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User Selection Criteria for Multiuser Systems With Optimal and Suboptimal LR Based Detectors

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***Abstract*—In this correspondence, we investigate user selection criteria for various multiple input multiple output (MIMO) detectors to exploit the multiuser diversity. Different user selection criteria are derived for various MIMO detectors, including the maximum likelihood (ML) detector and low complexity suboptimal detectors. It is shown that the user selection criterion plays a crucial role in exploiting both multiuser and receive (or spatial) diversity. We also show that the ML and even some low complexity suboptimal detectors (based on the lattice reduction (LR)) can achieve a full multiuser and receive diversity when the user selection criterion is properly chosen.**

***Index Terms*—Error probability, multiuser MIMO, user selection criterion.**

I. INTRODUCTION

A. Background

In multiuser wireless communication systems, an overall throughput can be improved using multiuser diversity that takes advantage of different channel gains of multiple users [1]. As shown in [2], the multiuser diversity can maximize throughput by allowing only one user who has the strongest channel gain to access a common channel. The multiuser diversity can be extended to the case of multiple antennas. Beamforming techniques for multiuser diversity are extensively investigated in [2] and antenna selection is considered in [3].

The signal-to-noise ratio (SNR) is usually considered as a user selection criterion to exploit the multiuser diversity. With beamforming or antenna selection, a user selection criterion based on SNR can be easily derived as in [2] and [3]. In general, the SNR based user selection criterion is directly related to the channel capacity based user selection criterion since the channel capacity increases with the SNR. Thus, when multiple input multiple output (MIMO) systems are considered (without antenna selection), the user who has the highest channel capacity can be chosen [4]. In [5], the achievable rate with the zero-forcing (ZF) receiver is considered for the user selection criterion. In addition, in [6], the minimum eigenvalue of user's MIMO channel is used as the user selection criterion.

B. Motivation

Although the channel capacity can be used as a user selection criterion to exploit the multiuser diversity in terms of the information-theoretic point of view, there might be different selection criteria that take into account practical issues and/or adopt non-information-theoretic approaches (e.g., a minimum error probability based approach for a given modulation scheme).

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The approach in [5] is interesting as the user selection criterion is devised when a receiver constraint is imposed. In addition to a receiver constraint, we may also need to consider the following non-ideal transmission issues. Since a user can decide his/her transmission rate depending on applications, the actual transmission rate is not necessary to be close to the channel capacity. Assuming that the transmission rate is R_k for user k, the throughput for user k can be expressed as follows: $U_k = R_k(1 - P_{k,\text{error}})$, where $P_{k,\text{error}}$ denotes the error probability of packet or symbol of user k. It is noteworthy that the error probability depends on channel conditions as well as detection/decoding methods (thus, receiver constraints can be accommodated into the error probability). Using the throughput expression, we can consider the user selection criterion in which the user who has the maximum throughput is chosen. Alternatively, the user who maximizes the normalized throughput or throughput efficiency, U_k/R_k, can be chosen. This is equivalent to choosing the user who has the smallest error probability. Note that although the throughput has been considered to see the impact of multiuser diversity, the error probability can also be considered as in [1] (in this case, as a selection diversity scheme, the diversity order in terms of the bit error rate[1] linearly increases with the number of users in a multiuser system). In this correspondence, we consider the user selection criterion based on error probability with MIMO detector constraints.

C. Notations

The superscripts T and H stand for the transpose and Hermitian transpose, respectively. For a vector or matrix, $\|\cdot\|_{\mathrm{F}}$ denotes the Frobenius norm. For a square matrix, $\mathrm{Det}(\cdot)$ and $\mathrm{Tr}(\cdot)$ denote the determinant and trace, respectively. For a function of x, denoted by $g(x)$, we write $g(x) = o(x)$ if $\lim_{x\to 0}(g(x))/x = 0$.

II. System Model

Suppose that there are K users (or transmitters) in a multiuser system and assume that the receiver[2] is equipped with N receive antennas and each user is equipped with Q transmit antennas. Furthermore, it is assumed that only one user can access a common radio channel at a time to exploit multiuser diversity. If user k is chosen, the received signal for an L-symbol duration is given by

$$\mathbf{Y}_k = \mathbf{H}_k\mathbf{S}_k + \mathbf{N} \tag{1}$$

where $\mathbf{H}_k$, $\mathbf{S}_k$, and $\mathbf{N}$ are the $N \times Q$ channel matrix, the $Q \times L$ transmitted signal matrix, and the $N \times L$ noise matrix, respectively. We assume that each column vector of $\mathbf{N}$ is an independent zero-mean circularly symmetric complex Gaussian (CSCG) random vector with $E[\mathbf{n}_l\mathbf{n}_l^{\mathrm{H}}] = N_0\mathbf{I}$, where $\mathbf{n}_l$ denotes the lth column vector of $\mathbf{N}$. In (1), we assume that the channel is not varying over L-symbol interval (i.e., block fading is considered). Each user would have statistically independent channel matrix $\mathbf{H}_k$.

In this correspondence, we devise user selection criteria to exploit the multiuser diversity effectively with various MIMO detectors. For convenience, we have the following assumptions.

A1) $[\mathbf{S}_k]_{q,l} \in \mathbb{Z} + j\mathbb{Z}$, where $\mathbb{Z}$ denotes the set of integer numbers and $j = \sqrt{-1}$, and a common signal alphabet, denoted by $\mathcal{S}$, is used for all the users.[3] Note that the signal constellation or alphabet is a subset of $\mathbb{Z} + j\mathbb{Z}$. For example, the signal alphabet of 16-QAM is a subset of $\mathbb{Z} + j\mathbb{Z}$, where there are 16 lattice points.

A2) The elements of the channel matrix $\mathbf{H}_k$ are independent zero-mean CSCG random variables with variance σ_h^2 (in this variance term, the signal power is absorbed for convenience). Note that in deriving user selection criteria, we do not need to use this assumption. However, in order to derive the diversity gain, we will use this assumption.

Throughout the correspondence, we focus on the case of uncoded signals. This implies that the user selection criteria are based on error probabilities of uncoded signals (not throughput). Although it is desirable to take into account channel coding, it would be reasonable to derive user selection criteria for uncoded signals if a common coding scheme is used for all the users.

III. User Selection Criteria

We can derive user selection criteria depending on the type of actually employed MIMO detector. In this section, the ML detector and two suboptimal detectors will be considered: one is the linear detector and the other is the successive interference cancellation (SIC) detector. For the two suboptimal detectors, the lattice reduction (LR) is applied for better performance [7], [8].

A. ML Detector

Assume that user k is selected, we omit the user index k for the sake of simplicity. From (1), the ML decoding is given by

$$\hat{\mathbf{S}}_{\mathrm{ml}} = \arg\min_{\mathbf{S}} \|\mathbf{Y} - \mathbf{HS}\|_{\mathrm{F}}^2. \tag{2}$$

To derive the selection criterion, we can consider the pairwise error probability (PEP). Suppose that $\mathbf{S}_{(1)}$ is transmitted, while $\mathbf{S}_{(2)}$ is erroneously detected. Then, from [11], the PEP is given by

$$\begin{aligned} P&\left(\mathbf{S}_{(1)} \to \mathbf{S}_{(2)}\right) \\ &= \Pr\left(\left\|\mathbf{Y} - \mathbf{HS}_{(2)}\right\|_{\mathrm{F}}^2 \le \left\|\mathbf{Y} - \mathbf{HS}_{(1)}\right\|_{\mathrm{F}}^2\right) \\ &= \mathcal{Q}\left(\sqrt{\frac{\|\mathbf{H\Delta}\|_{\mathrm{F}}^2}{2N_0}}\right) \end{aligned} \tag{3}$$

where $\mathcal{Q}(x) = \int_x^\infty (1)/(\sqrt{2\pi})e^{-z^2/2}\mathrm{d}z$ and $\mathbf{\Delta} = \mathbf{S}_{(1)} - \mathbf{S}_{(2)}$. For uncoded signals, let $L = 1$ and $\mathbf{S} \in \mathcal{S}^Q$ throughout the correspondence. Thus, $\mathbf{Y}$, $\mathbf{N}$, and $\mathbf{S}_{(i)}$ will be replaced with $\mathbf{y}$, $\mathbf{n}$, and $\mathbf{s}_{(i)}$, respectively. Then, the following upper bound can be obtained:

$$P\left(\mathbf{s}_{(1)} \to \mathbf{s}_{(2)}\right) \le \mathcal{Q}\left(\sqrt{\frac{\|\mathbf{H}\bar{\mathbf{d}}\|^2}{2N_0}}\right) \tag{4}$$

where

$$\bar{\mathbf{d}} = \arg\min_{\mathbf{d}\in\mathcal{D},\mathbf{d}\neq\mathbf{0}} \|\mathbf{Hd}\|^2. \tag{5}$$

Here, $\mathcal{D} = \{\mathbf{d} = \mathbf{s} - \mathbf{s}' \mid \mathbf{s},\ \mathbf{s}' \in \mathcal{S}^Q\} \subset \mathbb{Z}^Q + j\mathbb{Z}^Q$. For convenience, denote by $S(\mathbf{H})$ the length of the shortest non-zero vector of the lattice generated by $\mathbf{H}$. Then, we can see that

$$S(\mathbf{H}) = \|\mathbf{H}\bar{\mathbf{d}}\|.$$

From (4), if the ML detector is employed, the user selection criterion to minimize the error probability becomes

$$k^* = \arg\max_k S(\mathbf{H}_k). \tag{6}$$

[1]This is the bit error probability of the transmitted signal from the user who has the highest SNR.

[2]If we consider cellular uplink channels, the receiver becomes the base station (BS).

[3]Although it is possible that each user can have different signal constellation, this simplifies the derivation of user selection criteria.

Throughout this correspondence, the user selection criterion in (6) is referred to as the max-min distance (MDist) criterion as $S(\mathbf{H})$ is the minimum distance of the lattice generated by $\mathbf{H}$.

The problem to find a non-zero shortest vector in a lattice is called the shortest vector problem (SVP) and known to be NP-hard [9]. For an approximation, the LLL algorithm [10], which has a polynomial time complexity, can be used.

Another approximation can be considered by relaxing the constraint on $\mathbf{\Delta}$. We have

$$\|\mathbf{H}\mathbf{\Delta}\|^2 = \mathbf{\Delta}^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\mathbf{H}\mathbf{\Delta} \geq \|\mathbf{\Delta}\|^2 \lambda_{\min}(\mathbf{H}^{\mathrm{H}}\mathbf{H})$$

where $\lambda_{\min}(\mathbf{A})$ stands for the minimum eigenvalue of $\mathbf{A}$. This shows that the selection criterion can be based on the minimum eigenvalue of the channel matrix:

$$k^* = \arg\max_k \lambda_{\min}\left(\mathbf{H}_k^{\mathrm{H}}\mathbf{H}_k\right). \tag{7}$$

Thus, each user can feed back its minimum eigenvalue of the channel matrix and the user who has the maximum $\lambda_{\min}(\mathbf{H}_k^{\mathrm{H}}\mathbf{H}_k)$ can be selected to access the channel. This selection criterion is referred to as the max-min eigenvalue (ME) criterion throughout the correspondence.

It is well-known that the ML detector can achieve a full receive diversity when $\mathbf{\Delta}$ is fixed. Using the upper bound in (4), we can show that

$$E\left[P\left(\mathbf{s}_{(1)} \to \mathbf{s}_{(2)}\right)\right] \leq \mathrm{Det}\left(\mathbf{I} + \frac{\sigma_h^2}{4N_0}\mathbf{\Delta}\mathbf{\Delta}^{\mathrm{H}}\right)^{-N} \tag{8}$$

where the expectation is carried out with respect to random channel matrix $\mathbf{H}$ according to A2) [11]. The diversity gain is with the upper bound in (4), where $\bar{\mathbf{d}}$ is a function of the channel realization, $\mathbf{H}$, can also be found. In the following result, we can show that the ML detector can achieve a full receive diversity when $\bar{\mathbf{d}}$ is a function of $\mathbf{H}$ (although the proof is straightforward, we present it as it will be used to show that the LR based SIC detector can achieve a full receive diversity later).

Property 1: The ML detector can achieve a full receive diversity under A1) and A2). That is, from (4), the average PEP is given by

$$E\left[P\left(\mathbf{s}_{(1)} \to \mathbf{s}_{(2)}\right)\right] \leq \sum_{\mathbf{d}\in\mathcal{D},\mathbf{d}\neq\mathbf{0}} \mathrm{Det}\left(\mathbf{I} + \frac{\sigma_h^2}{4N_0}\mathbf{d}\mathbf{d}^{\mathrm{H}}\right)^{-N}. \tag{9}$$

Proof: From (4), under A1), the PEP is bounded as

$$P\left(\mathbf{s}_{(1)} \to \mathbf{s}_{(2)}\right) \leq \mathcal{Q}\left(\sqrt{\frac{\|\mathbf{H}\bar{\mathbf{d}}\|^2}{2N_0}}\right) \leq \sum_{\mathbf{d}\in\mathcal{D},\mathbf{d}\neq\mathbf{0}} \mathcal{Q}\left(\sqrt{\frac{\|\mathbf{H}\mathbf{d}\|^2}{2N_0}}\right). \tag{10}$$

Using the Chernoff bound, under Assumption A2), the average PEP is bounded as

$$E\left[P\left(\mathbf{s}_{(1)} \to \mathbf{s}_{(2)}\right)\right] \leq \sum_{\mathbf{d}\in\mathcal{D},\mathbf{d}\neq\mathbf{0}} E\left[\exp\left(-\frac{\|\mathbf{H}\mathbf{d}\|^2}{2N_0}\right)\right]. \tag{11}$$

Noting that $\|\mathbf{H}\mathbf{d}\|^2 = \mathrm{Tr}(\mathbf{H}^{\mathrm{H}}\mathbf{H}\mathbf{d}\mathbf{d}^{\mathrm{H}})$, from (8), we have

$$E\left[\exp\left(-\frac{\|\mathbf{H}\mathbf{d}\|^2}{2N_0}\right)\right] \leq \mathrm{Det}\left(\mathbf{I} + \frac{\sigma_h^2}{4N_0}\mathbf{d}\mathbf{d}^{\mathrm{H}}\right)^{-N}. \tag{12}$$

Substituting (12) into (11), we derive (9). Note that the sum in (9) is a finite sum for a finite size of signal constellation, where the number of terms in the summation is independent of N. ■

B. Linear Detectors

An estimate of $\mathbf{s}$ can be obtained by a linear transformation as follows:

$$\hat{\mathbf{s}} = \mathbf{W}\mathbf{y} \tag{13}$$

where $\mathbf{W}$ is a linear filter that is given by $\mathbf{W} = (\mathbf{H}^{\mathrm{H}}\mathbf{H} + c\mathbf{I})^{-1}\mathbf{H}^{\mathrm{H}}$. If $c = 0$, the linear detector corresponds to the ZF detector, while the minimum mean square error (MMSE) detector is obtained if $c = N_0/E_s$. Here, E_s is the symbol energy and it is assumed that $E[\mathbf{s}\mathbf{s}^{\mathrm{H}}] = E_s\mathbf{I}$.

As the SNR increases, we have $c \to 0$ (in this case, the MMSE detector becomes the ZF detector) and the PEP has the following upper bound [11]:

$$\begin{aligned} P\left(\mathbf{s}_{(1)} \to \mathbf{s}_{(2)}\right) &= \mathcal{Q}\left(\frac{\|\mathbf{\Delta}\|^2}{\sqrt{2N_0\mathbf{\Delta}^{\mathrm{H}}(\mathbf{H}^{\mathrm{H}}\mathbf{H})^{-1}\mathbf{\Delta}}}\right) \\ &\leq \mathcal{Q}\left(\sqrt{\frac{\lambda_{\min}(\mathbf{H}^{\mathrm{H}}\mathbf{H})}{2N_0}\|\mathbf{\Delta}\|^2}\right) \end{aligned} \tag{14}$$

because $\mathcal{Q}(\cdot)$ is a decreasing function and $\Delta^{\mathrm{H}}(\mathbf{H}^{\mathrm{H}}\mathbf{H})^{-1}\Delta \leq \lambda_{\max}((\mathbf{H}^{\mathrm{H}}\mathbf{H})^{-1})\|\mathbf{\Delta}\|^2 = (\|\Delta\|^2)/(\lambda_{\min}(\mathbf{H}^{\mathrm{H}}\mathbf{H}))$. Therefore, the ME criterion in (7) can be used for the user selection criterion.

It is important to note that this ME criterion is valid for the LR based linear detectors [7], [8]. To improve the performance of the detector, the LR is performed in the LR based detection. A complex valued matrix can be converted into a real valued matrix for the LR as in [8]. Alternatively, the LR can be directly performed with a complex valued matrix as in [12], [7]. For convenience, in this correspondence, we assume that the LR is performed with complex valued matrices.

For a given channel matrix $\mathbf{H}$, the LR basis can be found as follows:

$$\mathbf{H} = \mathbf{G}\mathbf{U}$$

where $\mathbf{U}$ is an (complex) integer unimodular matrix and $\mathbf{G}$ is a matrix whose column vectors are nearly orthogonal. The received signal can be rewritten as

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} = \mathbf{G}\mathbf{c} + \mathbf{n} \tag{15}$$

where $\mathbf{C} = \mathbf{U}\mathbf{s}$. Let $\mathbf{c}_{(i)} = \mathbf{U}\mathbf{s}_{(i)}$, $i = 1, 2$. Then, from (14), the PEP is bounded as

$$P\left(\mathbf{s}_{(1)} \to \mathbf{s}_{(2)}\right) \leq \mathcal{Q}\left(\sqrt{\frac{\lambda_{\min}(\mathbf{G}^{\mathrm{H}}\mathbf{G})}{2N_0}\|\mathbf{\Delta}_{\mathbf{U}}\|^2}\right) \tag{16}$$

where $\mathbf{\Delta}_{\mathbf{U}} = \mathbf{c}_{(1)} - \mathbf{c}_{(2)} = \mathbf{U}(\mathbf{s}_{(1)} - \mathbf{s}_{(2)})$. From (16), the selection criterion becomes

$$k^* = \arg\max_k \lambda_{\min}\left(\mathbf{G}_k^{\mathrm{H}}\mathbf{G}_k\right) \tag{17}$$

where $\mathbf{G}_k$ is the reduced basis from $\mathbf{H}_k$. This ME criterion is the same as that in (7) except that the channel matrix $\mathbf{H}_k$ is replaced by its reduced one $\mathbf{G}_k$.

C. SIC Detectors

A SIC detector is not a linear detector due to its cancellation operation. In [8], the LR based SIC detectors are proposed. To generalize the LR based SIC detector, define the extended channel matrix as $\mathbf{H}_{\mathrm{ex}} = \begin{bmatrix} \mathbf{H} \\ \sqrt{c}\mathbf{I} \end{bmatrix}$. The LR basis can be found as

$$\mathbf{H}_{\mathrm{ex}} = \mathbf{G}_{\mathrm{ex}}\mathbf{U}_{\mathrm{ex}} \tag{18}$$

where $\mathbf{U}_{\rm ex}$ is a complex integer unimodular matrix and $\mathbf{G}_{\rm ex}$ is a matrix whose column vectors are nearly orthogonal. If the LR basis is not used, $\mathbf{U}_{\rm ex} = \mathbf{I}$ (i.e., $\mathbf{G}_{\rm ex} = \mathbf{H}_{\rm ex}$).

Note that the size of $\mathbf{G}_{\rm ex}$ is the same as that of $\mathbf{H}_{\rm ex}$ which is $2N \times Q$. Let the QR factorization of $\mathbf{G}_{\rm ex}$ be $\mathbf{G}_{\rm ex} = \mathbf{QR}$, where $\mathbf{Q}$ is a matrix whose column vectors are orthonormal and $\mathbf{R}$ is upper triangular. Let $\mathbf{y}_{\rm ex} = [\mathbf{y}^{\rm T}\ \mathbf{0}]^{\rm T}$ and $\mathbf{n}_{\rm ex} = [\mathbf{n}^{\rm T}\ -\sqrt{c}\mathbf{s}^{\rm T}]^{\rm T}$. This results in $\mathbf{y}_{\rm ex} = \mathbf{H}_{\rm ex}\mathbf{s} + \mathbf{n}_{\rm ex}$. Then, the LR based SIC detection can be carried out with the following signal:

$$\mathbf{Q}^{\rm H}\mathbf{y}_{\rm ex} = \mathbf{Q}^{\rm H}\mathbf{G}_{\rm ex}\mathbf{U}_{\rm ex}\mathbf{s} + \mathbf{Q}^{\rm H}\mathbf{n}_{\rm ex} = \mathbf{Rc} + \bar{\mathbf{n}} \tag{19}$$

where $\mathbf{c} = \mathbf{U}_{\rm ex}\mathbf{s}$ and $\bar{\mathbf{n}} = \mathbf{Q}_{\rm ex}^{\rm H}\mathbf{n}_{\rm ex}$. Since the statistical properties of $\mathbf{n}$ and $\bar{\mathbf{n}}$ are the same, we will use $\mathbf{n}$ to denote $\bar{\mathbf{n}}$. Note that $\mathbf{n}$ also includes the self-interference as mentioned in [8].

The SIC detection can be carried out with (19). The elements of the last row, the Qth layer, are detected first. Then, their contributions in the second last row are cancelled and the signals of the $(Q-1)$th row are detected. This operation is repeated up to the first row.

As the LR is performed, the column vectors of $\mathbf{G}_{\rm ex}$ would be nearly orthogonal. In other words, the upper off-diagonal elements of $\mathbf{R}$ would be small. Thus, the SIC detection performance would mainly depend on the diagonal elements of $\mathbf{R}$. For convenience, let $c = 0$ (this is the case when $N_0 \to 0$ or high SNR). Let $r_{q,q}^{(k)}$ denote the (q,q)th element of $\mathbf{R}$ from the kth user's channel $\mathbf{H}_k$. Then, ignoring the interference terms (as they are cancelled when the detection of the lower layers is successfully carried out with no error), the SNR of the qth layer of $\mathbf{H}_k$ becomes $\gamma_q^{(k)} = (|r_{q,q}^{(k)}|^2)/(N_0)$. From this, the selection criterion can be given by

$$k^* = \arg\max_k \left\{ \min_q \left| r_{q,q}^{(k)} \right| \right\}. \tag{20}$$

This selection criterion is referred to as the max-min diagonal term (MD) criterion.

The MD criterion is also closely related to the minimum error probability criterion when the SNR is high. For convenience, let $\mathbf{x} = \mathbf{Q}^{\rm H}\mathbf{y}$. Then, (19) is rewritten as

$$\mathbf{x} = \mathbf{Rc} + \mathbf{n}. \tag{21}$$

Let n_q denote the qth element of $\mathbf{n}$. Then, the LR based SIC detection at the Qth layer does not have error if $(|n_Q|)/(|r_{Q,Q}|) < 1/2$ or $|n_Q|^2 < (|r_{Q,Q}|^2)/4$. Thus, the LR based SIC detection would have no error across all the layers if $|n_q|^2 < (|r_{q,q}|^2)/4$, for all q. The probability of no error can be lower bounded as

$$\begin{aligned}\Pr(\text{no error}) &\geq \Pr\left(|n_q|^2 < \frac{|r_{q,1}|^2}{4}, \forall q\right) \\ &= \prod_{q=1}^{Q} \Pr\left(|n_q|^2 < \frac{|r_{q,q}|^2}{4}\right).\end{aligned} \tag{22}$$

Since $|n_q|^2$ is a chi-square random variable with 2 degrees of freedom (or an exponential random variable), we have $\Pr(|n_q|^2 < (|r_{q,q}|^2)/4) = 1 - e^{-(|r_{q,q}|^2)/(4N_0)}$. Thus, from (22), the probability of error can be given by

$$\begin{aligned}\Pr(\text{error}) &\leq 1 - \prod_{q=1}^{Q}\left(1 - e^{-\frac{|r_{q,q}|^2}{4N_0}}\right) \\ &\simeq e^{-\min_q \frac{|r_{q,q}|^2}{4N_0}} \text{ as } N_0 \to 0.\end{aligned} \tag{23}$$

Therefore, to minimize the probability of error, the user who has the maximum $\min_q |r_{q,q}|$ can be selected.

Property 2: If the LLL reduced basis is used, under A1), we have

$$\min_q |r_{q,q}|^2 \geq \beta^{-Q+1} S^2(\mathbf{H}) \tag{24}$$

where $\beta > 4/3$ is a constant. In addition, the LR based SIC detector can achieve a full receive diversity for uncoded signals under A2). That is,

$$E\left[\exp\left(-\min_q \frac{|r_{q,q}|^2}{4N_0}\right)\right] \leq \sum_{\mathbf{d}\in\mathcal{D},\mathbf{d}\neq\mathbf{0}} \mathrm{Det}\left(\mathbf{I} + \frac{\sigma_h^2\beta^{-Q+1}}{4N_0}\mathbf{dd}^{\rm H}\right)^{-N}. \tag{25}$$

Proof: From [13] and [14], we can easily show (24). Applying the approach used to prove Property 1, we can derive (25). ■

Note that if the LLL reduced basis is not used (i.e., $\mathbf{U}_{\rm ex} = \mathbf{I}$), the (conventional) SIC detector cannot achieve a full receive diversity.

IV. Impact of the Number of Users on Diversity Gain

To see the impact of the number of users on the diversity gain for each user selection criterion, we consider the error probabilities of the ML and LR based SIC detectors. As the multiuser diversity is considered with MIMO systems, it is expected to have a better diversity gain by exploiting the multiuser diversity and receive diversity.

We derive two user selection criteria in Section III for the case that the ML detector is employed. With the MDist criterion, from (10) and (6), the PEP of the ML detector is bounded as

$$P\left(\mathbf{s}_{(1)} \to \mathbf{s}_{(2)}\right) \leq \mathcal{Q}\left(\sqrt{\frac{\max_k S^2(\mathbf{H}_k)}{2N_0}}\right). \tag{26}$$

On the other hand, if the ME criterion is used, the PEP is bounded as

$$P\left(\mathbf{s}_{(1)} \to \mathbf{s}_{(2)}\right) \leq \mathcal{Q}\left(\sqrt{\frac{\max_k \lambda_{\min}(\mathbf{H}_k^{\rm H}\mathbf{H}_k)\|\mathbf{\Delta}\|^2}{2N_0}}\right). \tag{27}$$

Property 3: Suppose that the ME criterion is employed and $N = Q$. Then, under A2), the order of the multiuser diversity with the ML detector is K.

Proof: Let $X = \lambda_{\min}/\sigma_h^2$, where $\lambda_{\min} = \lambda_{\min}(\mathbf{H}^{\rm H}\mathbf{H})$. When $N = Q$, the probability density function (pdf) of the smallest eigenvalue is given by [16] $f(x) = Qe^{-Qx}$. Let $V = \max\{X_1, X_2, \ldots, X_K\}$. Then, the pdf of V is given by

$$\begin{aligned}f_V(v) &= KQ(1-e^{-Qv})^{K-1}e^{-Qv} \\ &= KQ^K v^{K-1} + o(v^{K-1+\epsilon}),\ (v \to 0^+)\end{aligned} \tag{28}$$

where $\epsilon > 0$. Then, the upper bound on the PEP in (27) can be rewritten as

$$P\left(\mathbf{s}_{(1)} \to \mathbf{s}_{(2)}\right) \leq \mathcal{Q}\left(\sqrt{\frac{V\sigma_h^2\|\mathbf{\Delta}\|^2}{2N_0}}\right).$$

According to [15], we can show that

$$E\left[P\left(\mathbf{s}_{(1)} \to \mathbf{s}_{(2)}\right)\right] \leq c_1\gamma_\Delta^{-K} + o\left(\gamma_\Delta^{-(K+1)}\right) \tag{29}$$

where $\gamma_\Delta = (\sigma_h^2\|\mathbf{\Delta}\|^2)/(N_0)$ and $c_1 > 0$ is constant. ■

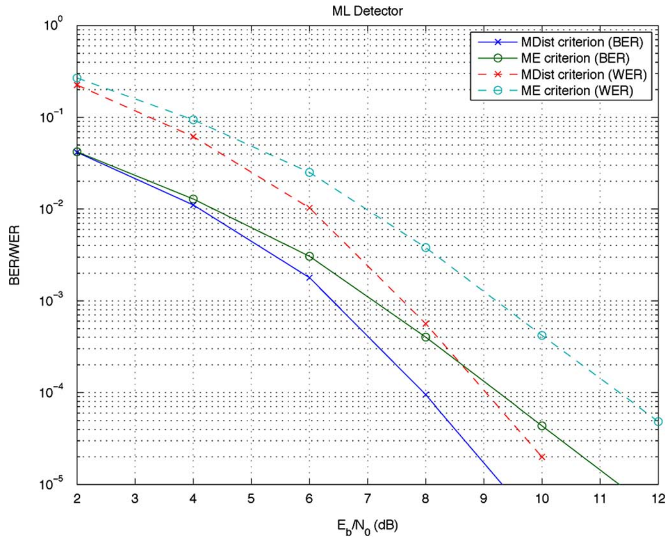

Fig. 1. Performance of the ML detector with multiuser diversity (16-QAM, $K = 10$, $Q = N = 4$).

From the result in Property 3, we can see that the ME criterion cannot fully exploit the receive diversity gain, but can exploit the multiuser diversity gain. If $N > Q$, we can show that the diversity order becomes $K(N-Q+1)$ using the result in [16].

Property 4: Suppose the MDist criterion is employed. Then, under A2), the order of the multiuser diversity with the ML detector is NK.

Proof: Since

$$\max_k S^2(\mathbf{H}_k) = \max_k \min_{\mathbf{d}\in\mathcal{D},\mathbf{d}\neq\mathbf{0}} \mathbf{d}^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}}\mathbf{H}_k\mathbf{d}$$

we can show that

$$P\left(\mathbf{s}_{(1)} \to \mathbf{s}_{(2)}\right) \le \sum_{\mathbf{d}\in\mathcal{D},\mathbf{d}\neq\mathbf{0}} \mathcal{Q}\left(\sqrt{\frac{\max_k \mathbf{d}^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}}\mathbf{H}_k\mathbf{d}}{2N_0}}\right).$$

Let $\mathbf{w}_k = \mathbf{H}_k\mathbf{d}$. Under A2), we can see that $\mathbf{w}_k$ is a CSCG random vector and $E[\mathbf{w}_k\mathbf{w}_k^{\mathrm{H}}] = \sigma_h^2\|\mathbf{d}\|^2\mathbf{I}$. From this, we show that $X_k = \|\mathbf{w}_k\|^2$ is a chi-square random variable with $2N$ degrees of freedom and its pdf becomes $f_X(x_k) = (1)/((\sigma_h^2\|\mathbf{d}\|^2)^N(N-1)!)x_k^{N-1}e^{-x_k/(\sigma_h^2\|\mathbf{d}\|^2)}$. The cumulative distribution function (cdf) is $F_X(x_k) = 1 - e^{(x_k)/(\sigma_h^2\|\mathbf{u}\|^2)}\sum_{q=0}^{N-1}((x_k/(\sigma_h^2\|\mathbf{d}\|^2))^q)/(q!)$. As the $\mathbf{w}_k$'s are independent, the pdf of $V = \max\{X_1, X_2, \ldots, X_K\}$ is given by

$$F_V(v) = KF_X^{K-1}(v)f_X(v) = c_2 v^{NK-1} + o(v^{NK-1+\epsilon}) \quad (30)$$

where $c_2 > 0$ is constant. Thus, according to [15], we can show that

$$\begin{aligned} E&\left[P\left(\mathbf{s}_{(1)} \to \mathbf{s}_{(2)}\right)\right] \\ &\le \sum_{\mathbf{d}\in\mathcal{D},\mathbf{d}\neq\mathbf{0}} E\left[\mathcal{Q}\left(\sqrt{\frac{\max_k \mathbf{d}^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}}\mathbf{H}_k\mathbf{d}}{2N_0}}\right)\right] \\ &= c_3\gamma_{\mathbf{d}}^{-NK} + o\left(\gamma_{\mathbf{d}}^{-(NK+1)}\right) \end{aligned} \quad (31)$$

where $\gamma_{\mathbf{d}} = (\|\sigma_h^2\mathbf{d}\|^2)/(N_0)$ and $c_3 > 0$ is a constant. This shows that the diversity order is NK and completes the proof. ■

For the SIC detector, the MD criterion is derived in Section III as the user selection criterion for the multiuser diversity. In the following result, we show that the LR based SIC detector can also have full diversity (i.e., the diversity order is NK).

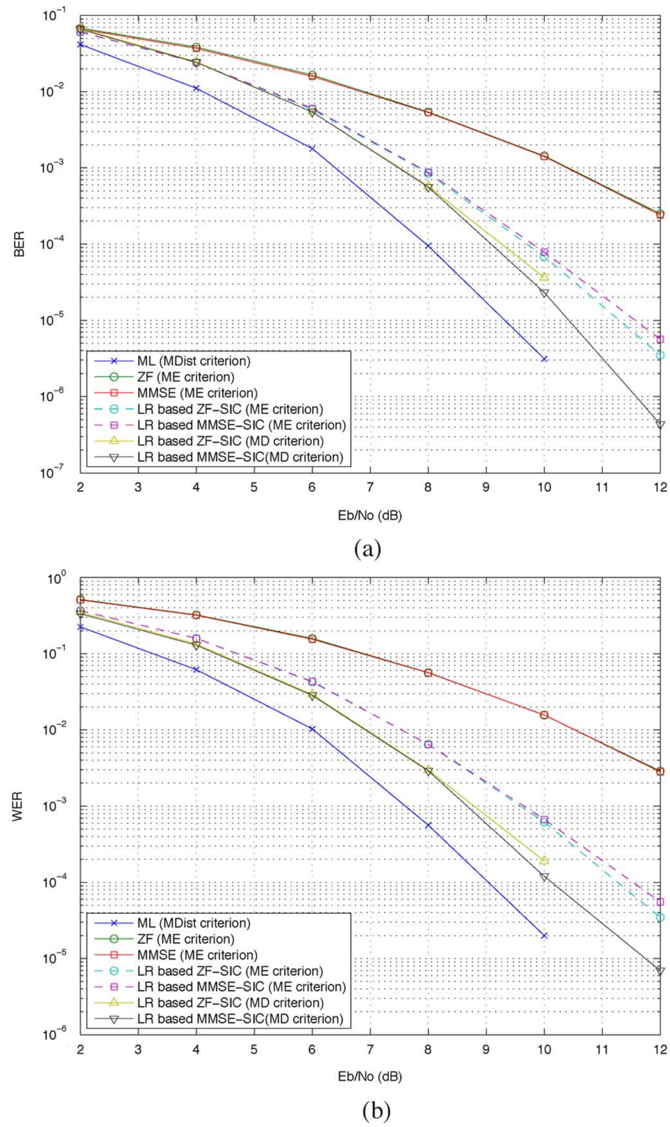

Fig. 2. (a) BER versus E_b/N_0; (b) WER versus E_b/N_0 (16-QAM, $K = 10, Q = N = 4$).

Property 5: Suppose that the MD criterion is employed for the user selection. In addition, the LR based SIC detector is used with the LLL reduced basis. Then, under A1) and A2), the diversity order becomes NK.

Proof: Using (23) and (24), we have

$$\Pr(\text{error}) \le \sum_{\mathbf{d}\in\mathcal{D},\mathbf{d}\neq\mathbf{0}} \exp\left(-\beta^{-Q+1}\frac{\max_k \mathbf{d}^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}}\mathbf{H}_k\mathbf{d}}{2N_0}\right). \quad (32)$$

Then, using the same approach in the proof of Property 4, we can show that the average probability of error becomes $E[\Pr(\text{error})] \le c_4\gamma_{\mathbf{d}}^{-NK} + o(\gamma_{\mathbf{d}}^{-(NK+1)})$, where $c_4 > 0$ is a constant. This completes the proof. ■

V. Simulation Results

In this section, we present simulation results to see the diversity gain from the multiuser diversity and MIMO systems. For simulations, we assume that 16-QAM is used for signaling and MIMO channels are independently generated according to A2).

Fig. 1 shows the bit error rate (BER) and word error rate (WER) of the ML detector when $N = Q = 4$ and $K = 10$. The WER is the

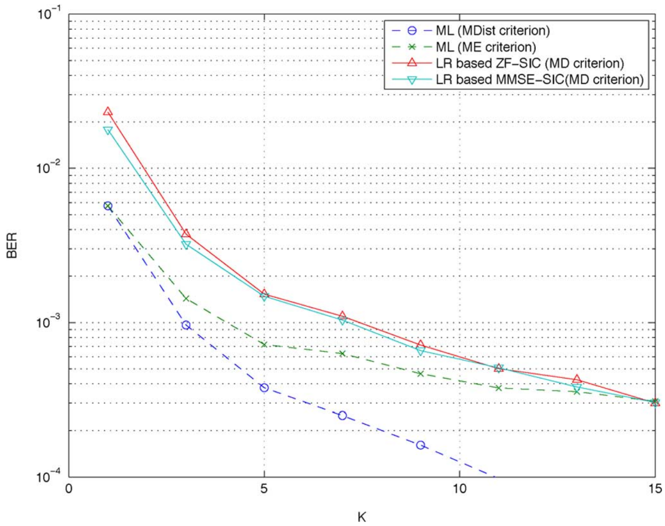

Fig. 3. BER versus K ($N = Q = 4$ and $E_b/N_0 = 8$ dB).

probability that the transmitted symbol vector $\mathbf{s}$ (when $L = 1$) is incorrectly detected. To implement the MDist criterion, the LLL algorithm is used to find $S(\mathbf{H}_k)$. It is shown that the performance with the MDist criterion is better than that with the ME criterion. In addition, we can see that the diversity gain is different: the MDist criterion can provide a better diversity gain. This result is predicted in Section IV. The ML detector with the MDist criterion can fully exploit the diversity gain from receive diversity and multiuser diversity, while only multiuser diversity is exploited when the ME criterion is used for the user selection.

In Fig. 2, we present the BER and WER results of various detectors with different user selection criteria, respectively, when $N = Q = 4$ and $K = 10$. We can see that the LR based SIC detector with the MD criterion can exploit a full diversity as the ML detector with the MDist criterion.

The conventional ZF and MMSE detectors with the ME criterion provide poor performance as they cannot fully exploit spatial diversity. To have a full receive diversity, the LR based detectors can be used. Then, the performance can be improved and the performance gap from the conventional ZF and MMSE detectors increases with the SNR due to a better diversity gain. A further improved performance can be achieved if the MD criterion is used for the user selection.

It is noteworthy that the LR basis has to be found when the LR based detector is used. In addition, when the MD criterion (and MDist criterion) is employed, each user has to find the LR basis. Thus, the computational complexity increases at both transmitter (i.e., mobile station) and receiver (i.e., BS) sides. Fortunately, as shown in [10], the LR basis can be found in a polynomial time and the increased computational complexity would not be significant.

Fig. 3 shows the performance for various numbers of users when $N = Q = 4$ and $E_b/N_0 = 8$ dB. We can see that the BER decreases with K as more multiuser diversity gain is achieved. As the ML detector with the MDist criterion can fully exploit the multiuser and spatial diversity, the performance improvement with increasing K is better. We can also see a performance improvement by increasing K when the LR based detectors are used with the MD criterion. However, as expected, the ML detector with the ME criterion has a slower performance improvement by increasing K as the spatial diversity is not fully exploited.

VI. Concluding Remarks

We derived various user selection criteria based on error probabilities for different MIMO detectors to exploit the multiuser diversity. The ML and suboptimal detectors were considered. It was shown that the user selection criterion is important to fully exploit both multiuser and receive diversity. For example, the ML detector was not able to exploit the receive diversity if the ME criterion is employed for the user selection. We also showed that low complexity suboptimal detectors (i.e., the LR based SIC detector) with the MD criterion for the user selection can fully exploit both multiuser and receive diversity and provide good performance even though their complexity is low.

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