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Sparse signal recovery with OMP algorithm using sensing measurement matrix

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Abstract: Orthogonal matching pursuit (OMP) algorithm with random measurement matrix (RMM), often selects an incorrect variable due to the induced coherent interference between the columns of RMM. In this paper, we propose a sensing measurement matrix (SMM)-OMP which mitigates the coherent interference and thus improves the successful recovery probability of signal. It is shown that the SMM-OMP selects all the significant variables of the sparse signal before selecting the incorrect ones. We present a mutual incoherent property (MIP) based theoretical analysis to verify that the proposed method has a better performance than RMM-OMP. Various simulation results confirm our proposed method efficiency.

Keywords: orthogonal matching pursuit (OMP), mutual incoherent property (MIP), sparse signal recovery, compressed sensing (CS), sensing measurement matrxi (SMM)

Classification: Science and engineering for electronics

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1 Introduction

Linear inverse problem often arises in applied mathmatics and engineering applications such as sparse channel estimation for wireless communication [1, 2]. In this case, accurate channel estimation with fewer designed training signal is a critical problem due to the scarcity of spectral resource. To acquire it, the training signal should be designed to statisfy restricted isometry property (RIP) [3] or mutual incoherent property (MIP) [4, 5] with high probability, e.g., the optimal design is that the probabiliy attains to 1. Traditionally, optimal training design can not implement due to coherent interference of colums in training signal. Hence, the problem based on a small number of measurements is fundamental problem in signal processing. Specifically, a typical complex system model is given as follows

$$\phi = X\beta + n, \tag{1}$$

where ϕ is an M-dimensional observed vector, X is an $M \times N$ random measurement matrix, and n is an M-dimensional measurement noise vector. The goal of sparse signal recovery is to reconstruct the unknown K-sparsity N-dimensional complex vector β, i.e., the number of nonzero variables of β is K and $K \ll N$. Conventional methods of signal recovery resolve overdetermind problem ($M \geq N$) with linear algorithms, such as least square (LS) and minimum mean square error (MMSE). In other words, accurate signal recovery acquires the number of measurements M to be larger than dimension N of the unknown signal. This results in resource waste of measurements. However, for high-dimensional signal recovery problem, it is very challenging if $M \ll N$. It is worth mentioning that if the measurements satisfies $M \geq C \cdot K \log(N/K)$, where C denots constant parameter, then the sparse signal is recovered with a very high probability [4].

Fortunately, in practical environments, most of high-dimensional signals have the inherent sparse structure. By exploiting this sparsity, we can improve the performance of signal recovery or utilize smaller number of measurements ($M \ll N$) to reconstruct a high-dimensional sparse signal, which is corrupted by measurement noise. Orthogonal matching pursuit (OMP) [4] is a canonical greedy algorithm for high-dimensional sparse signal recovery with the over-complete measurement matrix given that $M \ll N$. OMP algorithm combines the simplicity and the fastness for high-dimensional sparse

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signal recovery. Hence it is easy to implement in practice. Currently, there existes two kinds of theoretical analysis of OMP, namely mutual incoherence property (MIP) [4, 5] and restricted isometry property (RIP) [3].

For an unknown signal vector $\beta=[\beta_1,...,\beta_N]^T$ and random measurement matrix (RMM) $X = [X_1, X_2, ..., X_N]$, we define several concepts which will be utilized in the following analysis. $\mathrm{supp}(\beta) = \{i : \beta_i \neq 0\}$ denotes the support of signal vector β which is a K-sparse signal if $|\mathrm{supp}(\beta)| \leq K$. To evaluate the reconstruction performance of sparse signal, mutual incoherence property (MIP) [4, 5] of measurement matrix is a widely used tool. Commonly, the mutual incoherence between columns of measurement matrix X is defined by

$$\mu(X, X) = \max_{i \neq j} |\langle X_i, X_j\rangle| , \tag{2}$$

where $\langle\cdot,\cdot\rangle$ denotes inner product operation of two column vectors. Obviously, we can find that smaller mutual incoherence $\mu(X, X)$, means that the measurement matrix X has a better MIP. A previous work [5] has proved that if the measurement matrix X satisfieds $\mu(X, X) \leq (2K - 1)^{-1}$, then K-sparse signal can be recovered accurately. However, due to the signal sparsity, this information can not be utilized in practice. In this letter, we compare the K-sparse signal β with OMP by utilizing RMM and sensing measurement matrix (SMM) under the system model (1). OMP algorithm is an iterative greedy algorithm. It selects a column of measurement matrix which has the most correlation with current residuals at each step. Hence, the chosen variable is added into the set of selected variables. The algorithm updates the residuals by projecting the signal onto the variables which have already been selected and then the algorithm iterates.

2 OMP for sparse signal recovery

In this section, we describe the OMP algorithm with RMM and SMM from a MIP perspective. Consider the $M \times N$ complex RMM X, where $X_i, i = 1, ..., N$ denotes its i-th column vector and assume that each column of X is normalized so that $\|X_i\|_2 = 1$. We define $X(\xi)$ as a submatrix of X for any subset $\xi \subset \{1, 2, ..., N\}$ and term X_i and $X(\xi_i)$ as i-th column and selected ξ_i-th column of X, respectively.

2.1 RMM-OMP for sparse signal reovery

RMM-OMP iteratively selects a column X_i in X that correlates most strongly with the residual signal $r_i = \phi - X_{i-1}\beta_{i-1}$. At each iterative step i, the optimal column $X(\xi_i)$ is selected as

$$X(\xi_i) = \underset{\xi_i, i=1,2,..,N}{\arg\max} |\langle X_i, r_i\rangle| , \text{ for } i = 1, ..., N, \tag{3}$$

where $\langle\cdot,\cdot\rangle$ denotes inner product operation of two column vectors. If the accurate column is selected, we update the selected subset $X(\xi_i) = X(\xi_{i-1}) \cup \{X_{\xi_i}\}$. Set $r_0 = \phi$ and $r_{i+1} = (I_M - P_i)\phi$, where P_i denotes the projection into the linear space spanned by the elements of $X(\xi_i)$, $I_M$is an

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$M \times M$ identity matrix. As a result of the coherence of columns of X, e.g., $\xi^{(1)} = |\langle X, \phi\rangle| = |\langle X, X\beta\rangle| \neq \xi$ for $i = 1$. Now the question is how to mitigate this coherent interference in the OMP algorithm. We caculate the coherence between columns in X as shown in Fig. 1. We find that most digonal coefficients are close to 1 and some of off-diagonal coefficients are close to 0.4. Therefore, these off-diagonal coefficients result in interference while selecting the optimal column in the measurement matrix. To mitigate the interference, we design the SMM for OMP in the next section.

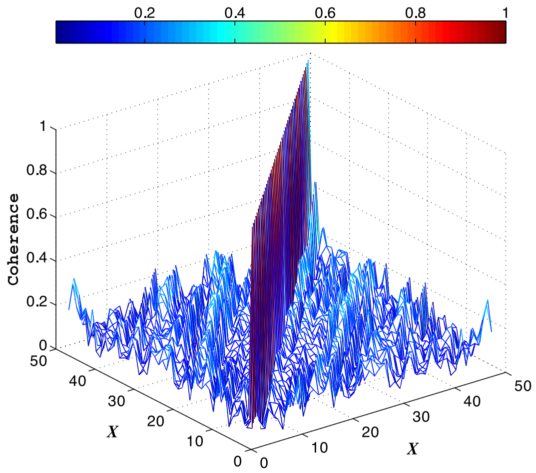

Fig. 1. Coherence betweeb columns in X.

2.2 SMM-OMP for sparse signal recovery

In order to identify the correct components in a coherent random measurement matrix, the improved OMP algorithm designs a $M \times N$ complex SMM $W = W_i, i = 1, ..., N$ and uses $\xi_i = \arg\max|\langle W_i, r_i\rangle|$ rather than $\xi_i = \arg\max|\langle X_i, r_i\rangle|$ in OMP. Obviously, when $W = X$, the OMP algorithm is a special case of the improved OMP. A good SMM should have a $\mu(W, X)$ as small as possible. In a straightforward way, we may calculate the correlation between X and W, i.e., the sensing vector W_i, as the solution to the following convex optimization problem:

$$\underset{W_i}{\text{minimize}} \left\{ \left\| W_i^H X(\xi) \right\|_\infty^2 + \lambda \cdot \|W_i\|_2^2 \right\} \quad \text{s.t.} \quad X_i^H W_i = 1, \tag{4}$$

where $\|a\|_\infty = \max\{|a_i|, i = 1, ..., N\}$ and $\|a\|_2^2 = \sum_{i=1}^N |a_i|^2$ with respect to signal vector a, $X(\xi)$ consists of correct columns corresponding to the Kth nonzero componments of β in (1) and λ is a regularized parameter which has a relationship with the noise level. The closed-form solution for (4) is

$$W_i = \Xi_i X_i, \text{ for } i = 1, 2, ..., N, \tag{5}$$

where Ξ_i is given by

$$\Xi_i = \frac{1}{X_i^H (X(\xi)X(\xi)^H + \alpha I_M)^{-1} X_i} \cdot (X(\xi)X(\xi)^H + \alpha I_M)^{-1}, \tag{6}$$

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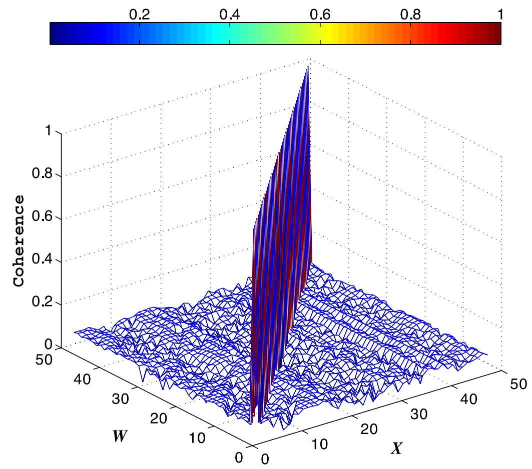

Fig. 2. Coherence between columns in X and W.

where α is a positive regularized parameter, $(\cdot)^{-1}$ and $(\cdot)^H$ denote inverse operation and pseudoinverse operation of matrix, respectivley. When X_i is a column vector of $X(\xi)$, i.e., a correct selected column, the minimum variance condition (4) will mitigate the correlation between the corresponding sensing vector W_i and other correct columns; Whereas the distortionless response constraint (5) will maintain the correlation between W_i and the correct selected X_i. As a result, the nonzero variables of β, which correspond to the correct columns, are estimated with a distortion as small as possible. On the other hand, when X_i is not a column vector of $X(\xi)$, the minimum variance condition (4) will prevent false columns being selected through mitigating the correlation between W_i and all the correct columns. The aforementioned fact is given as an example in Fig. 2. The detail of SMM-OMP algorithm lists as follows:

Input: Observation signal vector ϕ and SMM W.

Output: Sparse signal vector β_{SMM}

Step 1: Initialize the residual $r_0 = \phi$ and the set of selected variable $X(\xi_0) = \emptyset$. Let iteration counter $i = 1$;

Step 2: Find the variable X_i hat solves the maximization problem $\max_i |\langle W_i, r_i \rangle|$ and add the variable X_i to the set of selected variables. Update $X(\xi_i) = X(\xi_{i-1}) \cup \{X_i\}$.

Step 3: Let $P_i = X(\xi_i)(X(\xi_i)^H X(\xi_i))^{-1} X(\xi_i)^H$ denote the projection onto the linear space spanned by the elements of $X(\xi_i)$. Update $r_i = (I - P_i)\phi$.

Step 4: If the stopping condition is achieved, stop the algorithm. Otherwise, set $i = i + 1$ and return to Step 2.

3 Simulation result

To gain some insights into the effect of the proposed SMM-OMP on sparse signal recovery, we evaluate 10000 independent Monte-Carlo trials. The nonzero variables of sparse signal β are generated randomly from a Guassian distribution and subject to $\|\beta\|_2^2 = 1$. The signal length is set to $N = 48$ and the number of measurements are set from 16 to 40. The positions of nonzero

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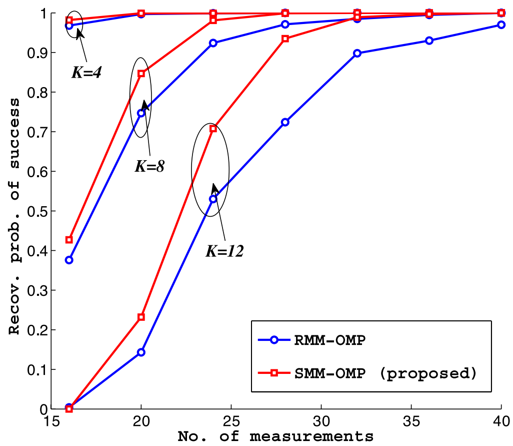

Fig. 3. Successful recovery probability versus number of measurements via sparsity $K = 4, 8, 12$.

variables of β are generated randomly. Consider the signal to noise ratio (SNR) as $\mathbf{SNR} = 10\,\mathbf{dB}$. Simulation result is shown in Fig. 3. We observe that the SMM-OMP has a better recovery performance than RMM-OMP on sparse signal recovery. Regarding the fact that the sparse signal recovery is a high-determind recovery problem, OMP algorithm can obtain a better perforemance if the signal is more sparser.

4 Conclusion

In this paper, we have investigated the OMP for sparse signal recovery while considering the coherent mitigation of measurement matrix. We proposed an SMM-OMP to mitigate the coherent interference between the columns of measurment matrix. Compared to conventional RMM-OMP sparse signal recovery method, the proposed method has obtained a higher probability of successful recovery than provious method under the same condition. In other words, the proposed SMM satisfies RIP or MIP with higher probability than RMM. For instance, on sparse channel estimation, by utilized SMM-based training signal obtain a better estimate performance than RMM-based training signal. Simulation results confirmed the proposed method.

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