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LETTER

Throughput Optimization in Rateless Coded Cooperative Relay Networks

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SUMMARY By performing the encoding operation on several message packets, rateless coding in cooperative networks has the potential risk of processing information already available to the receivers. In this paper, the intermediate packet decodability of rateless codes is exploited to mitigate such redundant packet processing at the cooperating nodes. The message packets that are already decoded at the receivers are eliminated from further processing by harnessing the back channel (from receiver to transmitter) for feedback. This reduces the required number of transmissions and optimizes the throughput of the network.

key words: *rateless coding, cooperative communication, feedback*

1. Introduction

Cooperative communication technique of sharing the resources among multiple nodes has the potential to cater to the growing demand of high-data rate wireless networks with wide coverage [1]. Relaying signals for each other in such networks creates a virtual antenna array and achieves spatial diversity. User cooperation thereby leads to an improvement in link reliability, range extension and power saving capability. Of the cooperating strategies, the incremental redundancy method of adapting the rate to suit time varying channel conditions, offers better spectral efficiency [1]. The near capacity performance along with the inherent incremental redundancy nature of rateless codes such as Luby Transform (LT) codes in [2], ideally suit such techniques.

The rateless code property of reconstructing the source data from any subset of encoded packets with sufficient mutual information is particularly attractive for cooperative networks. Traditionally in such systems, the intermediate nodes monitor the source transmissions virtually indefinitely until it acquires enough mutual information to decode [3]. The intermediate node then re-encodes the decoded source packets and cooperates in transmitting the information to the destination. Even though there are several such schemes that use rateless coding in cooperative networks, most of them neglect the inherent advantages of rateless coding or require considerable complexity and/or resources.

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2. Problem Formulation

Allowing the intermediate relay nodes in a cooperative rateless coded network to process all the received source packets may result in the transmission of redundant information across the links. The availability of additional information — the number of input symbols already decoded at the receiver — at the relay, will eliminate such risks. The relay can be intimated of the decodability at the receiver by harnessing the back channel (from the receiver to the relay) for feedback. Rateless codes with feedback have been shown to offer better data recovery [4], [5]. In [4] a feedback is used to transmit systematic symbols to revive the decoding process and in [5] the transmitter uses the feedback information to adjust the degree distribution in the encoding process. To the best of authors' knowledge, the proposed method of using a single feedback message to completely eliminate redundant transmission and processing by the relay nodes in rateless coded cooperative networks is first of its kind. In such a scheme, the packets recovered at the receiver are revealed to the relay nodes which results in significant savings in communication complexity, memory usage and overall energy efficiency.

3. Network Model & Proposed Scheme

A generic three node network comprising of a source node (*SN*), a relay node (*RN*) and a destination node (*DN*) is considered for the present analysis. Single antenna nodes with half duplex transmission constitute the network.

3.1 Received Signal & Channel Model

The continuous stream of information bits (u) generated by the *SN* are first grouped into blocks of size $p_k \times b$, where p_k is the number of packets per encoding set and b is the bit size of each data packet. Each block is then fed into the rateless encoder. The encoder continuously generates codewords c_i's of size $1 \times b$ until sufficient mutual information is received at the decoder for proper decoding. The channel is assumed to remain static for the entire codeword length (b bits). Then the received signal model over a quasi-static Rayleigh fading channel is

$$Y_q = \sqrt{\epsilon} H_{pq} X_p + N_q \quad (1)$$

where $p \in \{SN, RN\}$ and $q \in \{RN, DN\}$, $X_p \in \mathbb{C}^{1\times b}$ is the transmitted codeword matrix; $H_{pq} \in \mathbb{C}^{1\times 1}$ is the channel transfer matrix between transmitting node p and receiving node q, which is selected from a complex Gaussian distribution corresponding to Rayleigh fading amplitudes; $N_q \in \mathbb{C}^{1\times b}$ is the noise matrix which corresponds to additive white Gaussian noise (AWGN) with zero mean and variance N_0 and $Y_q \in \mathbb{C}^{1\times b}$ is the received signal matrix. The parameter ϵ corresponds to the average energy per symbol during transmission. The channel matrix H_{pq} has a variance of σ_{pq}, which captures the pathloss between nodes p and q (without loss of generality, shadowing is not considered in this paper). The variance σ_{pq} is dependent on the pathloss model and is proportionally related to the distance (d_{pq}) as $\sigma_{pq} \propto d_{pq}^{-\alpha}$, where α is the pathloss exponent.

Following Shannon's theorem of reliable communication, any codeword transmitted through such a channel can be decoded with vanishing error probability as the codeword length (b) grows, if the transmission rate R is less than the channel capacity C. And the channel is considered to be in outage whenever this constraint is violated. In an outage event, there is no guarantee that the transmitted encoded packet can be decoded without error, hence such packets are considered as erased. This enables the conversion of the wireless channel into an equivalent erasure channel and the analysis for the rateless coded system can be performed using the same approach as that in an erasure channel. For simplicity of exposition, binary phase shift keying (BPSK) modulation with coherent detection is used.

3.2 Proposed Adaptive Rateless Coding with Feedback

The proposed method employs rateless codes which has a low-complexity encoder/decoder, to recover the source packets with minimum number of encoded packets (slightly more than the number of source packets). In general, the number of recovered source packets depends on the number of received encoded packets and the degree distribution employed for encoding. In traditional rateless coded cooperative relay networks, feedback channels are required only to signal successful decoding at the receiver. If the *SN* receives the acknowledgment (ACK) from the *DN*, the next set of data packets are encoded and transmitted. Whereas if the *RN* decodes first, then the *SN* ceases transmission and the *RN* transmits the encoded packets (*RN* switches from reception to transmission) till the *DN* decodes correctly.

In most of the existing rateless coded cooperative systems, *RN* re-encodes the entire block of decoded source packets. However the broadcast nature of source transmissions facilitates the reception of some encoded packets at the *DN*, whose quality is dependent on the SD link. Even though the received codewords are not sufficient to decode all the source packets, recovery of some packets is still possible. This intermediate decodability of rateless codes at the *DN* is exploited in this paper.

In the proposed adaptive rateless coded system, the *DN* uses the received encoded packets from the *SN* to recover as many source packets as possible. This additional information — the source packets already decoded at the *DN*, is advertised to the *RN* by transmitting a feedback message of size p_k bits (equal to the number of source packets used for encoding). Then the n known input packets ($n \leq p_k$) are removed from the *RN* buffer and the remaining ($p_k - n$) packets are rateless coded and transmitted by the *RN*. This basically mitigates the redundant information processing and transmission by the *RN*. In rateless coding a significant performance improvement can be obtained by not transmitting any encoded packets that depend solely on source packets that are already known at the receiver. However the required feedback to achieve this is small compared to the information bit size as $p_k \ll b$. Throughout the sequel it is assumed that the decoding is successful once the received generator matrix is full rank i.e $Rank(G) = p_k$, where G is the generator matrix at the receiving node. The entire scheme can be summarized as in Algorithm 1. The drawback of the proposed scheme is the use of a single feedback message compared to other rateless coded cooperative relay networks. However the achievable bandwidth efficiency supersedes such a detriment which is validated by the analysis provided in the following sections.

Algorithm 1: Algorithm of the proposed scheme

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SN encodes information sequence and transmits coded packets;
if received generator matrix G is full rank at DN then
 | SN codes the next sequence and transmits;
else if received generator matrix G is full rank at RN then
 | DN transmits feedback (p_k bits) to signal packet recovery;
 | RN encodes the remaining (p_k − n) packets which are not
 | acknowledged by the DN. And transmits
else
 L SN transmits encoded packets
This process continues till all source packets are received at DN.
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4. Theoretical Analysis

In this section, the average number of packets ($\overline{n}$) recoverable at the *DN* before the *RN* starts cooperating is analyzed. Initially the source data is rateless coded and transmitted. Based on the erasure technique presented in Sect. 3, the probability of erasure in a Rayleigh fading channel is

$$P_e^{pq} = \Pr[C < R] = 1 - \exp\left(\frac{1 - 2^{2R}}{\rho_p \sigma_{pq}}\right) \tag{2}$$

where $\rho_p = \epsilon / N_0$ and R is the transmission rate.

4.1 Expected Number of Transmissions for Successful Relay Decoding ($\overline{T}_r$)

The *RN* receives the encoded packets from *SN* based on the channel quality across the SR link. The corresponding expected number of transmissions required for proper decoding at *RN* across the SR link can be computed as

$$\overline{T}_r = \sum_{t_r=p_k}^{\infty} t_r \Pr(T_r = t_r) \tag{3}$$

where T_r represents the number of transmissions required for the *RN* to decode successfully, and $\Pr(T_r = t_r)$ is the corresponding probability mass function (PMF). The required minimum number of transmissions is p_k, as at least p_k transmissions are required to decode all p_k source packets. And the upper bound goes till infinity, owing to the rateless property of the code. T_r basically captures the time when *RN* is ready for cooperation i.e. when the received generator matrix at the *RN* is full rank. The binary generator matrices from the rateless encoding operation are considered to be equiprobable (the same analysis can be performed for other cases). Then the probability of having a generator matrix of rank p_k after receiving i unerased encoded packets is [6]

$$P^{p_k}_{rank}(i) = \begin{cases} 0 & \text{if } i < p_k \\ \prod_{j=0}^{p_k-1}\left(1-2^{j-i}\right) & \text{if } i = p_k \\ \frac{2^{-i}(2^{p_k}-1)}{1-2^{p_k-i}} \prod_{j=0}^{p_k-1}\left(1-2^{j-i+1}\right) & \text{if } i > p_k \end{cases} \tag{4}$$

Being an erasure channel, packets are erased based on the link quality and the probability of receiving the ith encoded packet on t_r^{th} transmission across nodes p and q is

$$P_i^{t_r}\left(P_e^{SR}\right) = \left(1-P_e^{SR}\right)\binom{t_r-1}{i-1}\left(P_e^{SR}\right)^{t_r-i}\left(1-P_e^{SR}\right)^{(i-1)} \tag{5}$$

Using (4) and (5), the PMF of receiving a full rank matrix at *RN* after t_r transmissions can be computed as

$$\Pr(T_r = t_r) = \sum_{i=p_k}^{t_r} P^{p_k}_{rank}(i) P_i^{t_r}\left(P_e^{SR}\right) \tag{6}$$

Then, the expected number of transmissions required for relay decoding can be computed by substituting (6) into (3).

4.2 Expected Number of Packets Recoverable at *DN* with T_r Source Transmissions ($\overline{n}$)

Based on the SD link quality, the *DN* also receives encoded packets during the T_r source transmissions. In the proposed scheme, the source packets that are recoverable from the received encoded packets at the *DN* with these T_r transmissions are advertised to the *RN*. This can be done with the help of a feedback message from *DN* to *RN* of size p_k bits to signal which packets are recovered at the *DN*. This is done only once and just before the *RN* starts transmitting the encoded packets. The idea is to reduce the number of redundant packets being processed at the *RN* and thereby the relay transmissions supplement the mutual information at the *DN* for faster decoding. The number of packets that are recoverable at the *DN* is a function of the rank of the received generator matrix. The expected number of packets recovered at the *DN* with T_r source transmissions is given as

$$\overline{n} = \sum_{\tilde{n}=1}^{p_k} \tilde{n} \Pr(n = \tilde{n}) \tag{7}$$

where n represent the number of packets decoded correctly at the *DN* and $\Pr(n = \tilde{n})$ is the corresponding PMF. For analytical simplicity it is assumed that the number of packets decoded correctly is equal to the rank of the received generator matrix (in reality it is less than or equal to the rank due to the linear combination of packets being transmitted).

Two cases need to be considered to compute the PMF of the number of packets received at the *DN* with T_r source transmissions. For the first case, the *DN* is able to decode correctly all the source packets i.e. the received generator matrix is full rank with T_r or less number of channel uses. Then the corresponding PMF can be computed as

$$\Pr[Rank(G) = p_k] = \sum_{i=p_k}^{T_r} \sum_{j=p_k}^{i} P^{p_k}_{rank}(j) P_j^i\left(P_e^{SD}\right) \tag{8}$$

The first summation by the total probability theorem, represents the required number of transmission which can vary from the minimum limit (p_k) to the number of transmissions (T_r) required for proper decoding at the *RN*. The second summation represents the reception of a full rank matrix from the unerased packets received from corresponding source transmissions across the SD link. For the second case, the receiver is able to decode only a subset of p_k packets ($\tilde{n}$) in T_r source transmissions. The corresponding PMF of decoding $\tilde{n}$ packets is

$$\Pr[Rank(G) = \tilde{n}] = \sum_{j=\tilde{n}}^{T_r} P_{lr}(\tilde{n}, j, p_k) \times \binom{T_r}{j} \times \left(1-P_e^{SD}\right)^j \left(P_e^{SD}\right)^{T_r-j} \tag{9}$$

where $\tilde{n} \in \{1 \text{ to } (p_k-1)\}$ and $P_{lr}(\tilde{n}, j, p_k)$ is the PMF of receiving a low rank submatrix $\tilde{n}$ from a higher dimension matrix of order $p_k \times j$. The detailed steps to compute P_{lr} are shown in the Appendix. In (9), the summation signifies the reception of j packets from T_r source transmissions and the received generator matrix to be of rank $\tilde{n}$ to decode the $\tilde{n}$ source packets. Therefore the PMF of receiving $\tilde{n}$ packets after T_r source transmissions can be evaluated as

$$\Pr(n = \tilde{n}) = \begin{cases} \Pr[Rank(G) = p_k] & \text{if } \tilde{n} = p_k \\ \Pr[Rank(G) = \tilde{n}] & \text{else} \end{cases} \tag{10}$$

By evaluating (7) using (10), the expected number of packets recoverable at the *DN* can be computed.

4.3 Computation Complexity

The computational complexity of the proposed adaptive rateless coded cooperative system assisted by the feedback message differs from other schemes in the sum of operations required for (i) removing the known n input symbols from the *RN* buffer and, (ii) the subsequent operations for decoding an LT code comprised of $(p_k - n)$ input symbols across the RD link. From [2], the corresponding decoding complexity of an LT code for decoding $(p_k - n)$ input symbols is $O((p_k - n)\log(p_k - n))$. The proposed scheme thereby reduces the decoding complexity by exploiting a single feedback message from the *DN*. Eliminating redundant processing at the *RN* allocates more memory for further processing.

5. Simulation Results & Discussion

This section presents the numerical and empirical results to evaluate the performance of the proposed adaptive rateless coded cooperative network. The nodes analysed in this paper are connected with each other through wireless links where the channels between nodes are modelled as Rayleigh as explained in Sect. 3. A typical pathloss exponent (α) value of 3.5 is chosen to model an urban environment. For simplicity, the ACK and feedback message is assumed to be perfectly received. Without loss of generality, the transmitter and receiver are supposed to use a deterministic random-generator for fountain coding and the binary generator matrices are considered to be equi-probable. The receiver thereby easily synchronises with the transmitter. For simulations, the number of source packets (p_k) is taken as 12 and length of each packet (b) is taken as 256 bits. The received packets are erased based on the erasure technique presented in Sect. 3. The *RN* is considered to be placed half way between the source and destination. And the performance comparisons are done w.r.t. the average received SNR ($|H_{SD}|^2\epsilon/N_0$) across the direct link (SD link).

Figure 1 illustrates the savings in the number of packets that need to be processed at the *RN*. The numerical result obtained in the former section is compared with the simulated performance. As linear combination of packets are being transmitted, the numerical analysis derived in the previous section serves as the upper bound, as the number of packets decodable will be less than or equal to the rank of the received generator matrix. It can be seen that as SD link improves, more source packets are decodable at *DN* owing to the availability of more encoded packets. This results in lesser number of packets needing to be processed at the *RN*. At high SNRs, it is observed that the entire source packets can be recovered by direct transmissions and hence there is no need for cooperation.

The simulation results for the throughput obtained are depicted in Fig. 2. The performance of the proposed scheme is compared with normal rateless coded cooperative systems where the entire source packets are processed at the *RN*. The results confirm that the proposed scheme provides an improvement in throughput compared with normal relay transmissions at the cost of a single feedback message. The better performance of the proposed scheme can be attributed to the fact that in traditional rateless coded transmissions, the relay encodes packets that may be recoverable at the *DN*. However, the proposed scheme mitigates the redundant packet processing at the *RN* which causes the relay transmissions

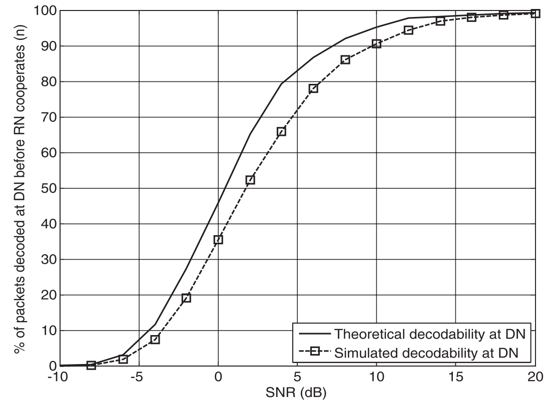

Fig. 1 Average percentage of packets recovered at *DN* vs. SNR, before *RN* starts cooperating.

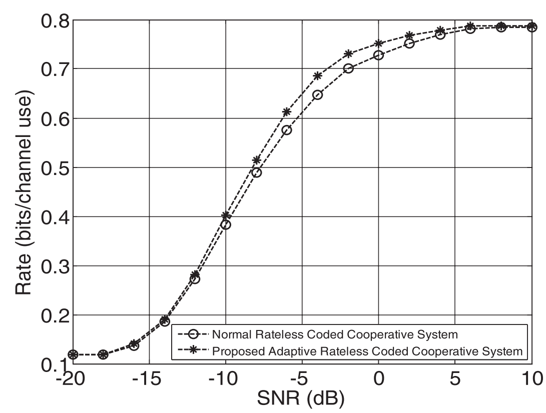

Fig. 2 Throughput performance of proposed scheme compared to the normal rateless coded cooperative transmissions for varying SNRs.

to improve the mutual information at the *DN*. This reduces the required number of transmissions and thereby gives an improvement in throughput.

6. Conclusion

In this paper, a novel spectrally efficient adaptive rateless coded cooperative transmission scheme assisted by a feedback message is proposed. Adaptation is used to mitigate redundant packet processing at the *RN* by harnessing the reverse feedback channel from the *DN* to the *RN*. Removal of recovered information from the encoding set ensures that relay transmissions supplements the mutual information to recover the remaining unknown information at the *DN*. Such a scheme reduces the communication complexity and processing at the *RN*. Analytical expressions for the expected number of packets recoverable at the receiver has been derived and verified through extensive simulation studies.

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Appendix: PMF of Receiving a Low Rank Submatrix from a Higher Dimension Matrix

Each new fountain coded packet is associated with an encoding vector over the Galois Field ($GF(2)$) of dimension p_k. Each packet is obtained by the linear combination of p_k source packets as explained in Sect. 3. The p_k source packets can be recovered if p_k linearly independent packets are received i.e. the $Rank(G)$ is p_k.

As an all zero column in G does not contain any information, it is assumed that an all zero column is not generated by the encoder. Then it is certain to receive a rank 1 matrix from a non-zero encoding set with one column. With the next encoding set there are 2 columns that are dependent among them, then the probability of having two linearly independent columns can be obtained as $\left(1 - \frac{1}{2^{p_k}}\right)$. By extending this property for R_p received packets, the probability of receiving l independent columns with $l = p_k = R_p$ is

$$P_{lr}\left(l, R_p, p_k\right) = \prod_{i=0}^{l-1}\left(1 - \frac{2^i - 1}{2^{p_k}}\right) \tag{A$\cdot$1}$$

Then as R_p increases, there are $2^{(R_p - l)}$ columns that are dependent among them. Then the probability of having l linearly independent columns from R_p columns is

$$P_{lr}\left(l, R_p, p_k\right) = C \times 2^{-(R_p - l)p_k} \times \prod_{i=0}^{l-1}\left(1 - \left(2^i - 1\right)2^{-p_k}\right) \tag{A$\cdot$2}$$

where C is a constant. By evaluation, the constant C can be approximated as a geometric series with a common ratio of 2 and initial term to be $2^{(R_p - l)}$. The number of terms between initial and final terms can be computed as

$$k = \left(R_p - l\right) \times (l - 1) + 1 \tag{A$\cdot$3}$$

Thus the constant C can be computed as

$$C \approx 2^{(R_p - l)} \times \left(2^k - 1\right) \tag{A$\cdot$4}$$