

## RESEARCH ARTICLE

# Iterative decision-directed estimation and compensation of nonlinear distortion effects for OFDM systems

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## ABSTRACT

Orthogonal frequency division multiplexing (OFDM) has been adopted for several wireless network standards due to its robustness against multipath fading. Main drawback of OFDM is its high peak-to-average power ratio (PAPR) that causes a signal degradation in a peak-limiting (e.g., clipping) channel leading to a higher bit error rate (BER). At the receiver end, the effect of peak limitation can be removed to some extent to improve the system performance. In this paper, a joint iterative channel estimation/equalization and clipping noise reduction technique based on minimum mean square error (MMSE) criterion is presented. The equalization weight that minimizes the mean square error (MSE) between the signal after channel equalization and feedback signal after clipping noise reduction is derived assuming imperfect channel state information (CSI). The MSE performance of the proposed technique is theoretically evaluated. It is shown that the BER performance of OFDM with proposed technique can be significantly improved in a peak-limited and doubly-selective (i.e., time- and frequency-selective) fading channel. Copyright © 2011 John Wiley & Sons, Ltd.

## KEYWORDS

OFDM; clipping; decision-feedback frequency domain equalization; fast fading

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## 1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been attracting considerable attention because of its robustness against a frequency-selective fading [1]. The OFDM signals, however, are known to suffer from a high peak-to-average power ratio (PAPR), caused by the addition of a large number of independently modulated subcarriers in parallel at the transmitter. Peak-limiting (i.e., amplitude clipping) is known to be the simplest PAPR reduction technique, but the clipped OFDM signal may undergo a significant nonlinear distortion that introduce an additional clipping noise term. Consequently, the bit error rate (BER) performance may significantly degrade if the clipping noise is left uncompensated.

There have been several iterative techniques that were introduced to reduce the nonlinear distortions due to clipping at the receiver end [2–10]. A maximum likelihood (ML) decision based nonlinear (i.e., clipping) noise reduction for multi-carrier signals (i.e., OFDM) was proposed in Reference [2] while taking into consideration the clipping noise at the receiver over a Gaussian channel (thus, channel equalizer was not considered). Later in 1999, decision-

aided reconstruction (DAR) algorithm was proposed to reduce the clipping noise [3]. ML sequence detector is used to compensate for the clipping noise at the OFDM receiver with perfect knowledge of channel state information (CSI) in Reference [4]. Clipping noise reduction and cancelation technique based on DAR algorithm was proposed in Reference [5]. Iterative reduction of clipping noise was presented in Reference [6] as an extension of Reference [4] to multiple-input multiple-output (MIMO) OFDM based on perfect knowledge of CSI, where the authors use a complex computation of clipping noise estimate using sub-optimal linear minimum mean square error (LMMSE) filtering based on the iterative algorithm given in Reference [5]. The knowledge of noise and distortion variance is required *a priori* to compute the weight of LMMSE filter, which requires an update of noise and distortion variance during each iteration. Moreover, the clipping noise removal algorithm and channel equalization are done independently in Reference [6]. In References [7–9], channel equalization and iterative clipping noise reduction techniques are used independently since the authors only focused on solving the problem caused by nonlinear degradation. In Reference [10], an algorithm was proposed in order to eliminate both

clipping noise and inter-modulation noise, but the model is not accurate enough for a low clipping levels (e.g., 0–4 dB). In particular, the residual clipping noise still cause a degradation of the BER performance and channel equalization is not designed to cope with the residual clipping noise.

Iterative channel estimation/equalization can be applied to further improve the OFDM system performance in a doubly selective (i.e., time and frequency selective) fading channel with imperfect CSI. The clipping noise may not be perfectly compensated for, even if iterative clipping noise reduction techniques presented in References [4–10] are considered—the residual clipping noise errors remain. Consequently, the OFDM system will not be able to cope with a problem of clipping error propagation when iterative channel estimation/equalization is applied to improve the system performance with imperfect CSI. Unlike the previous work [2–10], where channel equalization and non-linear noise reduction are independent, we take a different approach to take into consideration the residual clipping noise errors.

This paper proposes a robust iterative technique to further improve the BER performance of clipped OFDM signal in a doubly-selective (i.e., time- and frequency-selective) fading channel with imperfect CSI. The contribution of this paper is threefold: (1) unlike References [2–10], where equalizer and iterative clipping noise reduction are applied independently, we present a joint iterative channel estimation/equalization and clipping noise reduction based on minimum mean square error (MMSE) criterion to overcome the residual clipping noise error propagation by using the Bussgang theorem [11–13]; (2) we derive an iterative equalization weight based on MMSE criterion by taking into consideration the imperfect CSI and residual clipping noise errors; and (3) unlike References [2–10], where perfect knowledge of CSI is assumed, we present the performance analysis of the proposed method assuming imperfect CSI with clipping noise reduction and imperfect CSI with ideal clipping noise reduction (without residual clipping noise errors). Unlike work in Reference [6], the equalization weight derived based on MMSE criterion does not require *a priori* knowledge of noise and distortion variance. It is shown that the proposed technique can improve the BER performance in a doubly-selective fading channel due to more accurate clipping noise reduction and compensation of the residual clipping noise errors.

The paper is organized as follows. Section 2 presents the system model. The proposed technique is presented in Section 3. The receiver performance is analyzed in Section 4. In Section 5, the simulation results and discussions are presented. Section 6 concludes the paper.

## 2. SYSTEM MODEL

The OFDM system model is illustrated in Figure 1. Throughout this work,  $T_c$ -spaced discrete time representa-

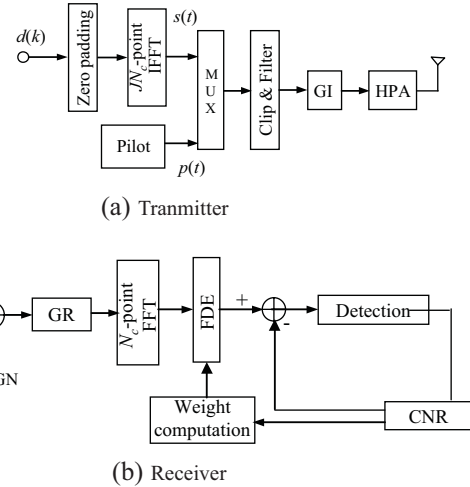


Figure 1. System model.

tion is used, where  $T_c$  represents the fast Fourier transform (FFT) sampling period.

### 2.1. Transmit signal representation

A block data-modulated symbol sequence,  $\{d_m(k); k = 0 \sim N_c - 1\}$  with  $E[|d_m(k)|^2] = 1$ , is transmitted during the  $m$ th ( $m = \dots -1, 0, 1 \dots$ ) signaling interval ( $E[\cdot]$  denotes the ensemble average operation).  $\{d_m(k)\}$  is zero-padded and fed to  $JN_c$ -point inverse FFT (IFFT) to generate an interpolated OFDM signal  $\{s_m(t); t = 0 \sim JN_c - 1\}$  with  $N_c$  subcarriers. Then, the signal amplitude is clipped and filtered [14] to predetermined clipping level  $\beta$  that is known at the receiver. For the sake of brevity, henceforth we refer to clipping and filtering only by clipping. The clipped signal is fed to power amplifier, where the amplifier saturation level equals to the clipping level  $\beta$ . After insertion of guard interval (GI), the signal is power amplified and transmitted over a frequency-selective fading channel.

### 2.2. Received signal representation

At the receiver, after GI removal (GR), the  $JN_c$ -point FFT is applied to decompose the  $m$ th frame's received signal into  $JN_c$  frequency domain components  $\{R_m(k); k = 0 \sim JN_c - 1\}$ , from which the first  $N_c$  signal components are selected.  $R_m(k)$  is given by

$$R_m(k) = \sqrt{2P}d_m(k)H_m(k) + C_m(k) + N_m(k) \quad (1)$$

for  $k = 0 \sim N_c - 1$ , where  $P (= 2E_s/T_c N_c)$ ,  $H_m(k)$ ,  $C_m(k)$ , and  $N_m(k)$ , respectively, denote the average power, FFT of the channel gain, the clipping noise, and additive white Gaussian noise (AWGN) having double-sided power spectral density  $2N_0/T_c N_c$  at the  $k$ th subcarrier.

The distortion in the channel changes the phase and amplitude of each subcarrier.  $R_m(k)$  is corrected by the

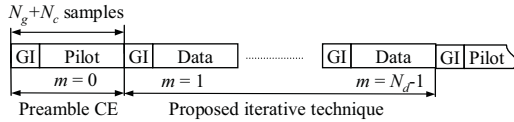


Figure 2. Frame structure.

single tap equalizer done in an iterative fashion to  $R_m(k)$  as

$$\hat{R}_m^{(n)}(k) = R_m(k)w_m^{(n)}(k), \quad (2)$$

where  $w_m^{(n)}(k)$  denotes the equalization weight in the  $m$ th signaling interval at the  $n$ th ( $n = 0 \sim N-1$ ) iteration that minimizes MSE with respect to residual clipping noise errors as derived in subsection 3.2.

### 2.3. Channel estimation and detection

As illustrated in Figure 2, channel estimation (CE) operates in two modes. In the first mode ( $m = 0$ )—preamble CE—the channel gain estimate  $\{H_{0,e}(k); k = 0 \sim N_c-1\}$  at the  $k$ th subcarrier is obtained by reverse modulation, where as a pilot we utilize Chu sequence [15] to avoid a nonlinear distortion (practically the use of Chu sequence with relatively low PAPR will reduce nonlinear distortion effect). In the second mode ( $m = 1 \sim N_d$ )—proposed iterative technique—a joint iterative channel estimation/equalization and clipping noise reduction based on MMSE criterion is applied to compute the channel gain estimate  $\{H_{m,e}^{(n)}(k); k = 0 \sim N_c-1\}$  at the  $k$ th subcarrier during the  $n$ th iteration as presented below.

Finally, after  $N$  iterations, the  $m$ th block decision variables  $\{\hat{d}_m^{(N)}(k); k = 0 \sim N_c-1\}$  are obtained by finding the minimum Euclidean distance between  $\hat{R}_m^{(N)}(k)$  and candidate signal for  $k = 0 \sim N_c-1$ .

## 3. PROPOSED TECHNIQUE

In this section, first the improved clipping noise reduction technique is presented. Later we derive the equalization weight that minimizes MSE between the received signal and feedback signal for each iteration stage. Block diagram and detailed scheme of the proposed method are illustrated in Figures 3 and 4, respectively. The channel gain estimates obtained during the preamble CE mode ( $m = 0$ ) are used during the initial equalization stage. We consider two equalization stages:

- (1) Initial equalization stage, where ZF equalization is used. The ZF weight for the initial equalization stage is derived based on the condition given by  $H(n)w(n) = 1$  [16], which cannot take into consideration the residual clipping noise errors.
- (2) After initial equalization stage, a joint iterative channel estimation/equalization and clipping noise reduction technique is applied.

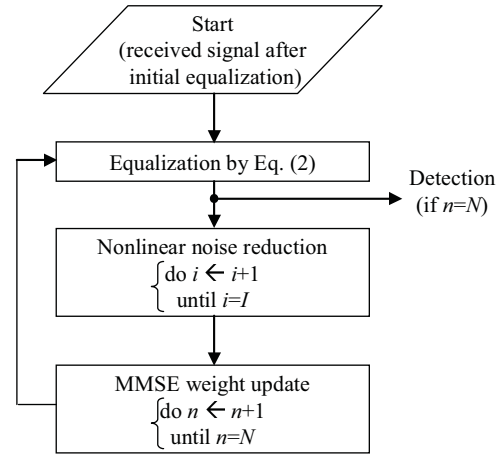


Figure 3. Algorithm block diagram.

### 3.1. Nonlinear noise reduction

The  $m$ th block decision variables after initial ZF equalization, for  $m = 1 \sim N_d$ , at the  $i$ th iteration  $\{\hat{d}_m^{(i)}(k); k = 0 \sim N_c-1\}$  are obtained by finding the minimum Euclidean distance between  $\hat{R}_m^{(n)}(k)$  and all candidate signals with the initial condition  $\hat{R}_m^{(0)}(k) = \hat{R}_m^{(n)}(k)$ . Thus,  $\hat{d}_m^{(i)}(k)$  can be expressed as

$$\hat{d}_m^{(i)}(k) = d_m(k) + \Lambda_{m,c}^{(i)}(k) \quad (3)$$

where  $\Lambda_{m,c}^{(i)}(k)$  denotes the residual clipping noise at the  $i$ th iteration.  $\{\hat{d}_m^{(i)}(k)\}$  is fed to a  $JN_c$ -point IFFT to generate the OFDM signal replica:

$$\hat{s}_m^{(i)}(t) = s_m(t) + \lambda_{m,c}^{(i)}(t) \quad (4)$$

where  $\lambda_{m,c}^{(i)}(t)$  denotes the time domain noise due to  $\Lambda_{m,c}^{(i)}(k)$ . Then, the signal replica is passed through two branches.

The upper branch 1 in Figure 4 regenerates the oversampled transmitted signal in the same fashion as presented in Section 2. The signal is clipped according to predetermined clipping level  $\beta$  and fed to power amplifier with saturation level equal to clipping level  $\beta$  (in Figure 4 clipping block includes the power amplifier for the sake of clear illustration). Using the Bussgang theorem [11], an OFDM signal

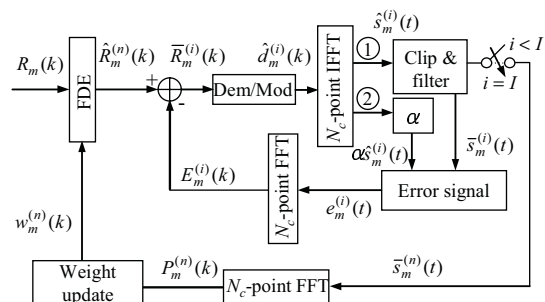


Figure 4. Proposed technique.

after nonlinear device can be expressed as [12,13]

$$\tilde{s}_m^{(i)}(t) = \alpha \hat{s}_m^{(i)}(t) + \tilde{s}_m^{(i)}(t) \quad (5)$$

where  $\tilde{s}_m^{(i)}(t)$  denotes the nonlinear noise and  $\alpha = 1 - \exp(-\beta^2) + (\sqrt{\pi}\beta/2) \operatorname{erfc}(\beta)$  is the attenuation constant [12] with  $\operatorname{erfc}(\cdot)$  representing the complementary error function [11].

The lower branch 2 in Figure 4 regenerates the attenuated time-domain signal replica  $\alpha \hat{s}_m^{(i)}(t)$ . The error signal at the  $i$ th iteration is generated as

$$e_m^{(i)}(t) = \tilde{s}_m^{(i)}(t) - \alpha \hat{s}_m^{(i)}(t) = \tilde{s}_m^{(i)}(t) \quad (6)$$

for  $t = 0 \sim N_c - 1$ . The estimation error  $e_m^{(i)}(k)$  in Reference [10] is generated as

$$e_m^{(i)}(k) = \tilde{s}_m^{(i)}(k) - \hat{s}_m^{(i)}(k) = (\alpha - 1) \hat{s}_m^{(i)}(k) + \tilde{s}_m^{(i)}(k) \quad (7)$$

where in comparison with our model given by Equation (6) an extra nonlinear term  $(\alpha - 1) \hat{s}_m^{(i)}(k)$  is evident. This additional nonlinear noise term can be neglected only for a large clipping level (e.g.,  $\beta > 7$  dB) when  $\alpha \rightarrow 1$ . The work done in Reference [10] can be seen as a special case of our proposal. Equation (7) is a consequence of Bussgang theorem [11–13] and consequently, by using the error model given by Equations (6) and (7), we are able to take into consideration the residual clipping noise errors and derive an iterative equalization weight based on the MMSE criterion, which was not considered in References [2–10]. Thus, our contribution is not just to propose an improved clipping noise reduction method, but also to present the joint iterative channel estimation/equalization and clipping noise reduction method to deal with residual clipping noise errors with imperfect CSI.

$e_m^{(i)}(t)$  given by Equation (6) is fed to  $JN_c$ -point FFT, from which the first  $N_c$  signal frequency components are extracted, to obtain clipping noise error  $\{E_m^{(i)}(k); k = 0 \sim N_c - 1\}$  in the frequency-domain. The estimated clipping noise  $E_m^{(i)}(k)$  is subtracted from  $\hat{R}_m^{(i)}(k)$  to obtain the improved signal reference as

$$\bar{R}_m^{(i)}(k) = \hat{R}_m^{(i)}(k) - E_m^{(i)}(k) \quad (8)$$

Now we increment  $i \leftarrow i + 1$  and compute  $\{\hat{d}_m^{(i+1)}(k)\}$  using  $\bar{R}_m^{(i)}(k)$ . This is repeated  $I$  times to reduce the clipping noise error term in Equation (3).

In the next subsection, the equalization weights update based on MMSE criterion is presented.

### 3.2. MMSE equalization weight update

Assuming imperfect CSI, the ZF weight for the initial equalization stage is derived based on the condition given by  $H(n)w(n) = 1$  [16], which cannot take into consideration the residual clipping noise errors. In order to consider the residual errors, we derive an iterative equalization weight

based on the MMSE criterion using the feedback signal after clipping noise reduction.

To further reduce residual clipping noise, iterative equalization is applied where equalization weights are computed to minimize MSE. Using Equations (3) and (5), a pilot signal generated by decision feedback can be expressed as

$$P_m^{(n)}(k) = \alpha d_m(k) - \tilde{\Lambda}_{m,c}^{(l)}(k) \quad (9)$$

for  $m = 1 \sim N_d - 1$ , where  $P_m^{(n)}(k)$  and  $\tilde{\Lambda}_{m,c}^{(l)}(k)$  denote the decision-feedback pilot and the residual clipping noise after the  $l$ th iteration, respectively. The attenuation factor  $\alpha$ , introduced in Equation (5), is a constant depending only on the clipping level and, thus, it has the same value either in Equation (5) (time domain) or in Equation (9) (frequency domain). Then, the iterative equalization weight is chosen to minimize the mean square error (MSE) term  $E[|\hat{R}_m(k) - P_m^{(n)}(k)|^2]$  during the  $n$ th iteration at the  $k$ th subcarrier, with assumption that the received signal  $R_m(k)$  is corrupted only with clipping noise (i.e., no AWGN noise term in Equation (3)) since feedback signal is not corrupted by AWGN (except the decision errors that are caused by AWGN). Thus, we can write

$$\begin{aligned} \text{MSE}^{(n)}(k) &= \alpha^2 E[|d_m(k)|^2] |\hat{H}_m^{(n)}(k)|^2 \\ &+ E[|C_m^{(n)}(k)|^2] |\hat{H}_m^{(n)}(k)|^2 + 1 \\ &- 2\alpha \Re\{E[|d_m(k)|^2] \hat{H}_m^{(n)*}(k)\} \\ &- 2\Re\{E[d_m(k) \hat{H}_m^{(n)*}(k) C_m^{(n)*}(k)]\} \quad (10) \end{aligned}$$

where  $\Re\{z\}$  and  $(\cdot)^*$  denote the real part of the complex number  $z$  and complex conjugate operation. We note here that the clipping noise is assumed to be a zero-mean complex random variable [12] and, thus, the expectation is taken over the clipping noise to obtain

$$\begin{aligned} \text{MSE}^{(n)}(k) &= \frac{\alpha^2}{N_c} |H_m^{(n)}(k) w_m^{(n)}(k)|^2 + 1 \\ &+ \frac{1 - \exp(-\beta^2) - \alpha^2}{N_c} |H_m^{(n)}(k) w_m^{(n)}(k)|^2 \\ &- \frac{2\alpha}{N_c} \Re\{H_m^{(n)*}(k) w_m^{(n)}(k)\} \quad (11) \end{aligned}$$

By solving  $\{\partial \text{MSE}^{(n)}(k) / \partial w_m^{(n)}(k)\} = 0$ , we obtain the iterative equalization weight as

$$w_m^{(n)}(k) = \frac{[1 - \exp(-\beta^2) + \frac{\sqrt{\pi}\beta}{2} \operatorname{erfc}(\beta)] H_m^{(n)*}(k)}{[1 - \exp(-\beta^2)] |H_m^{(n)}(k)|^2} \quad (12)$$

For large  $\beta$ , i.e.,  $\beta > 7$  dB,  $\exp(-\beta^2) \approx 0$  and  $\alpha \approx 1$  and, therefore, Equation (12) reduces to equalization weight for the system without nonlinear distortion.

ZF equalization based on the condition given by  $H(n)w(n) = 1$  [16] cannot bring us to the solution

represented by Equation (12) since the residual clipping noise errors cannot be considered. The channel gain  $H_m(n)$  in Equation (12) is replaced by the channel gain estimate  $H_{m,e}^{(n)}(k)$ .

#### 4. PERFORMANCE ANALYSIS

In this section, the MSE performance between the signal after FDE and feedback signal after clipping noise reduction of the channel estimator is analyzed. Unlike References [2–6], where perfect knowledge of CSI is assumed, we evaluate the MSE of proposed method assuming (1) imperfect CSI with clipping noise reduction and (2) imperfect CSI with ideal clipping noise reduction (without residual clipping noise errors).

The presence of nonlinear noise degrades the iterative channel estimator performance as shown by Equation (9). The channel gain, for  $m \neq 0$  (in the following we omit the frame index  $m$  and the iteration index  $n$  for the sake of brevity), can be expressed as

$$\begin{aligned} \hat{H}(k) &= \frac{\hat{R}(k)}{P(k)} \\ &= H(k) \frac{\alpha d(k) + E(k)}{\alpha d(k) + \bar{E}(k)} + \frac{N(k)}{\alpha d(k) + \bar{E}(k)} \\ &= H(k) \left[ 1 + \frac{\Delta(k)}{\alpha d(k) + \bar{E}(k)} \right] \\ &\quad + \frac{N(k)}{\alpha d(k) + \bar{E}(k)} \end{aligned} \quad (13)$$

for  $k = 0 \sim N_c - 1$  with  $\bar{E}(k) = E(k) - \Delta(k)$ , where  $P(k)$  and  $\Delta(k)$  denote the feedback pilot given by Equation (9) and the amount of nonlinearity reduced by iterative process, respectively.  $\text{MSE}(k) = E[|\hat{H}(k) - H(k)|^2]$  of the channel estimator is given as

$$\begin{aligned} \text{MSE}(k) &= E \left\{ \left| H(k) \left[ 1 + \frac{\Delta(k)}{\alpha d(k) + \bar{E}(k)} \right] \right|^2 \right\} \\ &\quad + E \left\{ \left| \frac{N(k)}{\alpha d(k) + \bar{E}(k)} \right|^2 \right\} \\ &\quad + E \left\{ \frac{H^*(k)N(k)}{\alpha d(k) + \bar{E}(k)} \left[ 1 + \frac{\Delta(k)}{\alpha d(k) + \bar{E}(k)} \right] \right\} \\ &\quad + 1 - 2\Re \left\{ E[H(k)H^*(k)] + E[H(k)H^*(k)] \right. \\ &\quad \left. \times \frac{\Delta^*(k)}{\alpha d(k) + \bar{E}(k)} \right\} \end{aligned} \quad (14)$$

We assume that  $\bar{E}(k)$  can be approximated as a zero-mean complex random variable. If we assume that the transmit signal power  $P \gg \sigma_c^2$ , which is usually the case, we can expand  $|N(k)/(\alpha d(k) + \bar{E}(k))|^2$  in a polynomial sum. Note

that the IFFT (and/or FFT) does not alter the properties of the clipping noise and that the knowledge of the shape of probability density function (pdf) of the clipping noise is not required since we are not considering the BER analysis. Then, the expectation is done over expanded terms and after some manipulations the MSE averaged over the  $N_c$  subcarriers can be approximated as

$$\text{MSE} \approx \frac{1}{\alpha^2} \frac{\sigma_n^2}{P} \left[ 1 + \frac{3}{\alpha^2} \frac{\sigma_c^2}{P} + \frac{5}{\alpha^4} \left( \frac{\sigma_c^2}{P} \right)^2 + \dots \right] \quad (15)$$

where  $\sigma_n^2$  and  $\sigma_c^2$  denote the variance of AWGN and nonlinear noise, respectively. We note here that MSE given by Equation (15) is not an exact MSE, but an approximate one based on the assumption of no propagation errors due to nonlinear amplifier. The exact MSE analysis taking into consideration the propagation errors may be very difficult, if not impossible.

To evaluate the impact of clipping noise during iterative CE only we assume that the nonlinearity is completely removed by clipping noise reduction process, i.e.,  $\bar{E}(k) = 0$ . In this case Equation (13) can be expressed as

$$\hat{H}(k) = H(k) + \frac{E(k)}{\alpha^2 d(k)} + \frac{N(k)}{\alpha^2 d(k)} \quad (16)$$

and consequently,  $\text{MSE}_c$  can be written as

$$\text{MSE}_c = \frac{1}{\alpha^2} \frac{1}{P} (\sigma_n^2 + \sigma_c^2) \quad (17)$$

The above theoretical expressions for MSE of the iterative channel estimator with clipping noise estimation given by Equations (15) and (17) are illustrated in Figure 5. The performance is illustrated for three cases: (1) without clipping (w/o clip), (2) with clipping (with clip) given by Equation (17), and (3) iterative FDE with clipping noise reduction (Iter) given by Equation (15). We note here that the computer simulation results (denoted by black circles in Figure 5) are only plotted for the case without clipping (i.e., (i)). As shown in References [7–9], due to iterative decision-directed estimation and nonlinear amplifier it is not possible to compare theoretical and computer simulation results. It can be seen from the figure that the MSE performance of the channel estimator with clipping significantly degrades since the feedback pilot signals are corrupted with the clipping noise, while the proposed estimator gives a better performance due to less clipping noise. This is also confirmed by evaluating the BER performance in the following section.

#### 5. SIMULATION RESULTS

The computer simulation conditions are given in Table I. We assume an OFDM signal with  $N_c = 256$  subcarriers, GI length of  $N_g = 32$  samples, oversampling ratio  $J = 4$ , and ideal coherent 16 quadrature amplitude modulation (QAM).

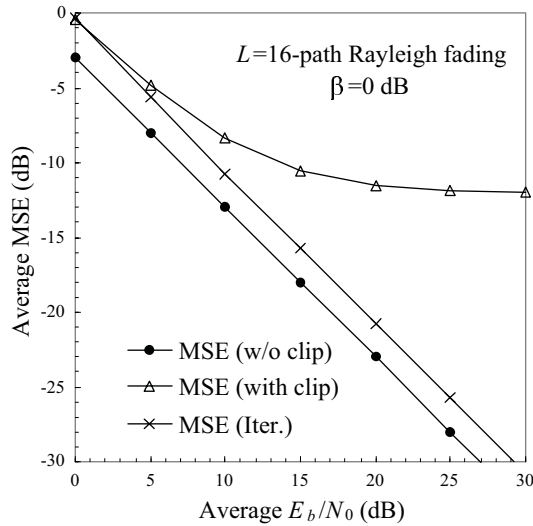


Figure 5. Average MSE performance.

Power amplifier following the Rapp model [17] with nonlinear coefficient  $p = 10$  is assumed. As the propagation channel, we assume an  $L = 16$ -path block Rayleigh fading channel with uniform power delay profile. It is assumed that the maximum time-delay difference is less than the GI length. Initial channel estimates during the pilot mode with  $N_d = 3$  are obtained using Chu sequence [15] to avoid nonlinear distortions (practically the Chu sequence with relatively low PAPR will not eliminate the nonlinear distortion effect completely). In Reference [10], it was shown that the performance improvement is negligible if the number of iterations is larger than or equal to 4 and thus, we set  $N = I = 4$ .

### 5.1. BER performance

Figure 6 shows the average BER performance as a function of the average signal energy per bit-to-AWGN power spectrum density ratio  $E_b/N_0 = (1/\log_2 M) \times (E_s/N_0) \times (1 + N_g/N_c) \times (1 + 1/N_d)$  for the clipping level  $\beta = 3$  and 5 dB. The normalized Doppler frequency  $f_D T_s = 7 \times 10^{-3}$ , where  $1/T_s = 1/[T_c(1 + N_g/N_c)]$  is the transmission symbol rate ( $f_D T_s = 7 \times 10^{-3}$  corresponds to a mobile terminal moving speed of 70 km/h for the 5 GHz carrier frequency and the transmission data rate of  $1/T_c = 100$  M

Table I. Simulation parameters.

Transmitter	Data modulation	16QAM
	No. of subcarriers	$N_c = 256$
	GI	$N_g = 32$
Channel	$L = 16$ -path frequency-selective Rayleigh fading	
Receiver	Initial equalization	ZF
	Iterative equalization	MMSE
	Preamble CE	Reverse modulation
	Iterative CE	Proposed technique

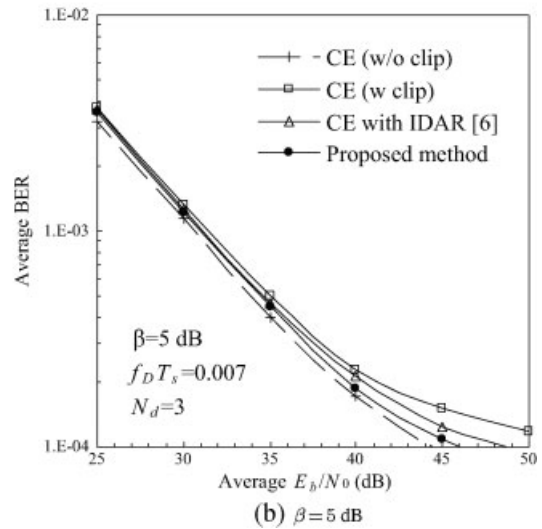
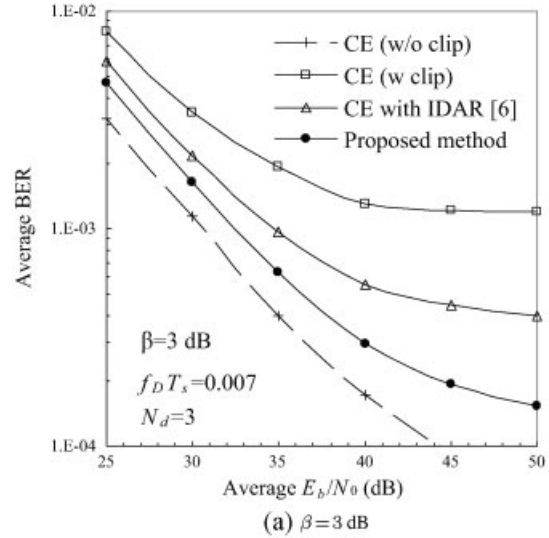


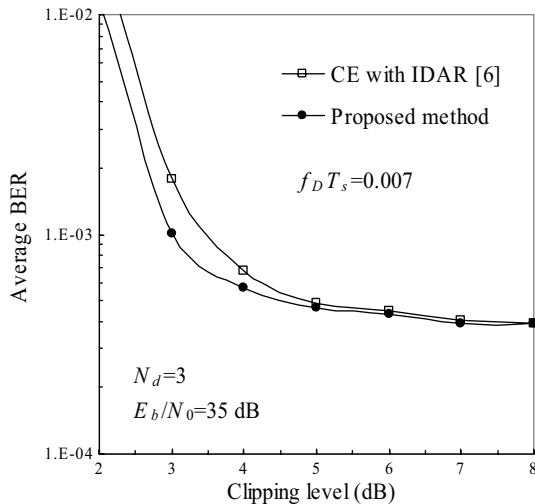
Figure 6. BER versus  $E_b/N_0$ .

symbols/s). As a reference we consider work in Reference [10] which uses the similar clipping noise reduction structure as our proposal presented in Section 3. Moreover, unlike References [2–9], our proposed technique is able to reduce and filter inter-modulated frequency components produced by clipping as shown in Reference [10].

The figure shows that the method presented in Section 3 improves the BER performance in comparison with technique presented in Reference [10]; for average  $BER = 10^{-3}$ , the required  $E_b/N_0$  is reduced by about 2.5 dB when  $\beta = 3$  dB. This is because the proposed method can mitigate the negative effect of the clipping noise through more accurate clipping noise model and in addition reduces the residual clipping noise through iterative nonlinear equalization. For  $\beta = 5$  dB the proposed method achieves almost the same BER performance for  $E_b/N_0 < 25$  dB, while for  $E_b/N_0 > 25$  dB the BER performance with the proposed technique slightly improves. In both cases improved

**Table II.** Computational complexity.

Technique	Number of complex multiplications
Proposed method	$3 \times l \times N \times N_c \log_2 N_c$
Reference [10]	$2 \times N \times N_c \log_2 N_c$

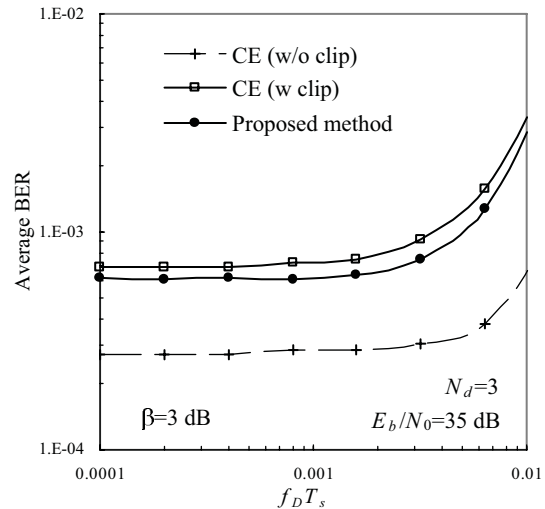
**Figure 7.** BER versus clipping level  $\beta$ .

BER performance is achieved in comparison with the method presented in Reference [10]. Note that the BER performance approaches the floor value due to two factors (1) residual clipping noise and (2) error propagation due to decision-feedback equalization. In the above discussion, only uncoded performance is presented, while the performance can be improved by channel coding. The performance with channel coding is left as future work.

The computational complexity comparison of the proposed method and iterative receiver in Reference [10] is shown in Table II. Note that computational complexity is defined by the required number of complex multiplications. It can be seen from the table that the BER performance improvement of the proposed algorithm in comparison with Reference [10] is achieved at an increased computational complexity.

### 5.2. Impact of clipping level

Figure 7 shows the average BER performance as a function of the amplitude clipping level  $\beta$  for the  $E_b/N_0 = 35$  dB and  $f_D T_s = 7 \times 10^{-3}$ . It can be seen from the figure that the proposed technique achieves a lower BER if the clipping level is less than 6 dB. Further increase in  $\beta$  does not affect the BER. This is due to the fact that the proposed model will converge to the model presented in Reference [10] because the attenuation factor  $\alpha \rightarrow 1$  for  $\beta > 6$  dB. In this work, the impact of nonlinearity on the signal spectrum is not considered since we do not assume the power spectrum mask (i.e., out of band spectrum limitations [18]).

**Figure 8.** BER versus  $f_D T_s$ .

### 5.3. Impact of fading rate

Since decision-feedback equalization is applied, the BER performance depends on the fading rate (i.e.,  $f_D T_s$ ). In the following the impact of fading rate is discussed.

Figure 8 shows the average BER as a function of  $f_D T_s$  when the clipping level  $\beta = 3$  dB for  $N_d = 3$  and  $E_b/N_0 = 35$  dB. In Figure 8,  $f_D T_s = 10^{-4}$  corresponds to a mobile terminal moving speed of about 10 km/h for 5 GHz carrier frequency and transmission data rate of 100 M symbols/s. For comparison, the BER without clipping is also plotted. It can be seen from the figure that the average BER remains constant until  $f_D T_s$  reaches about 0.001 and starts to degrade due to tracking error as  $f_D T_s$  value increases.

## 6. CONCLUSION

In this paper, we presented a joint channel estimation/equalization and clipping noise reduction technique to further improve the performance of OFDM with imperfect CSI in a doubly-selective fading channel. The iterative equalization weight that minimized MSE between the received signal after FDE and feedback signal after clipping noise reduction was derived for each iteration stage by taking into consideration the residual clipping noise errors. It was shown that the performance of OFDM with the proposed technique can be significantly improved in a nonlinear and doubly-selective fading channel.

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