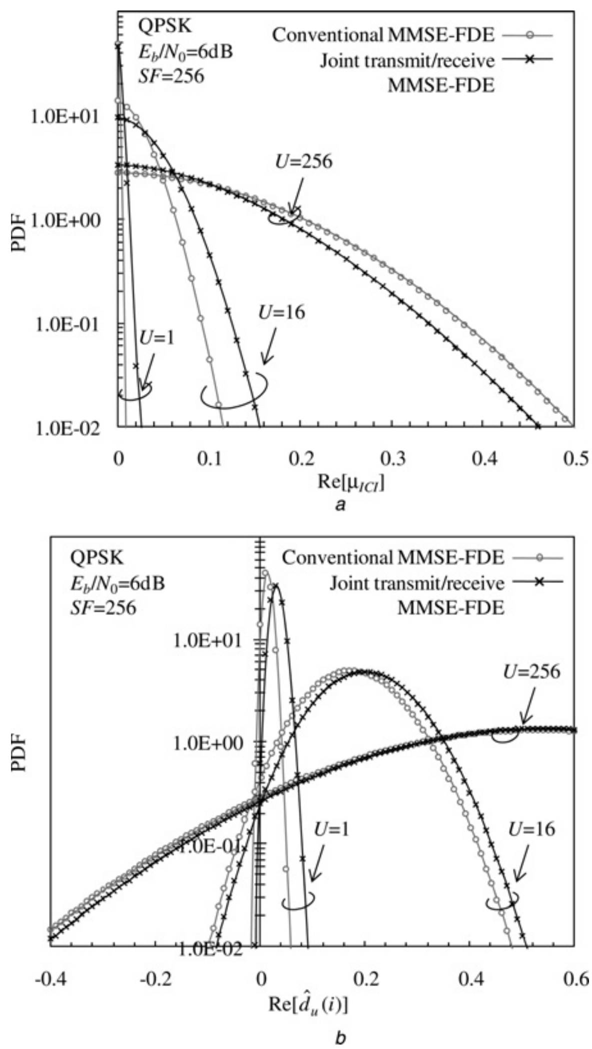

Figure 4 *Probability density functions of Re[μ_{ICI}] and Re[$\hat{d}_u(i)$]*

a Re[μ_{ICI}]
b Re[$\hat{d}_u(i)$]

5.4 Uncoded BER performance

First, we evaluate the uncoded BER performance by Monte-Carlo numerical computation method. The set of path gains $\{h_l;\ l = 0 \sim L-1\}$ is generated and the conditional BER for the given average transmit E_s/N_0 is computed using (36). The average BER is obtained by averaging the conditional BER over all possible channel realisations of $\boldsymbol{H}$. Fig. 5 compares the BER performances of DS-CDMA using conventional receive MMSE-FDE and proposed joint transmit/receive MMSE-FDE as a function of the average transmit E_b/N_0. The computer simulation results are also plotted to confirm the validity of our BER analysis based on the Gaussian approximation of the residual ICI. A fairly good agreement between the numerically computed and computer simulated results is seen. It can be seen from Fig. 5 that the proposed joint transmit/receive MMSE-FDE provides better BER performance than the conventional receive MMSE-FDE irrespective of the modulation level and the code multiplexing order U. As the total energy per transmit signal block

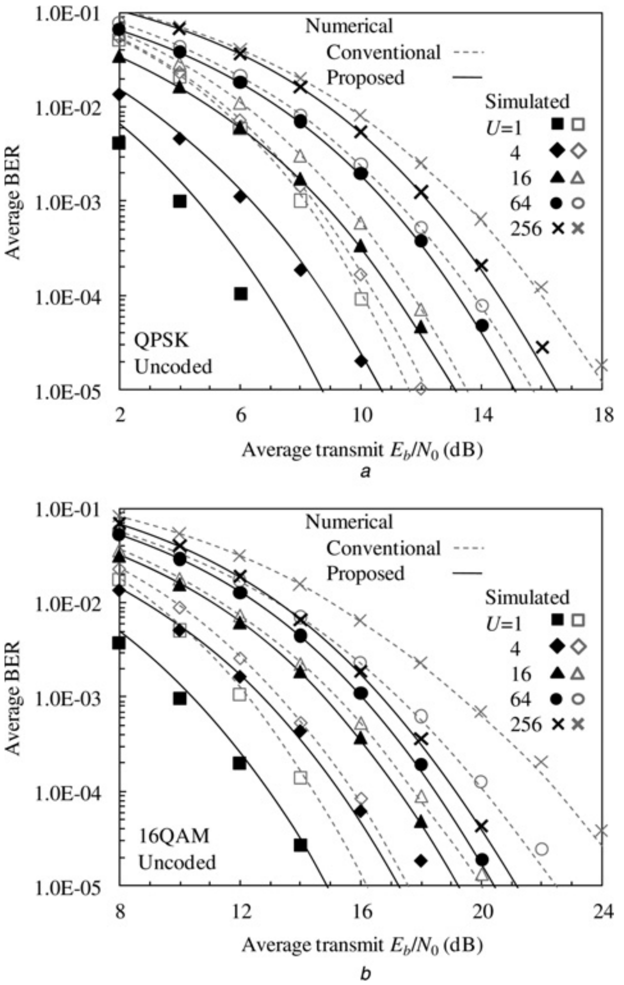

Figure 5 *Uncoded BER performance*

a QPSK
b 16QAM

increases (i.e. the modulation level or code multiplexing order U increases for the fixed value of E_b/N_0), the channel variations can be more likely mitigated by using the joint transmit/receive MMSE-FDE and thus, the residual ICI is better suppressed. On the other hand, as the total energy per transmit signal block decreases, most of the total energy is allocated to the frequencies having good condition. Although the residual ICI is enhanced by using the joint transmit/receive MMSE-FDE, the received SNR improves. In the case of small U, since not the residual ICI but the noise is a dominant factor in the performance degradation, the BER performance can be significantly improved.

5.5 Turbo-coded BER performance

Turbo coding [32] is a powerful error correction scheme. The transmission condition with turbo coding is summarised in Table 2. The information bit length of each packet is set as $K = 1018$ and the coding rate $R = 1/2$. Fig. 6 plots the average BER performance of turbo-coded multicode DS-CDMA using the proposed joint transmit/receive MMSE-FDE. The coded BER performance using conventional receive MMSE-FDE is also plotted in Fig. 6 for comparison. It can be seen from Fig. 6 that when using turbo coding, the proposed scheme further improves the BER performance compared to the conventional receive MMSE-FDE for a small U. On the other hand, for a large U, the proposed scheme suppresses the variations in the equivalent channel gain but not that much, and therefore only slightly suppresses the residual ICI compared to the conventional receive MMSE-FDE.

Table 2 Transmission condition with turbo-coding

information bit length	$K = 1018$
encoder	(13, 15) two RSCs
coding rate	$R = 1/2$
channel interleaver	block
decoder	log-MAP with eight iterations

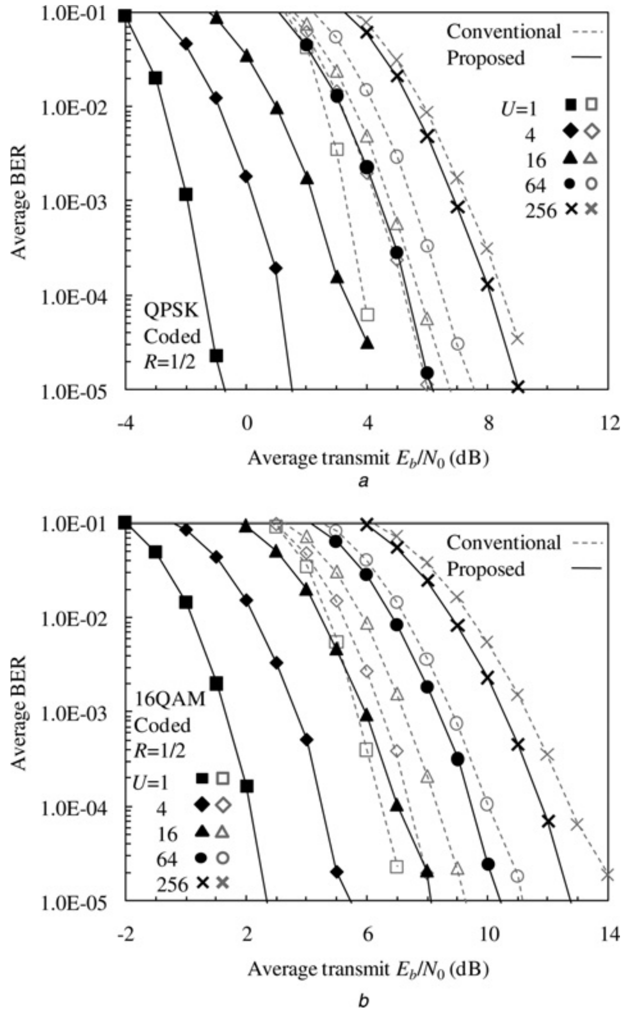

Figure 6 *Turbo-coded BER performance*

a QPSK
b 16QAM

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The reason for the pronounced performance improvement by the proposed scheme for a small U is given below. In a low E_b/N_0 region, where the coded transmission system is operated, the noise is the predominant cause of decision errors. The proposed scheme allocates the transmit power more to the frequencies having good condition and consequently, achieves the increased received SNR. The channel decoding can reduce the BER more effectively if the received SNR is improved. This results in the pronounced improvement of the BER performance for a small U.

6 Conclusions

In this paper, we proposed a joint transmit/receive MMSE-FDE to improve the multicode DS-CDMA signal transmission performance. A suboptimal set of transmit and receive FDE weights based on the MMSE criterion was theoretically derived and the BER performance improvement over the conventional receive MMSE-FDE was confirmed by the computer simulation. As the code multiplexing order U gets smaller, the proposed scheme tends to allocates the transmit power more to the frequencies having good condition while the conventional receive MMSE-FDE allocates the transmit power always equally over all the frequencies. By doing this, the proposed scheme can improve the received SNR, while reducing the residual ICI by using receive FDE and despreading when U is small. This SNR improvement yields the performance improvement by the proposed scheme over the conventional receive MMSE-FDE.

In this paper, it was assumed that the perfect CSI is available at both a transmitter and a receiver. Many studies for estimating the CSI at the transmitter can be found [33–37]. How the proposed joint transmit/receive MMSE-FDE is sensitive to the CSI error is an important future study topic.

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9 Appendix 1: A necessary condition for the value of *m*

Below, the necessary condition for the value of m in (27) is presented. Since $\boldsymbol{P}_0$ has $(N_c - m)$ zero elements, that is, $\boldsymbol{P}_0 = \text{diag}\{P_0(0), \ldots, P_0(m-1), 0, \ldots, 0\}$ with

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$\mathrm{tr}[\boldsymbol{P}_0] = N_c$, we can rewrite $e(\boldsymbol{P}_0)$ as

$$e(\boldsymbol{P}_0) = \sum_{q=0}^{m-1} \frac{\Omega N_c}{P_0(q)|G(q)|^2 + \Omega N_c} + (N_c - m) \tag{38}$$

The value of m is chosen so that the total MSE is minimised. The following inequalities hold

$$\begin{cases} e(\boldsymbol{P}_{-1}) \geq e(\boldsymbol{P}_0) < e(\boldsymbol{P}_{+1}), & \text{if } m = 1 \sim N_c - 1 \\ e(\boldsymbol{P}_{-1}) \geq e(\boldsymbol{P}_0), & \text{if } m = N_c \end{cases} \tag{39}$$

with $\boldsymbol{P}_{-1}\mathrm{diag}\{P_0(0), \ldots, P_0(m-2), 0, \ldots, 0\}$ and $\boldsymbol{P}_{+1} = \mathrm{diag}\{P_0(0), \ldots, P_0(m-2), P_0(m-1), X, 0, \ldots, 0\}$, where

$$X = \theta_{m-1} \frac{\sqrt{\Omega}}{|G(m)|} - \frac{\Omega N_c}{|G(m)|^2} \tag{40}$$

Using (40), (39) can be rewritten as

$$\begin{cases} \dfrac{\theta_{m-1}}{|G(m)|} - \dfrac{\sqrt{\Omega} N_c}{|G(m)|^2} < 0 \leq \dfrac{\theta_{m-1}}{|G(m-1)|} - \dfrac{\sqrt{\Omega} N_c}{|G(m-1)|^2}, \\ \quad \text{if } m = 1 \sim N_c - 1 \\ 0 \leq \dfrac{\theta_{N_c-1}}{|G(N_c-1)|} - \dfrac{\sqrt{\Omega} N_c}{|G(N_c-1)|^2}, \\ \quad \text{if } m = N_c \end{cases} \tag{41}$$

Equation (41) is a necessary condition for the value of m. This condition is explained in Fig. 7. In Fig. 7, $P_0(q)$ is given as a difference between the two curves for $q = 0 \sim m-1$, where $P_0(q) = 0$ for $q = m \sim N_c - 1$.

There is only one integer value of m between 1 and N_c that satisfies (41) (Appendix 3). By substituting $m = 1 \sim N_c$ into m of (41) and by checking whether the inequality of (41) is met or not, we can find the value of m.

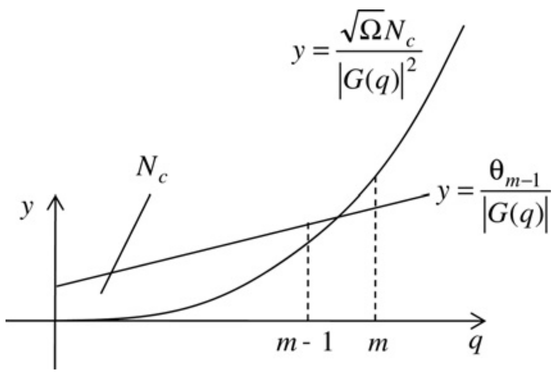

Figure 7 *Necessary condition for the value of m*

10 Appendix 2: Finding the value of *m*

We present a simple iterative algorithm for searching the value of m. Below, a integer n is introduced, where $m \leq n \leq N_c$. First, n is set to N_c. Then, it is checked whether

$$0 \leq \frac{\theta_{n-1}}{|G(n-1)|} - \frac{\sqrt{\Omega} N_c}{|G(n-1)|^2} \tag{42}$$

is met or not. If it is met, $m = n = N_c$. If not, $m < n = N_c$ and the following steps (a) and (b) are repeated.

Step (a): An integer value of δ that satisfies

$$\frac{\theta_{n-1}}{|G(n-\delta)|} - \frac{\sqrt{\Omega} N_c}{|G(n-\delta)|^2} < 0 \leq \frac{\theta_{n-1}}{|G(n-\delta-1)|} - \frac{\sqrt{\Omega} N_c}{|G(n-\delta-1)|^2} \tag{43}$$

with $1 \leq \delta \leq N_c - 1$ is found.

Step (b): Since

$$\theta_{m-1} \sum_{q=0}^{m-1} \frac{\sqrt{\Omega}}{|G(q)|} - \sum_{q=0}^{m-1} \frac{\Omega N_c}{|G(q)|^2} > \theta_{m-1} \sum_{q=0}^{m} \frac{\sqrt{\Omega}}{|G(q)|} - \sum_{q=0}^{m} \frac{\Omega N_c}{|G(q)|^2} \quad \text{for } m = 1 \sim N_c - 1 \tag{44}$$

and

$$\theta_{m-1} \sum_{q=0}^{m-1} \frac{\sqrt{\Omega}}{|G(q)|} - \sum_{q=0}^{m-1} \frac{\Omega N_c}{|G(q)|^2} = \theta_m \sum_{q=0}^{m} \frac{\sqrt{\Omega}}{|G(q)|} - \sum_{q=0}^{m} \frac{\Omega N_c}{|G(q)|^2} \quad \text{for } m = 1 \sim N_c - 1 \tag{45}$$

we obtain

$$\theta_{m-1} < \cdots < \theta_{n-1} < \cdots < \theta_{N_c-1} \tag{46}$$

and therefore the following inequalities hold

$$\begin{cases} \dfrac{\theta_{n-\delta-1}}{|G(n-\delta)|} - \dfrac{\sqrt{\Omega} N_c}{|G(n-\delta)|^2} < \dfrac{\theta_{n-1}}{|G(n-\delta)|} - \dfrac{\sqrt{\Omega} N_c}{|G(n-\delta)|^2} \\ \dfrac{\theta_{n-\delta-1}}{|G(n-\delta-1)|} - \dfrac{\sqrt{\Omega} N_c}{|G(n-\delta-1)|^2} < \dfrac{\theta_{n-1}}{|G(n-\delta-1)|} \\ \quad - \dfrac{\sqrt{\Omega} N_c}{|G(n-\delta-1)|^2} \end{cases} \tag{47}$$

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After finding the value of δ in step (a), it is checked whether

$$\frac{\theta_{n-\delta-1}}{|G(n-\delta)|} - \frac{\sqrt{\Omega}N_c}{|G(n-\delta)|^2} < 0 \leq \frac{\theta_{n-\delta-1}}{|G(n-\delta-1)|} - \frac{\sqrt{\Omega}N_c}{|G(n-\delta-1)|^2} \quad (48)$$

is met or not. If (48) is met, $m = n - \delta$. If not, $m < n - \delta$ and step (a) is repeated again by replacing n with $n - \delta$.

11 Appendix 3: Proof of the uniqueness value of *m*

Let us consider the case when $1 \leq m < N_c$. Assume that a non-zero integer β ($|\beta| < N_c - m$) exists that satisfies

$$\frac{\theta_{m+\beta-1}}{|G(m+\beta)|} - \frac{\sqrt{\Omega}N_c}{|G(m+\beta)|^2} < 0 \leq \frac{\theta_{m+\beta-1}}{|G(m+\beta-1)|} - \frac{\sqrt{\Omega}N_c}{|G(m+\beta-1)|^2} \quad (49)$$

On the other hand, from (27) and (28), we have

$$\left\{\theta_{m+\beta-1}\sum_{q=0}^{m+\beta-1}\frac{\sqrt{\Omega}}{|G(q)|} - \sum_{q=0}^{m+\beta-1}\frac{\Omega N_c}{|G(q)|^2}\right\} - \left\{\theta_{m-1}\sum_{q=0}^{m-1}\frac{\sqrt{\Omega}}{|G(q)|} - \sum_{q=0}^{m-1}\frac{\Omega N_c}{|G(q)|^2}\right\}$$
$$= \sqrt{\Omega}\left[\sum_{q=0}^{m-1}\frac{\theta_{m+\beta-1}-\theta_{m-1}}{|G(q)|} + \sum_{q=m}^{m+\beta-1}\left\{\frac{\theta_{m+\beta-1}}{|G(q)|} - \frac{\sqrt{\Omega}N_c}{|G(q)|^2}\right\}\right] = 0 \quad (50)$$

For the case of $\beta > 0$, from (50), the first term on the left-hand side of (50) is always positive. However, under the assumption of (49), the second term is also always positive. Therefore it can be understood that the assumption (49) is a contradiction to (52) and therefore there is no integer β. Similarly, it can be proved that there is no integer β for the case of $\beta > 0$.

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