

## PAPER

# Joint Iterative Transmit/Receive FDE & FDIC for Single-Carrier Block Transmissions

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**SUMMARY** In this paper, we propose a novel iterative transmit/receive equalization technique for single-carrier (SC) block transmission in a severe frequency-selective fading channel. Iterative frequency-domain inter-symbol interference (ISI) cancellation (FDIC) is introduced to the previously proposed joint iterative transmit/receive frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion. 1-tap FDE is employed at the transmitter. At the receiver, a 1-tap FDE and FDIC are jointly used and they are updated in an iterative manner. The transmit FDE weight is derived based on the MMSE criterion by taking into account the reduction of residual ISI in the receiver. To derive the weight, the transmitter assumes that the receiver can partially reduce the residual ISI after the FDIC. We conduct a computer simulation to investigate the achievable bit error rate (BER) performance to confirm the effectiveness of our proposed technique.

**key words:** frequency-domain equalization, interference cancellation, single-carrier

## 1. Introduction

High-speed and high-quality data services are demanded in the next generation wireless communication systems [1]. As the data rate increases, the number of resolvable propagation paths tends to increase and the channel frequency-selectivity gets stronger, thereby producing severe inter-symbol interference (ISI) [2]. An important problem is how to overcome the severe channel frequency-selectivity.

Orthogonal frequency division multiplexing (OFDM) [3], [4] has been considered as a promising transmission technique for the next generation wireless communication systems [5]. OFDM converts the problem of broadband data transmission over a frequency-selective channel into the problem of parallel data transmissions using a number of narrowband orthogonal subcarriers. However, one disadvantage of OFDM transmission is its high peak-to-average power ratio (PAPR) [6]. Therefore, the SC transmission still remains as an important transmission technique, particularly for uplink transmissions [7], but the use of some advanced equalization techniques is indispensable.

Frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion is able to overcome the SC transmission problem arising from the severe channel frequency-selectivity [8]. The SC signal transmission using FDE is generally performed over a block.

By inserting a cyclic prefix (CP) into the guard interval (GI) of each transmit signal block, the received signal block can be transformed into frequency-domain signals by fast Fourier transform (FFT). One-tap receive FDE weight computed based on the MMSE criterion is multiplied to each component of frequency-domain signal to compensate the spectrum distortion. When no channel coding is used, SC with MMSE-FDE achieves much better transmission performance than OFDM since it can exploit the channel frequency-selectivity [9]. When channel coding is used, it can achieve almost the same transmission performance as OFDM [9].

However, SC with MMSE-FDE still suffers from the residual ISI. The BER performance of SC with MMSE-FDE is a few dB away from the matched-filter (MF) bound [2]. Reducing the residual ISI can further improve the BER performance. One way to reduce the residual ISI is to introduce an ISI cancellation technique. In [10]–[12], frequency-domain iterative ISI cancellation (FDIC) was proposed. The residual ISI replica is generated using the log-likelihood ratio (LLR) of the received signal after channel decoding and is subtracted from the received signal after the receive FDE. The receive FDE & FDIC and channel decoding are repeated for a sufficient number of times. In each iteration, the receive FDE weight is updated by taking the reliability of the residual ISI replica into account. Joint iterative equalization and decoding was originally proposed as turbo equalization in [13], [14].

Another way for reducing the residual ISI is to introduce the transmit FDE into the SC with receive FDE. In [13], we proposed a joint transmit/receive MMSE-FDE for SC transmission. Channel state information (CSI) is shared by the transmitter and the receiver and the FDE is jointly carried out at both the transmitter and receiver. The set of transmit/receive FDE weights is optimized based on the MMSE criterion. We showed by theoretical analysis and computer simulation that the joint transmit/receive MMSE-FDE provides better BER performance than the receive MMSE-FDE.

In this paper, we introduce the FDIC into the previously proposed joint iterative transmit/receive MMSE-FDE (called joint iterative Tx/Rx FDE & FDIC in this paper) to further improve the BER performance of SC block transmissions. Similar to [10]–[12], the receive FDE weight and the residual ISI replica are updated in each iteration. In contrast to the receiver, transmit FDE weight cannot be updated. To compute the transmit FDE weight, it is assumed that the re-

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ceiver can partially reduce the residual ISI after the receive FDE and FDIC. Then, the transmit FDE weight is matched to the assumed receiver. We evaluate the bit error rate (BER) performance improvement by computer simulation to confirm the effectiveness of our proposed technique.

The rest of the paper is organized as follows. Section 2 presents the transmission system model of SC with joint iterative Tx/Rx FDE & FDIC. The transmit and received signal representations are given in Sect. 3. The transmit and receive FDE weights are derived in Sect. 4. The simulation results are presented in Sect. 5. Section 6 concludes this paper.

## 2. System Model of SC Using Joint Iterative Tx/Rx FDE & FDIC

In this paper, we use a symbol-spaced discrete-time signal representation. The propagation channel is assumed to be an  $L$ -path frequency-selective block fading channel with the impulse response given as

$$h(\tau) = \sum_{l=0}^{L-1} h_l \delta(\tau - \tau_l), \quad (1)$$

where  $h_l$  and  $\tau_l$  are the complex-valued path gain and the delay time of the  $l$ th path, respectively. We assume  $\sum_{l=0}^{L-1} E[|h_l|^2] = 1$ .

The system model of SC using the joint iterative Tx/Rx FDE & FDIC is illustrated in Fig. 1. We assume perfect knowledge of CSI is shared by the transmitter and the receiver. Any type of coding scheme can be applied to the proposed joint iterative Tx/Rx FDE & FDIC. In this paper, for channel coding, we apply turbo coding with rate-1/3 having two recursive systematic convolutional (RSC) encoders [16], which has been widely recognized as one of the most powerful error correction schemes and has been adopted for the third generation cellular mobile communication systems.

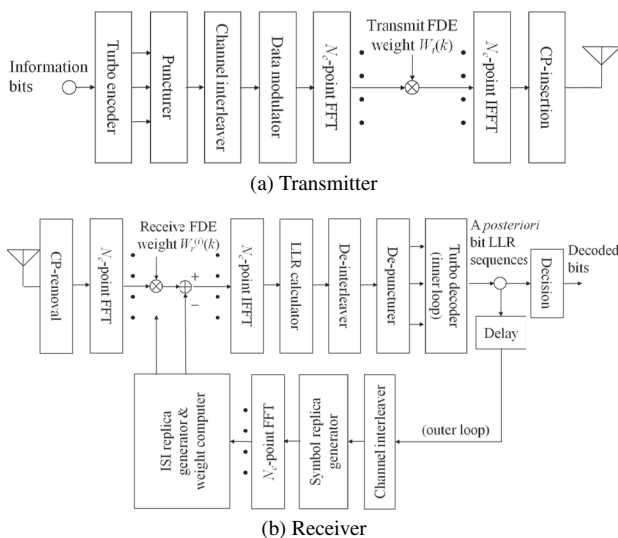


Fig. 1 SC using the joint iterative Tx/Rx FDE & FDIC.

At the transmitter, two parity bit sequences generated by the encoders are punctured to form the codeword of length  $K$ . The coding rate is represented by  $R$ . The codeword is then data-modulated. The data-modulated sequence is divided into  $N_c$ -symbol blocks and each block is transformed into the frequency-domain signal by  $N_c$ -point FFT. After multiplying with the transmit FDE weight, the frequency-domain signal is transformed back into the time-domain signal by  $N_c$ -point inverse FFT (IFFT). Then, the  $N_g$ -symbol CP is added to each  $N_c$ -symbol block and the resultant blocks are transmitted over the frequency-selective channel.

At the receiver, after removing the CP, the series of receive FDE, FDIC, and channel decoding is carried out in an iterative fashion. Let us consider the  $i$ th iteration stage with  $i = 1 \sim I$ , where  $I$  denotes the number of outer loop iterations of the receiver. Note that the number of inner loop iterations per outer loop iteration is one. The outer/inner loop iteration process will be described later. The received signal block is transformed into the frequency-domain signal by FFT. Each component of the frequency-domain signal is multiplied with the receive FDE weight and then the residual ISI replica generated after the turbo decoding in the  $(i-1)$ th iteration stage is subtracted. The frequency-domain signal is transformed into the time-domain signal by IFFT. After obtaining all the decision variables associated with the data-modulated symbols in a block, bit LLRs are computed and input to the turbo decoder.

The turbo decoder is composed of two maximum *a posteriori* probability (MAP) component decoders connected via interleaver/de-interleaver [16]. There are two iteration loops in the receiver: the outer loop (between equalizer and turbo decoder) and the inner loop (between component decoders in the turbo decoder). In this paper, only one iteration is carried out in turbo decoder in each inner loop. The extrinsic information from the first component decoder is passed to the second component decoder in the current iteration stage of the outer loop, while the extrinsic information from the second component decoder is passed to the first component decoder in the next iteration stage of the outer loop. The output of the turbo decoder is fed back and used for generating an updated residual ISI replica in the next iteration stage.

## 3. Signal Representation

The joint iterative Tx/Rx FDE & FDIC is performed block-by-block and hence, we consider the signal representation of one  $N_c$ -symbol block in this section.

### 3.1 Transmit Signal

The data symbol block is represented using a vector form as  $\mathbf{d} = [d(0), \dots, d(t), \dots, d(N_c - 1)]^T$ .  $N_c$ -point FFT is carried out on  $\mathbf{d}$  to obtain the frequency-domain transmit signal  $\mathbf{D} = [D(0), \dots, D(k), \dots, D(N_c - 1)]^T$  as

$$\mathbf{D} = \mathbf{F}\mathbf{d} \quad (2)$$

with

$$\mathbf{F} = \frac{1}{\sqrt{N_c}} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{-j2\pi\frac{(1)\times(1)}{N_c}} & \cdots & e^{-j2\pi\frac{(1)\times(N_c-1)}{N_c}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j2\pi\frac{(N_c-1)\times(1)}{N_c}} & \cdots & e^{-j2\pi\frac{(N_c-1)\times(N_c-1)}{N_c}} \end{bmatrix} \quad (3)$$

being an  $N_c \times N_c$  FFT matrix.

The transmit FDE weight  $\mathbf{W}_t = \text{diag}\{W_t(0), \dots, W_t(k), \dots, W_t(N_c - 1)\}$  is multiplied to  $\mathbf{D}$  as

$$\begin{aligned} \mathbf{S} &= [S(0), \dots, S(k), \dots, S(N_c - 1)]^T \\ &= \mathbf{W}_t \mathbf{D} \end{aligned} \quad (4)$$

with

$$\text{tr}[\mathbf{W}_t \mathbf{W}_t^T] = N_c. \quad (5)$$

Equation (5) is the constraint that keeps the transmit signal power unchanged. An  $N_c$ -point IFFT is applied to obtain the time-domain transmit signal block  $\mathbf{s} = \mathbf{F}^H \mathbf{S} = [s(0), \dots, s(t), \dots, s(N_c - 1)]^T$ . After the insertion of  $N_g$ -sample CP into the GI, the signal block is transmitted.

### 3.2 Received Signal

The CP-length is assumed to be equal to or longer than the maximum channel time delay. The received signal block  $\mathbf{r} = [r(0), \dots, r(t), \dots, r(N_c - 1)]^T$  after the CP-removal can be expressed as

$$\mathbf{r} = \sqrt{\frac{2E_s}{T_s}} \mathbf{h} \mathbf{s} + \mathbf{n}, \quad (6)$$

where  $E_s$  and  $T_s$  are the average transmit symbol energy and symbol duration, respectively,  $\mathbf{h}$  is an  $N_c \times N_c$  circulant channel matrix given by

$$\mathbf{h} = \begin{bmatrix} h_0 & & & h_{L-1} & \cdots & h_1 \\ h_1 & \ddots & & & \ddots & \vdots \\ \vdots & & h_0 & \mathbf{0} & & h_{L-1} \\ h_{L-1} & h_1 & \ddots & & & \\ & \ddots & \vdots & \ddots & & \\ \mathbf{0} & h_{L-1} & \cdots & \cdots & h_0 & \end{bmatrix}, \quad (7)$$

and  $\mathbf{n} = [n(0), \dots, n(t), \dots, n(N_c - 1)]^T$  is the noise vector with  $n(t)$  being a zero-mean additive white Gaussian noise (AWGN) having variance  $2N_0/T_s$  ( $N_0$  is the one-sided noise power spectrum density).

An  $N_c$ -point FFT is carried out on  $\mathbf{r}$  to obtain the frequency-domain received signal block  $\mathbf{R} = [R(0), \dots, R(k), \dots, R(N_c - 1)]^T$  as

$$\begin{aligned} \mathbf{R} &= \mathbf{F} \mathbf{r} = \sqrt{\frac{2E_s}{T_s}} \mathbf{H} \mathbf{S} + \mathbf{N} \\ &= \sqrt{\frac{2E_s}{T_s}} \mathbf{H} \mathbf{W}_t \mathbf{D} + \mathbf{N}, \end{aligned} \quad (8)$$

where  $\mathbf{N} = \mathbf{F} \mathbf{n}$  and  $\mathbf{H} = \mathbf{F} \mathbf{h} \mathbf{F}^H$ . Due to the circulant property of  $\mathbf{h}$ , the channel gain matrix  $\mathbf{H}$  of size  $N_c \times N_c$  is diagonal. The  $k$ th diagonal element of  $\mathbf{H}$  is given by

$$H(k) = \sum_{l=0}^{L-1} h_l \exp\left(-j2\pi k \frac{\tau_l}{N_c}\right). \quad (9)$$

The iterative receive FDE & FDIC is performed on the frequency-domain received signal. Here we consider the receive FDE & FDIC at the  $i$ th iteration stage ( $i = 1 \sim I$ ) of the outer loop. The frequency-domain received signal block after the receive FDE & FDIC,  $\hat{\mathbf{D}}^{(i)} = [\hat{D}^{(i)}(0), \dots, \hat{D}^{(i)}(k), \dots, \hat{D}^{(i)}(N_c - 1)]^T$ , can be written as

$$\begin{aligned} \hat{\mathbf{D}}^{(i)} &= \mathbf{W}_r^{(i)} \mathbf{R} - \mathbf{\Theta}^{(i)} \\ &= \sqrt{\frac{2E_s}{T_s}} \mathbf{W}_r^{(i)} \mathbf{H} \mathbf{W}_t \mathbf{D} - \mathbf{\Theta}^{(i)} + \mathbf{W}_r^{(i)} \mathbf{N}, \end{aligned} \quad (10)$$

where  $\mathbf{W}_r^{(i)} = \text{diag}\{W_r^{(i)}(0), \dots, W_r^{(i)}(k), \dots, W_r^{(i)}(N_c - 1)\}^T$  is an  $N_c \times N_c$  diagonal receive FDE weight matrix for the  $i$ th iteration.  $\mathbf{\Theta}^{(i)}$  is an  $N_c \times 1$  vector representing the residual ISI replica block, given as

$$\mathbf{\Theta}^{(i)} = \sqrt{\frac{2E_s}{T_s}} \{\mathbf{W}_r^{(i)} \mathbf{H} \mathbf{W}_t - \mathbf{I}\} \tilde{\mathbf{D}}^{(i-1)} \quad (11)$$

with  $\tilde{\mathbf{D}}^{(i-1)} = [\tilde{D}^{(i-1)}(0), \dots, \tilde{D}^{(i-1)}(k), \dots, \tilde{D}^{(i-1)}(N_c - 1)]^T$  being the frequency-domain data symbol replica block. For the first iteration (i.e., when  $i = 1$ ),  $\tilde{\mathbf{D}}^{(0)} = \mathbf{0}$ .  $\tilde{\mathbf{D}}^{(i-1)}$  for  $i > 1$  will be presented in Sect. 3.4.

$\hat{\mathbf{D}}^{(i)}$  is transformed by an  $N_c$ -point IFFT to the time-domain decision variable block  $\hat{\mathbf{d}}^{(i)}$  associated with the data symbol block  $\mathbf{d}$  as

$$\begin{aligned} \hat{\mathbf{d}}^{(i)} &= [\hat{d}^{(i)}(0), \dots, \hat{d}^{(i)}(t), \dots, \hat{d}^{(i)}(N_c - 1)]^T \\ &= \mathbf{F}^H \hat{\mathbf{D}}^{(i)} \\ &= \sqrt{\frac{2E_s}{T_s}} \mathbf{d} + \sqrt{\frac{2E_s}{T_s}} \{\mathbf{F}^H \mathbf{W}_r^{(i)} \mathbf{H} \mathbf{W}_t \mathbf{F} - \mathbf{I}\} \\ &\quad \times \{\mathbf{d} - \tilde{\mathbf{d}}^{(i-1)}\} + \mathbf{F}^H \mathbf{W}_r^{(i)} \mathbf{F} \mathbf{n}, \end{aligned} \quad (12)$$

where  $\tilde{\mathbf{d}}^{(i-1)} = [\tilde{d}^{(i-1)}(0), \dots, \tilde{d}^{(i-1)}(t), \dots, \tilde{d}^{(i-1)}(N_c - 1)]^T = \mathbf{F}^H \tilde{\mathbf{D}}^{(i-1)}$  and the first, second, and third terms denote the desired signal, residual ISI after the receive FDE & FDIC at the  $i$ th iteration, and noise, respectively.

### 3.3 Turbo Decoding

Turbo decoding is carried out using the bit LLRs obtained from  $\hat{\mathbf{d}}^{(i)}$ . The bit LLR,  $\Lambda_n^{(i)}(x)$ , associated with the  $x$ th bit of the  $n$ th data symbol in a block is computed as

$$\begin{aligned} \Lambda_n^{(i)}(x) &\approx \frac{\left| \hat{d}^{(i)}(n) - \sqrt{\frac{2E_s}{T_s}} a_{b_{n,x}=0}^{\min} \right|^2}{2\{\sigma^{(i)}\}^2} \\ &\quad - \frac{\left| \hat{d}^{(i)}(n) - \sqrt{\frac{2E_s}{T_s}} a_{b_{n,x}=1}^{\min} \right|^2}{2\{\sigma^{(i)}\}^2}, \end{aligned} \quad (13)$$

where  $x = 0 \sim \log_2 M - 1$  and  $n = 0 \sim N_c - 1$  ( $M$  is the modulation level),  $b_{n,x}$  denotes the  $x$ th bit of the  $n$ th data symbol and denotes the most probable symbol that gives the minimum Euclidean distance from  $\tilde{d}^{(i)}(n)$  among all the candidate symbols with  $b_{n,x} = 0$  (or  $b_{n,x} = 1$ ).  $2\{\sigma^{(i)}\}^2$  represents the variance of the noise plus the residual ISI at the  $i$ th iteration stage.

The bit LLR sequence is then de-interleaved and de-punctured. Let us denote the bit LLR sequences associated with the systematic and the two parity bit sequences as  $\{\Lambda_s^{(i)}\}$ ,  $\{\Lambda_{p1}^{(i)}\}$ , and  $\{\Lambda_{p2}^{(i)}\}$ , respectively.

The turbo decoding is carried out by two component decoders. At first,  $\{\Lambda_s^{(i)}\}$  and  $\{\Lambda_{p1}^{(i)}\}$  are input to the first component decoder. Together with  $\{\Lambda_s^{(i)}\}$  and  $\{\Lambda_{p1}^{(i)}\}$ , the extrinsic information from the second component decoder in the previous iteration stage is used as *a priori* information to compute the *a posteriori* LLRs. Then, the second component decoder computes the improved *a posteriori* LLRs using  $\{\Lambda_s^{(i)}\}$ ,  $\{\Lambda_{p2}^{(i)}\}$ , and the extrinsic information from the first component decoder as *a priori* information.

The resultant *a posteriori* bit LLR sequences after the decoding are represented by  $\{\lambda_s^{(i)}\}$ ,  $\{\lambda_{p1}^{(i)}\}$ , and  $\{\lambda_{p2}^{(i)}\}$ , respectively, where  $\{\lambda_{p1}^{(i)}\}$  is obtained by the first component decoder and  $\{\lambda_s^{(i)}\}$  and  $\{\lambda_{p2}^{(i)}\}$  are obtained by the second component decoder. They will be used as *a priori* information to generate the residual ISI replica in the next iteration stage (see Sect. 3.4). The *a posteriori* bit LLR sequence  $\{\lambda_s^{(i)}\}$  from the second component decoder is used to generate the extrinsic information for the first component decoder in the next iteration stage [16].

The bit LLR sequences are interleaved to generate the data symbol replicas. The bit LLR, associated with the  $x$ th bit of the  $n$ th data symbol of the data symbol block, can be expressed as

$$\lambda_n^{(i)}(x) = \ln \frac{p^{(i)}(b_{n,x} = 1)}{p^{(i)}(b_{n,x} = 0)}, \quad (14)$$

where  $p^{(i)}(b_{n,x} = 1$  (or  $b_{n,x} = 0))$  represents the *a posteriori* probability of  $b_{n,x} = 1$  (or  $b_{n,x} = 0$ ) after the decoding.

### 3.4 Data Symbol Replica Generation

The frequency-domain data symbol replica block  $\tilde{\mathbf{D}}^{(i-1)}$  of Eq. (11) is obtained similar to [11], [12]. The  $n$ th data symbol replica,  $\tilde{d}^{(i-1)}(n)$ , is given as

$$\tilde{d}^{(i-1)}(n) = \sum_{d \in Y} d \prod_{b_{n,x} \in d} p^{(i-1)}(b_{n,x} | \lambda_n^{(i-1)}(x)), \quad (15)$$

where  $d$  represents the candidate symbol having  $b_{n,x} = 0$  or  $b_{n,x} = 1$  in the symbol set  $Y$  and  $p^{(i-1)}(b_{n,x} = 0)$  and  $p^{(i-1)}(b_{n,x} = 1)$  are given by

$$\begin{cases} p^{(i-1)}(b_{n,x} = 0 | \lambda_n^{(i-1)}(x)) = -\frac{1}{2} \tanh\left(\frac{\lambda_n^{(i-1)}(x)}{2}\right) + \frac{1}{2} \\ p^{(i-1)}(b_{n,x} = 1 | \lambda_n^{(i-1)}(x)) = \frac{1}{2} \tanh\left(\frac{\lambda_n^{(i-1)}(x)}{2}\right) + \frac{1}{2}, \end{cases} \quad (16)$$

since

$$p^{(i-1)}(b_{n,x} = 0 | \lambda_n^{(i-1)}(x)) + p^{(i-1)}(b_{n,x} = 1 | \lambda_n^{(i-1)}(x)) = 1. \quad (17)$$

According to Eqs. (16) and (17),  $\tilde{d}^{(i-1)}(n)$  is given as

$$\tilde{d}^{(i-1)}(n) = \begin{cases} \frac{1}{\sqrt{2}} \left( \tanh\left(\frac{\lambda_n^{(i-1)}(0)}{2}\right) + j \tanh\left(\frac{\lambda_n^{(i-1)}(1)}{2}\right) \right) & \text{for QPSK,} \\ \frac{1}{\sqrt{10}} \left( \tanh\left(\frac{\lambda_n^{(i-1)}(0)}{2}\right) \left( 2 + \tanh\left(\frac{\lambda_n^{(i-1)}(1)}{2}\right) \right) \right) & \\ + j \frac{1}{\sqrt{10}} \left( \tanh\left(\frac{\lambda_n^{(i-1)}(2)}{2}\right) \left( 2 + \tanh\left(\frac{\lambda_n^{(i-1)}(3)}{2}\right) \right) \right) & \text{for 16QAM.} \end{cases} \quad (18)$$

Using Eq. (18), the frequency-domain symbol replica is generated as  $\tilde{\mathbf{D}}^{(i-1)} = \mathbf{F} \tilde{\mathbf{d}}^{(i-1)}$ .

## 4. Transmit and Receive FDE Weights

### 4.1 Receive FDE Weight

The receive FDE weight  $\mathbf{W}_r^{(i)}$  at the  $i$ th iteration stage is derived. A concatenation of the transmit FDE and the propagation channel is viewed as the equivalent channel. The error vector between  $\hat{\mathbf{d}}^{(i)}$  and  $\mathbf{d}$  is given as

$$\begin{aligned} \mathbf{e}^{(i)} &= \frac{\hat{\mathbf{d}}^{(i)}}{\sqrt{2E_s/T_s}} - \mathbf{d} \\ &= \{\mathbf{F}^H \mathbf{W}_r^{(i)} \mathbf{H} \mathbf{W}_t \mathbf{F} - \mathbf{I}\} \{\mathbf{d} - \tilde{\mathbf{d}}^{(i-1)}\} \\ &\quad + \left(\frac{2E_s}{T_s}\right)^{-\frac{1}{2}} \mathbf{F}^H \mathbf{W}_r^{(i)} \mathbf{F} \mathbf{n}. \end{aligned} \quad (19)$$

The mean square error (MSE),  $e^{(i)}$ , is given as

$$e^{(i)} = \text{tr}[E\{\mathbf{e}^{(i)} \{\mathbf{e}^{(i)}\}^H\}]. \quad (20)$$

Using Eq. (19), Eq. (20) is rewritten as

$$\begin{aligned} e^{(i)} &= \text{tr}[\{\mathbf{W}_r^{(i)} \mathbf{H} \mathbf{W}_t - \mathbf{I}\} \\ &\quad \times E\{\{\mathbf{D} - \tilde{\mathbf{D}}^{(i-1)}\} \{\mathbf{D} - \tilde{\mathbf{D}}^{(i-1)}\}^H\} \\ &\quad \times \{\mathbf{W}_r^{(i)} \mathbf{H} \mathbf{W}_t - \mathbf{I}\}^H] \\ &\quad + \gamma^{-1} \cdot \text{tr}[\mathbf{W}_r^{(i)} \{\mathbf{W}_r^{(i)}\}^H] \end{aligned} \quad (21)$$

with  $\gamma = (E_s/N_0)$ . Equation (21) can be simplified to

$$\begin{aligned} e^{(i)} &= \text{tr}[\{\mathbf{W}_r^{(i)} \mathbf{H} \mathbf{W}_t - \mathbf{I}\} \rho^{(i-1)} \{\mathbf{W}_r^{(i)} \mathbf{H} \mathbf{W}_t - \mathbf{I}\}^H] \\ &\quad + \gamma^{-1} \cdot \text{tr}[\mathbf{W}_r^{(i)} \{\mathbf{W}_r^{(i)}\}^H], \end{aligned} \quad (22)$$

where

$$E\{(\mathbf{D} - \tilde{\mathbf{D}}^{(i-1)})\{\mathbf{D} - \tilde{\mathbf{D}}^{(i-1)}\}^H\} = \rho^{(i-1)} \cdot \mathbf{I} \quad (23)$$

with [11]

$$\begin{aligned} \rho^{(i-1)} &= E[|D(k) - \tilde{D}^{(i-1)}(k)|^2] \\ &= E[|d(t) - \tilde{d}^{(i-1)}(t)|^2] \\ &\sim \begin{cases} \frac{1}{N_c} \sum_{n=0}^{N_c-1} (1 - |\tilde{d}^{(i-1)}(n)|^2) & \text{for QPSK,} \\ \frac{1}{N_c} \sum_{n=0}^{N_c-1} \left( \frac{4}{10} \tanh\left(\frac{\lambda_n^{(i-1)}(1)}{2}\right) \right. \\ \quad \left. + \frac{4}{10} \tanh\left(\frac{\lambda_n^{(i-1)}(3)}{2}\right) + 1 - |\tilde{d}^{(i-1)}(n)|^2 \right) & \text{for 16QAM,} \end{cases} \end{aligned} \quad (24)$$

for  $i > 1$ , where at the first stage ( $i = 1$ ),  $\rho^{(0)} = 1$ .

Solving  $\partial e^{(i)} / \partial \mathbf{W}_r^{(i)} = \mathbf{0}$  gives

$$\begin{aligned} \mathbf{w}_r^{(i)} &= \rho^{(i-1)} \mathbf{W}_r^H \mathbf{H}^H \\ &\times (\rho^{(i-1)} \mathbf{H} \mathbf{W}_r \mathbf{W}_r^H \mathbf{H}^H + \gamma^{-1} \mathbf{I})^{-1}. \end{aligned} \quad (25)$$

## 4.2 Transmit FDE Weight

From [11] and the previous subsection, it can be understood that the receive FDE weight is updated in each iteration stage. As seen in Eq. (25), the receive FDE weight  $\mathbf{W}_r^{(i)}$  at the  $i$ th iteration includes  $\rho^{(i-1)}$  which indicates the accuracy of the symbol replicas generated in the previous iteration. When  $i = 1$ ,  $\tilde{\mathbf{D}}^{(0)} = \mathbf{0}$  and  $\rho^{(0)} = 1$ . As the number  $i$  of iterations increases,  $\rho^{(i-1)}$  gets smaller and  $\mathbf{W}_r^{(i)}$  approaches the maximal-ratio combining (MRC) weight  $\mathbf{W}_r^H \mathbf{H}^H$ . This receive FDE weight maximizes the frequency diversity gain.

In contrast to the receiver side, transmit FDE weight cannot be updated. In [15], we derived the transmit FDE weight for the given receive FDE weight. However, the previously proposed transmit FDE weight did not consider the FDIC of the receiver. If the use of FDIC of the receiver is considered at the transmitter, the transmit FDE can provide a much higher frequency diversity gain than the previously proposed transmit FDE.

In this paper, to derive the transmit FDE weight, we assume a virtual receiver having receive FDE with the weight matrix  $\mathbf{W}_r^{tx} = \text{diag}\{W_r^{tx}(0), \dots, W_r^{tx}(k), \dots, W_r^{tx}(N_c - 1)\}$ . The residual ISI after the receive FDE of the virtual receiver is assumed to be reduced by a factor of  $1 - \sqrt{\rho^{tx}}$ , where  $\rho^{tx}$  is a parameter between 0 and 1. The error vector  $\mathbf{e}^{tx} = [e^{tx}(0), \dots, e^{tx}(t), \dots, e^{tx}(N_c - 1)]^T$  is given as

$$\begin{aligned} \mathbf{e}^{tx} &= \sqrt{\rho^{tx}} (\mathbf{F}^H \mathbf{W}_r^{(i)} \mathbf{H} \mathbf{W}_r \mathbf{F} - \mathbf{I}) \mathbf{d} \\ &+ \left( \frac{2E_s}{T_s} \right)^{-\frac{1}{2}} \mathbf{F}^H \mathbf{W}_r^{tx} \mathbf{F} \mathbf{n}, \end{aligned} \quad (26)$$

where the first term represents the residual ISI of the above virtual receiver. Similar to Eqs. (19)–(25), the receive FDE weight in the virtual receiver is derived as

$$\mathbf{w}_r^{tx} = \rho^{tx} \mathbf{W}_r^H \mathbf{H}^H$$

$$\times (\rho^{tx} \mathbf{H} \mathbf{W}_r \mathbf{W}_r^H \mathbf{H}^H + \gamma^{-1} \mathbf{I})^{-1}. \quad (27)$$

The total MSE,  $e^{tx} = \text{tr}[E(\mathbf{e}^{tx} \mathbf{e}^{tx}{}^H)]$ , for the given  $\mathbf{W}_r^{tx}$  is given using Eqs. (26) and (27) as

$$\begin{aligned} e^{tx} &= \text{tr}\{[\mathbf{W}_r^{tx} \mathbf{H} \mathbf{W}_r - \mathbf{I}] \rho^{tx} \{\mathbf{W}_r^{tx} \mathbf{H} \mathbf{W}_r - \mathbf{I}\}^H\} \\ &+ \gamma^{-1} \cdot \text{tr}\{\mathbf{W}_r^{tx} \mathbf{W}_r^{tx}{}^H\} \\ &= \gamma^{-1} \rho^{tx} \cdot \text{tr}[(\rho^{tx} \mathbf{H} \mathbf{W}_r \mathbf{W}_r^H \mathbf{H}^H + \gamma^{-1} \mathbf{I})^{-1}]. \end{aligned} \quad (28)$$

The optimal transmit FDE weight that minimizes  $e^{tx}$  in Eq. (28) under the transmit power constraint of Eq. (5) is obtained, similar to [15], as

$$W_t(k) = \max \left( \sqrt{\frac{1}{\mu \sqrt{\gamma} |H(k)|} - \frac{1}{\rho^{tx} \gamma |H(k)|^2}}, 0 \right), \quad (29)$$

where  $\mu$  is a constant determined to satisfy the constraint given by Eq. (5).

Equation (29) indicates that the transmit FDE weight is always real valued weight and depends on  $\rho^{tx}$  which indicates the reliability of the interference cancellation at the receiver, predicted by the transmitter side. If the transmitter believes that the receiver can perfectly cancel the residual ISI,  $\rho^{tx}$  is set to be small. On the other hand, if the transmitter believes that the receiver cannot cancel the residual ISI at all,  $\rho^{tx}$  is set to be 1. The optimal  $\rho^{tx}$  may depend on the instantaneous channel condition, the average transmit  $E_s/N_0$ , data modulation, the number of iterations of the receive FDE & FDIC and so on. Therefore, it is quite difficult to analytically find the optimal  $\rho^{tx}$  and hence, in this paper, by computer simulation, we will find  $\rho^{tx}$  such that the average BER is minimized for the given transmit  $E_s/N_0$ .

## 5. Performance Evaluation

The BER performance of SC using the joint iterative Tx/Rx FDE & FDIC is evaluated by computer simulation. The channel is assumed to be an  $L=16$ -path frequency-selective block Rayleigh fading channel having uniform power delay profile. Ideal channel estimation is assumed.  $N_c=256$  and  $N_g=32$  are used. Data modulation is either QPSK or 16QAM. In the following, the uncoded case is discussed first, followed by the coded case.

### 5.1 Transmit Weight Example

Figure 2 illustrates an example of  $|W_t(k)|$  for different values of  $\rho^{tx}$ .  $E_s/N_0$  is set to 10 dB. When  $\rho^{tx} = 1$ , transmit FDE assumes that the receiver cannot reduce residual ISI at all. As  $\rho^{tx}$  decreases, the transmit FDE tends to allocate more signal power to the frequencies having good condition since the transmitter believes that a larger amount of the residual ISI can be reduced at the receiver. As can be seen from Fig. 2(d), if the transmitter sets  $\rho^{tx} = 0.1$  (the residual ISI after the receive FDE can be reduced by 1/10), the transmit FDE weights ( $k = 0 \sim N_c - 1$ ) are replaced by zeros at 1/4 of the total frequencies.

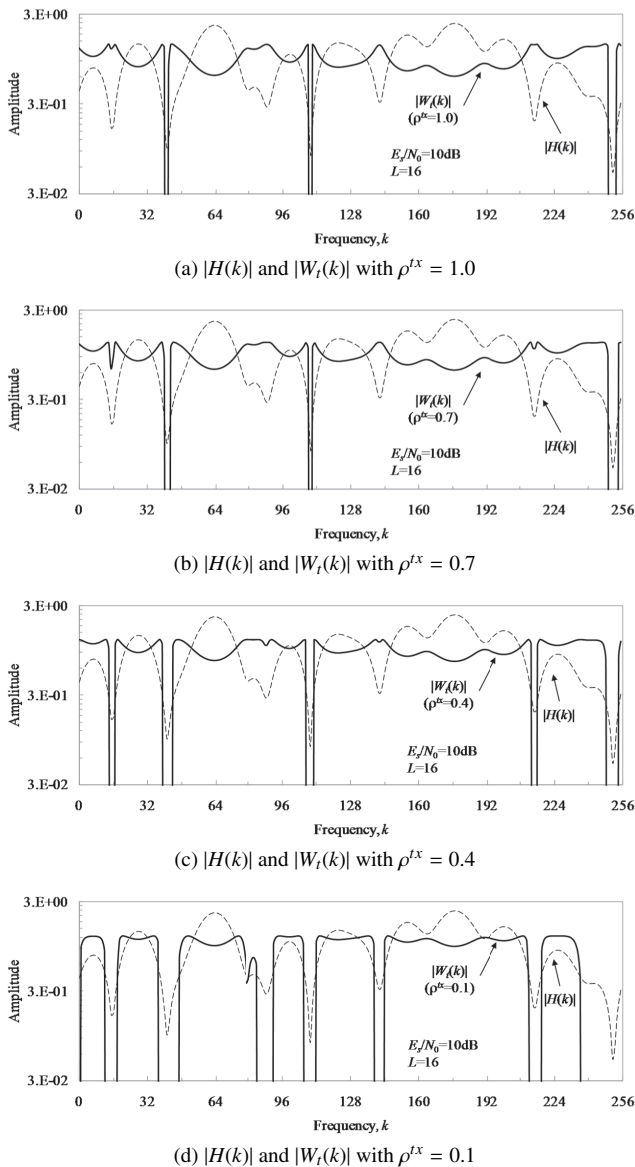


Fig. 2 Channel gain and transmit FDE weight.

## 5.2 Uncoded BER

At first, to simply discuss the impacts of the value of  $\rho^{tx}$  on the performance, we evaluate the uncoded average BER in Fig. 3 as a function of  $\rho^{tx}$  with the average transmit bit energy-to-noise power spectrum density ratio  $E_b/N_0$  ( $= 1/\log_2 M(E_s/N_0)(1 + N_g/N_c)$ ) as a parameter ( $M$  represents the modulation level). In the case of uncoded transmission, the LLRs computed from the decision variables are directly used as the a priori information in the next iteration of the receive FDE & FDIC, i.e., Eq. (13) is substituted into Eq. (18) [11].  $I = 1$  (without FDIC) and  $I = 6$  are considered. It can be seen that the optimal value of  $\rho^{tx}$  that minimizes the average BER depends on  $I$  and  $E_b/N_0$ . It is obvious that when  $I = 1$ , the optimal value of  $\rho^{tx}$  is 1 irrespective of  $E_b/N_0$  since the FDIC is not used. When  $I = 6$ ,

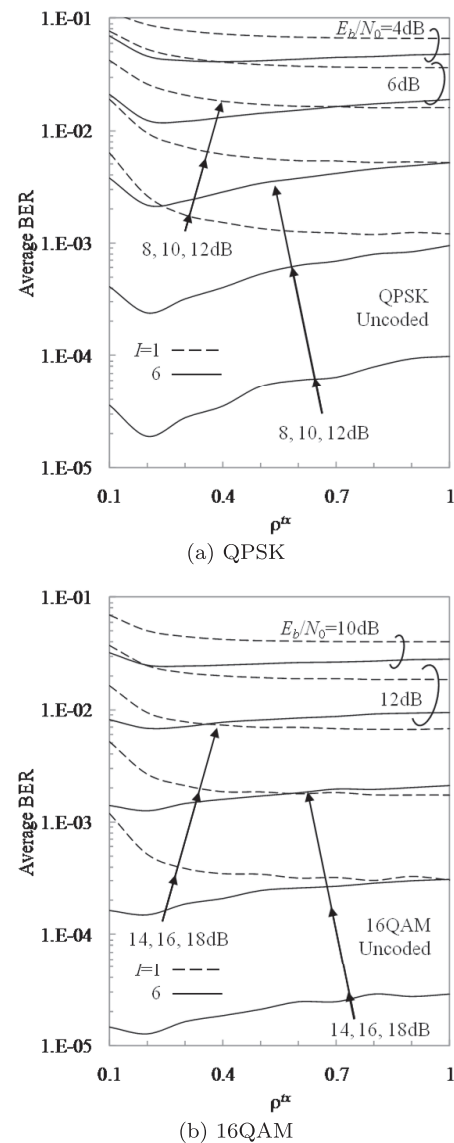
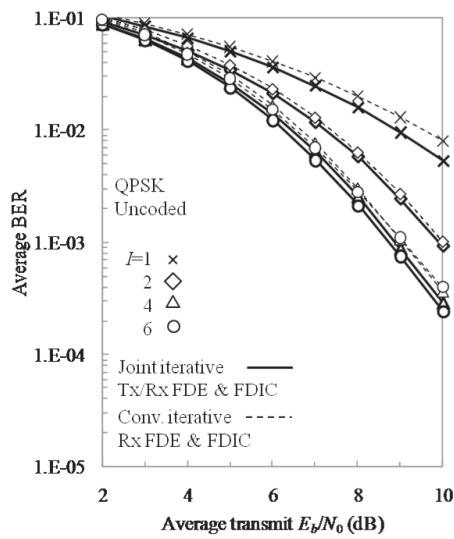


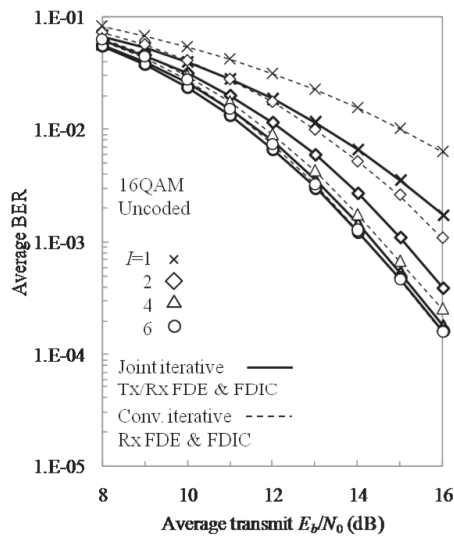
Fig. 3 Average BER versus  $\rho^{tx}$  for the uncoded case.

the receiver is updated in each iteration to improve the BER by further exploiting frequency diversity gain. However, if  $\rho^{tx}$  is set close to 1, sufficient frequency diversity gain cannot be obtained. The use of smaller  $\rho^{tx}$  results in allocating more power to the frequencies having a good condition (see Fig. 2(d)) and obtains larger frequency diversity gain. However, too small  $\rho^{tx}$  makes the ISI stronger and degrades the BER performance. Therefore, it can be said that there is a tradeoff between the frequency diversity gain and the residual ISI reduction in the joint iterative Tx/Rx FDE & FDIC.

Figure 4 compares the uncoded achievable average BER performances of the joint iterative Tx/Rx FDE & FDIC and the conventional iterative receive FDE & FDIC for various number of iterations,  $I = 1, 2, 4$ , and 6. The optimal  $\rho^{tx}$  is used at each  $E_b/N_0$  for the joint iterative Tx/Rx FDE & FDIC. As the number of iterations  $I$  increases, the BER performance can be significantly improved; however,



(a) QPSK



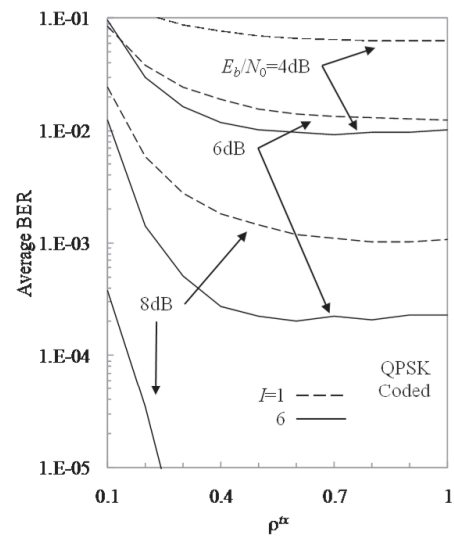
(b) 16QAM

Fig. 4 BER performance comparison for the uncoded case.

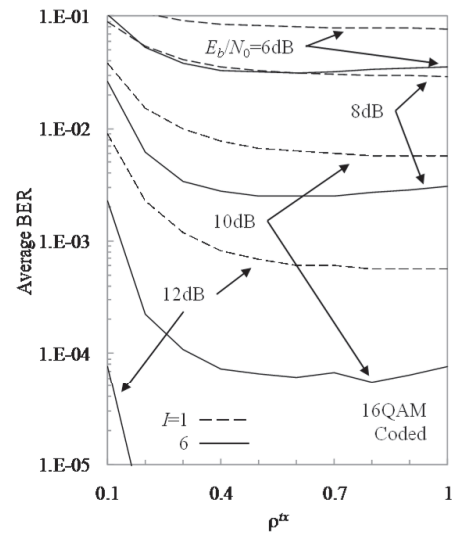
the performance improvement compared to the conventional iterative receive FDE & FDIC gets smaller. For example, when QPSK data modulation is used, the required  $E_b/N_0$  for achieving  $BER = 10^{-3}$  using the joint iterative Tx/Rx FDE & FDIC is reduced by about 0.5 dB compared to the conventional receive FDE & FDIC when  $I = 6$ . In the case of 16QAM, no  $E_b/N_0$  reduction is possible by the joint iterative Tx/Rx FDE & FDIC. This is because the iterative receive FDE & FDIC exploits the frequency diversity gain enough and offers sufficiently improved BER performance when  $I = 6$ .

### 5.3 Coded BER

Figure 5 plots the coded average BER as a function of  $\rho^{tx}$  with the average  $E_b/N_0$  as a parameter.  $K = 2048$  and  $R = 1/2$  are assumed. Similar to the uncoded case shown in



(a) QPSK



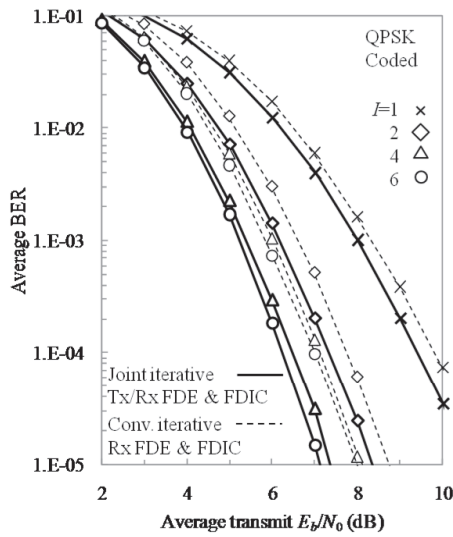
(b) 16QAM

Fig. 5 Average BER versus  $\rho^{tx}$  for the coded case.

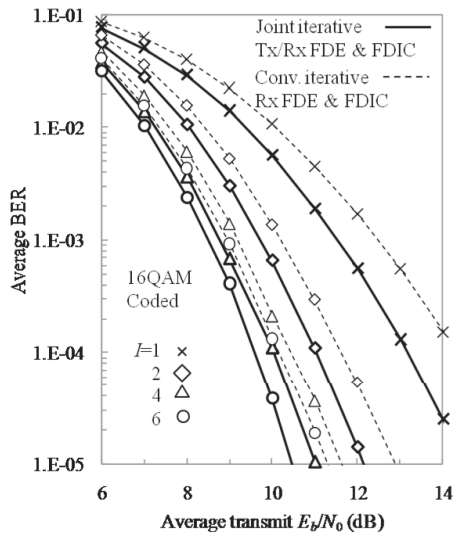
Fig. 3, optimal  $\rho^{tx}$  exists for  $I = 6$ , while  $\rho^{tx} = 1$  is optimal for  $I = 1$ . However, due to the coding gain, low BER is achieved even for a large  $\rho^{tx}$ .

Figure 6 compares the average coded BER performances of the joint iterative Tx/Rx FDE & FDIC and the conventional iterative receive FDE & FDIC for various number of iterations,  $I = 1, 2, 4,$  and  $6$ . The optimal  $\rho^{tx}$  is used at each  $E_b/N_0$  for the joint Tx/Rx FDE & FDIC.

Unlike the uncoded case, the coded BER of the joint iterative Tx/Rx FDE & FDIC can be improved compared to the conventional iterative receive FDE & FDIC when  $I = 6$ . This is because in the coded transmission, the LLRs, used for computing the residual ISI replicas for the FDIC, are improved by the decoder. Exploiting not only the frequency diversity gain but also the channel coding gain, it provides better BER performance than the conventional iterative receive FDE & FDIC.



(a) QPSK



(b) 16QAM

Fig. 6 BER performance comparison for the coded case.

When QPSK data modulation is used, the joint iterative Tx/Rx FDE & FDIC can reduce the required  $E_b/N_0$  for achieving  $BER = 10^{-3}$  by about 1.0 dB compared to the conventional iterative receive FDE & FDIC. When 16QAM data modulation is used, it can reduce the required  $E_b/N_0$  by about 0.8 dB.

#### 5.4 PAPR Level

The transmit signal PAPR was measured by performing 4-time oversampling for the transmit  $E_s/N_0 = 0-20$  dB assuming 16QAM. The PAPR level (denoted by PAPR 1% level) which the measured PAPR exceeds at 1% probability is plotted in Fig. 7 as a function of  $\rho^{tx}$  with transmit  $E_s/N_0$  as a parameter. The PAPR 1% level of SC with transmit FDE is lower than OFDM. However, it is higher than that of SC without transmit FDE. To suppress the PAPR increase,

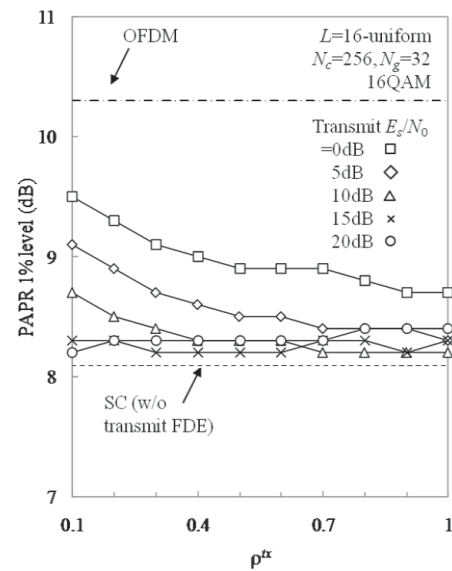


Fig. 7 PAPR 1% level (dB).

PAPR reduction schemes proposed for OFDM [6] can be applied. How the PAPR can be suppressed is left as a future important study topic.

#### 6. Conclusion

In this paper, we proposed joint iterative Tx/Rx FDE & FDIC for SC block transmission. The receive FDE weight and the residual ISI replica are updated in each iteration. We assume a virtual receiver having FDE & FDIC at the transmitter to compute the transmit FDE weight based on the MMSE criterion. We investigated the BER performance improvement by computer simulation. It was shown that when turbo coding is used, the joint iterative Tx/Rx FDE & FDIC provides much better BER performance than the conventional iterative receive FDE & FDIC.

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