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Full Length Research Paper

Estimation of multipath time delays based on bi-sparse constraint

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In this paper, a novel method to estimate the multi-path time delays of broadband signal transmission is proposed. This new method extends the existed basis selection methods. The difference between the problems discussed here and that of basis selection is the occurrence of an unknown weight matrix in the measurement equation, which represents the frequency characteristics of the received signal. Existed basis selection methods are insufficient to solve the problem. We introduced an iterative algorithm in consideration of bi-sparse constraint on solutions of a constructed equation and developed a method based on order recursive matching pursuit (ORMP) to obtain the solution. Both solution vectors and weight matrix are updated iteratively in our algorithm. Validation of the proposed method is confirmed by simulations.

Key words: Multi-path time delay, bi-sparse constraint, order recursive matching pursuit (ORMP).

INTRODUCTION

Time difference of arrival (TDOA) of the signal received by multiple sensors can be used to determine the location of the source (Smith and Jonathan, 1987), such as electronic warfare and mobile terminal positioning. Due to the multipath transmission of wireless signals, in many propagation environments, each sensor receives not only a direct-path signal but also one or more multipath replicas of the signal. There are many methods for estimating multipath time delay (Belanger, 1996; Cater, 1987; Hou and Wu, 1982; Chen et al., 2010).

The problem addressed in this paper is the estimation of direct-path and multipath TDOA in the presence of correlated multipath interference. We extended the classical basis selection methods (Adler et al., 1997; Chen and Wigger, 1995; Irina et al., 1997) to solve this.

Problem formulation

It is assumed that the unknown transmitted signal can be problem represented in complex envelope form as a sample from a band-limited, stationary and Gaussian stochastic process (Belanger, 1996). Suppose there are two sensors and the direct-path signal on sensor 1 is considered to be the reference signal, the received signals $x_1(n)$ and $x_2(n)$ on these two sensors can be expressed using the following equations:

$$x_1(n) = a(n)\mathrm{e}^{j\omega n} + \sum_{m=1}^{L_1} a(n-\tau_{1m})\mathrm{e}^{j\omega(n-\tau_{1m})} + v_1(n) \quad (1)$$

$$x_2(n) = \sum_{m=1}^{L_2} a(n-\tau_{2m})\mathrm{e}^{j\omega(n-\tau_{2m})} + v_2(n) \quad (2)$$

where $a(n)$, $n = 1,\cdots,N$ is the complex envelope of the reference signal. τ_{1m} and τ_{2m} represent the time delays to be estimated between multi-path replicas and the reference signal on sensors 1 and 2, respectively. The simply system model is shown in Figure 1. It should be noted that we treat the direct signal on sensor 2 as a multi-path one and estimate its TDOA with respect to the reference signal. L_1 and L_2 are the sampling number of multi-path signals on sensor 1 and sensor 2, respectively. $v_1(n)$ and $v_2(n)$ are white Gaussian random processes characterizing the additive noise.

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If we ignore the noise and make the assumption that $\alpha = \max(\tau_{1m}, \tau_{2m})/N << 1$ is satisfied, we arrive at to the following representation of the received signals in frequency domain:

$$X_1(k) \approx A(2\pi k - N\omega)\left[1 + \sum_{m=1}^{L_1} e^{-j2\pi k \tau_{1m}/N}\right] \quad (3)$$

$$X_2(k) \approx A(2\pi k - N\omega)\sum_{m=1}^{L_2} e^{-j2\pi k \tau_{2m}/N} \quad (4)$$

where $A(2\pi k - N\omega) = \sum_{n=1}^{N} a(n)e^{j(\omega - 2\pi k/N)n}$, $k = 0, ..., K-1$ and N denotes the number of Discrete Fourier Transform (DFT). K is the number of frequency samplings. A direct result derived from Equations (3) and (4) is:

$$X_2(k)\left[1 + \sum_{m=1}^{L_1} e^{-j2\pi k \tau_{1m}/N}\right] \approx X_1(k)\sum_{m=1}^{L_2} e^{-j2\pi k \tau_{2m}/N} \quad (5)$$

which is important for estimating the multi-path time delays. Based on this equation, searching algorithm can be derived to find τ_{1m} and τ_{2m}. In the following section, we develop an algorithm to estimate τ_{1m} and τ_{2m} based on the basis selection.

MULTI-PATH TIME-DELAY ESTIMATION BASED ON BI-SPARSE CONSTRAINT ON SOLUTIONS

The matrix formulations corresponding to Equations (3) to (5) are the following:

$$\vec{\mathbf{X}}_1 = \mathbf{WE}\vec{\mathbf{s}}_1 \quad (6)$$

$$\vec{\mathbf{X}}_2 = \mathbf{WE}\vec{\mathbf{s}}_2 \quad (7)$$

$$\mathbf{A}_1\vec{\mathbf{s}}_1 = \mathbf{A}_2\vec{\mathbf{s}}_2 \quad (8)$$

where $\vec{\mathbf{X}}_i \in R^{K\times 1}, i = 1, 2$ are column vectors constructed by DFT of received signal having the form of $\vec{\mathbf{X}}_i = [X_i(0), ..., X_i(K-1)]^T$. The matrix $\mathbf{W} \in R^{K\times K}$ is a diagonal matrix whose diagonal elements are $A(2\pi k - N\omega | 1, N)$, $k = 0, \cdots, K-1$. The matrix $\mathbf{E} \in R^{K\times M}$ $K < M$ denotes the over-complete dictionary made up of atoms of $\vec{\mathbf{e}}_i^T = [1, .., e^{-j2\pi(N-1)\tau_i/N}]$, $i = 1, \cdots, M$. The matrices $\mathbf{A}_1 \in R^{K\times M}$ and $\mathbf{A}_2 \in R^{K\times M}$ are considered as over-complete dictionaries, respectively. They have the form of $\mathbf{A}_1 = \mathbf{X}_2 \odot \mathbf{E}$ and $\mathbf{A}_2 = \mathbf{X}_1 \odot \mathbf{E}$, where $\odot$ denotes element by element product and $\mathbf{X}_1 = [\vec{\mathbf{X}}_1, ..., \vec{\mathbf{X}}_1]$, $\mathbf{X}_2 = [\vec{\mathbf{X}}_2, .., \vec{\mathbf{X}}_2]$. The element of the solutions s_{ij}, $i = 1, 2$ is nonzero if and only if its corresponding τ_j is one of the true multi-path time delays. Therefore, the time-delays can be obtained from the nonzero elements of $\vec{\mathbf{s}}_i$.

Since $\mathbf{W}$ and $\vec{\mathbf{s}}_i$ are all unknown in Equations (4) and (5), the classical basis selection methods are no longer and capable of solving the problem. However, as the earlier analysis, these two equations are related by Equation (6). It is noted that both $\vec{\mathbf{s}}_1$ and $\vec{\mathbf{s}}_2$ have the property of sparsity in Equation (6), we name this property as bi-sparse. If we have obtained one of the solutions $\vec{\mathbf{s}}_i$, then the other solution $\vec{\mathbf{s}}_j$, $j \neq i$ can be achieved based on (6). Thus, the iteration process is illuminated from this idea and is stated as follows:

Step 1: Generate over-complete dictionary $\mathbf{E}$ in terms of all possible multi-path time delays;
Step 2: Initialization: $k = 0$, $\vec{\mathbf{W}}^{(02)}$ and $\vec{\mathbf{s}}_i^{(0)}$, $i = 1, 2$;
Step 3: Iteration.

(1) Update $\vec{\mathbf{s}}_1$: based on $\begin{bmatrix}\mathbf{E}\\ \mathbf{A}_1\end{bmatrix}\vec{\mathbf{s}}_1^{(k+1)} = \begin{bmatrix}\vec{\mathbf{X}}_1 / \vec{\mathbf{W}}^{(k2)}\\ \mathbf{A}_2\vec{\mathbf{s}}_2^{(k)}\end{bmatrix}$, using the method of order recursive matching pursuit (ORMP) (Adler et al., 1997; Chen and Wigger, 1995), $\vec{\mathbf{s}}_1^{(k+1)}$ is calculated.

(2) Update $\vec{\mathbf{W}}$: $\vec{\mathbf{W}}^{(k1)} = \vec{\mathbf{X}}_1 / (\mathbf{E}\vec{\mathbf{s}}_1^{(k+1)})$.

(3) Update $\vec{\mathbf{s}}_2$: use $\begin{bmatrix}\mathbf{E}\\ \mathbf{A}_2\end{bmatrix}\vec{\mathbf{s}}_2^{(k+1)} = \begin{bmatrix}\vec{\mathbf{X}}_2 / \vec{\mathbf{W}}^{(k1)}\\ \mathbf{A}_1\vec{\mathbf{s}}_1^{(k+1)}\end{bmatrix}$ and ORMP to update $\vec{\mathbf{s}}_2$. $k = k+1$;

Step 4: If the location of the nonzero elements of $\vec{\mathbf{S}}_1$ and $\vec{\mathbf{S}}_2$ do not change, exit; else, go to Step 3.

In the earlier iterations, $\vec{\mathbf{W}}$ is a column vector whose elements are composed of diagonal elements of $\mathbf{W}$. The operation "/" is element by element division.

In our method, we use ORMP (Adler et al., 1997; Chen and Wigger, 1995) to derive sparse solution. The advantage of using ORMP is its capability to achieve adjustable number of nonzero elements of solution. Thus, if the numbers of multi-paths replicas, that is, L_1 and L_2, are known, running ORMP at the corresponding times, we can achieve required number of time delay estimations.

SIMULATION RESULTS

To give an indication of the behavior of the method discussed in the previous section, we devised the following test scenario. The number of DFT is supposed to be 2048 and the number of frequency sampling is 128. We use 512 τ_i to generate the over-complete dictionary and the time interval is assumed to be unit. Two multi-paths with respect to two sensors are considered, that is, $L_1 = 1$ and $L_2 = 2$ and the true time delays are $\tau_{11} = 0$, $\tau_{12} = 135$ and $\tau_{21} = 85$, $\tau_{22} = 355$. The complex amplitude of the received signal is assumed to be a Gaussian random process with variance equal to 0.2.

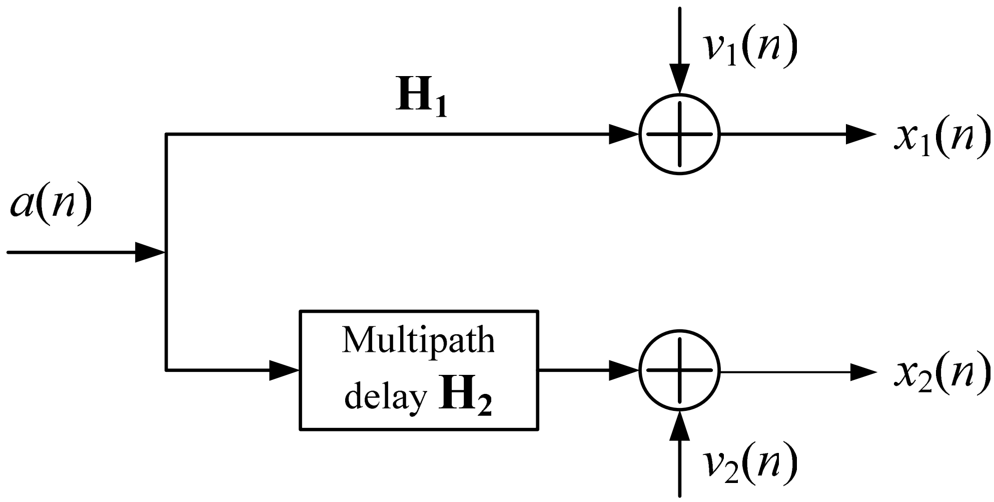

Figure 1. System model with one source and two sensors.

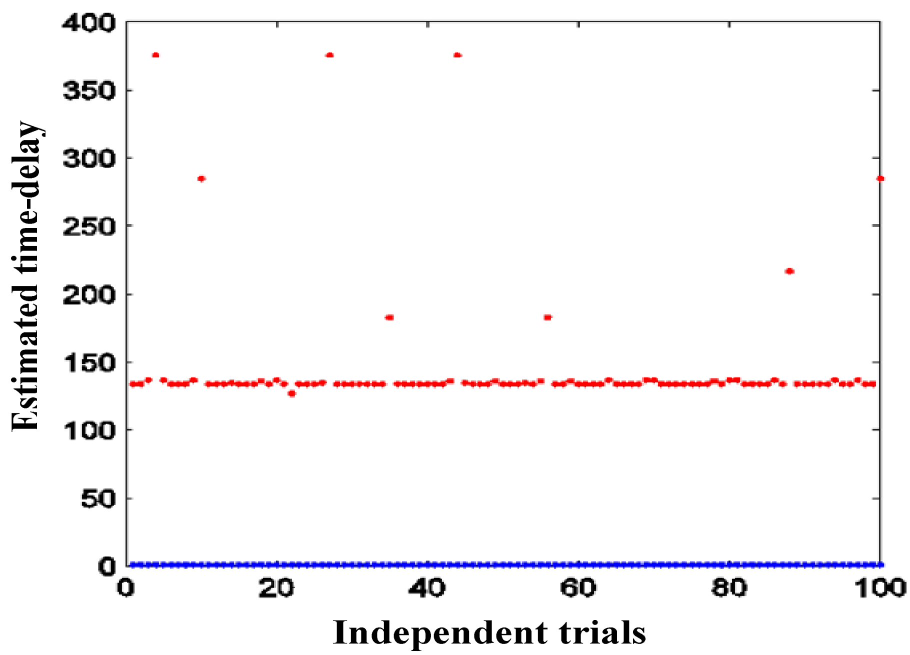

Figure 2. The estimation of time-delays on sensor 1 (noiseless).

Since the objective function of deriving sparse solution is non-convex, local minimums do exist and the algorithm may converge to a wrong solution (Gorodnitsky and Rao, 1997). However, this does not always happen as shown in simulations. Actually, we may calculate the residue of (6) to determine whether a minimum is reached. If the residue is beyond a preset threshold, the current estimation should be thrown away and we may run the algorithm with another initialization.

In the first experiment, we tried the proposed method without consideration of noise and 100 independent trials which were conducted. The result is shown in Figures 2 and 3. It is observed that the estimation result is satisfactory. In the second experiment, noise is taken into account and the signal-to-noise ratio (SNR) is assumed to be 13dB. The result of 100 independent trials is given in Figures 4 and 5. Under the distortion of additive noise, the successful rate of the algorithm decreases, but the estimation result is still acceptable. In these two experiments, we did not preset threshold to throw away the bad estimation. As for those bad estimations, if the threshold was used, successful estimation would be

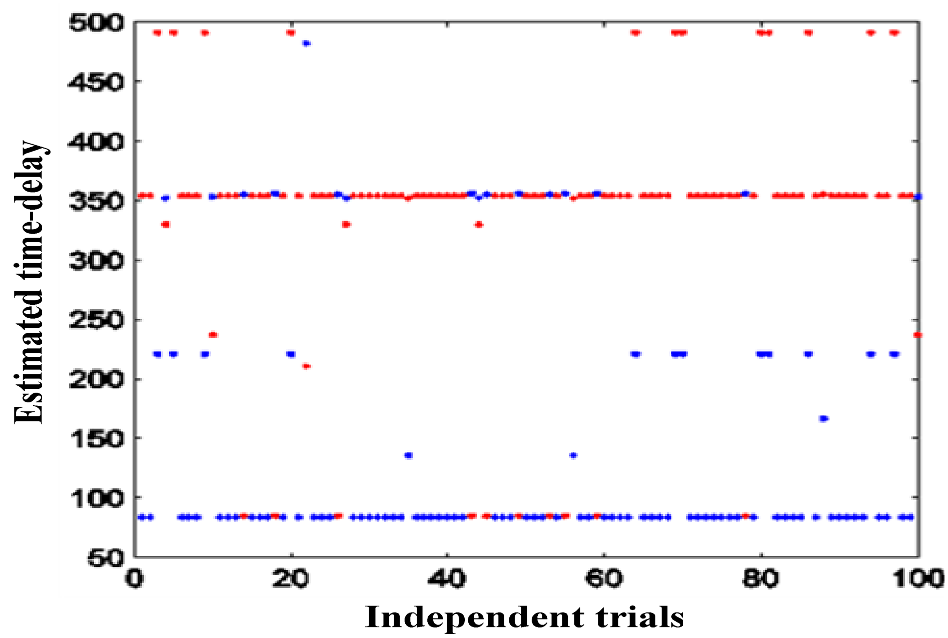

Figure 3. The estimation of time-delays on sensor 2 (noiseless).

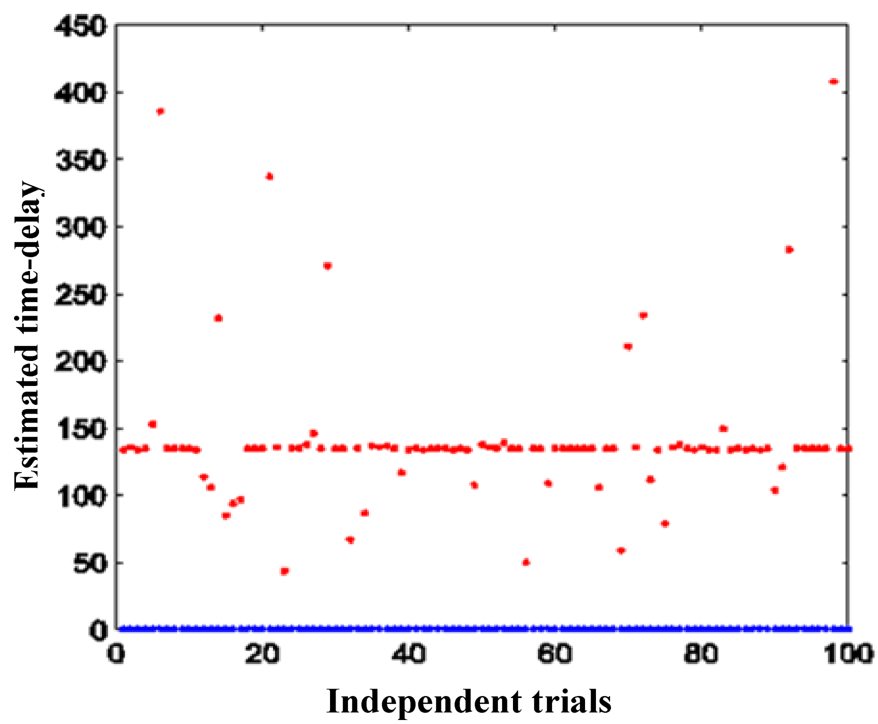

Figure 4. The estimation of time-delays on sensor 1 (SNR = 13 dB).

ready to get.

We also run our algorithm when the number of multi-path replicas increased to three and four. The success rate decreased at this time due to the increase of number of basins (Gorodnitsky and Rao, 1997). However, as mentioned earlier, we may run the algorithm with another initialization whenever the residue of (6) of current trial is beyond the preset threshold. By doing this, a successful

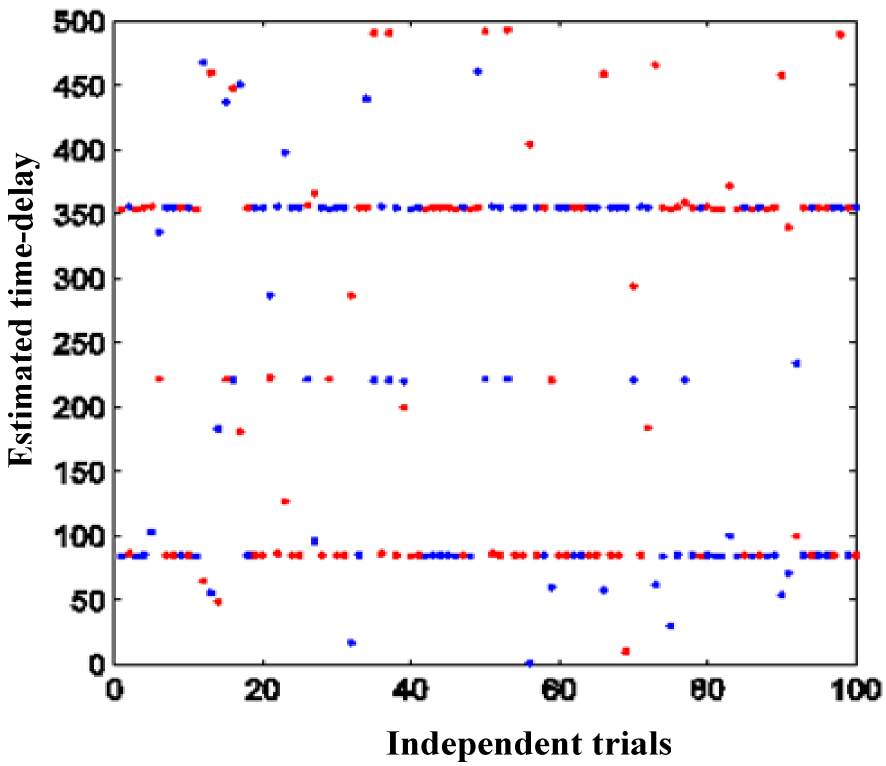

Figure 5. The estimation of time-delays on sensor 2 (SNR = 13 dB).

estimation can be achieved. We only mention the result here instead of giving the figure out and it is stated that our method is still effective as the number of multi-paths increases.

Conclusion

In this paper, we proposed a brand new method to estimate TDOA in multi-path propagation environments. The iterative process is based on two measurement equations and a constructed equation having the property of bi-sparsity. An iterative process is presented to derive the sparse solution based on the principle of ORMP. Simulations verify the validation of our method.

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