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Compressed Channel Estimation of Two-Way Relay Networks Using Mixed-Norm Sparse Constraint

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Abstract: In this study, compressed channel estimation method for sparse multipath two-way relay networks is investigated. Conventional estimation methods, e.g., Least Square (LS) and Minimum Mean Square Error (MMSE), are based on the dense assumption of relay channel and cannot exploit channel sparsity which has been verified by lots of channel measurements. Unlike the previous methods, we propose a compressed channel estimation method by using bi-sparse constraint which can exploit the sparsity and hence provide significant improvements in MSE performance when compared with conventional LS-based estimation method. Simulation results confirm the superiority of proposed method.

Keywords: Channel State Information (CSI), compressive channel estimation, compressive sensing, sparse multipath channel, Two-Way Relay Networks (TWRN)

INTRODUCTION

In Two-Way Relay Network (TWRN), two terminals,T_1 and T_2, exchange information based on the assistance of a relay *R* via Amplify-and-Forward (AF). TWRNs have been intensively studied due to its capability of enhancing the transmission capacity and providing spatial diversity for single-antenna wireless transceivers by employing the relay nodes as virtual antennas (Gao *et al*., 2009). A major difficulty is how to effectively recover the data transmitted over an unknown sparse multipath fading channel. Demodulation and coherence detection at each terminal requires the Channel State Information (CSI) from relay to itself and the CSI from the other terminal to relay. Training Sequence (TS)-based linear channel estimation methods were proposed in Gao *et al*. (2009) for the TWRN. The authors considered optimal training sequence design and linear probing methods based on the implicit assumption of a rich underlying multipath environment. In other words, TS-based methods proposed (Gao *et al*., 2009) employs linear reconstruction techniques such as Least Square (LS), Minimum Mean Square Error (MMSE) and Linear Maximum Signal-Noise Ratio (LMSNR), thus the problem of channel estimation accurately reduces to the designing of optimal training sequences (Gao *et al*., 2009).

In recent years, numerous channel measurements have shown that multipath fading channels tend to exhibit clustering sparse structures in which majority of the channel taps end up being either zero or below the noise floor (Yan *et al*., 2007). However, Two-way Relay Channels (TWRCs) differ from the conventional sparse channels and so far, there is no paper to investigate its sparsity. In this study, TWRC is also confirmed as a sparse channel by using Monte Carlo method. In our previous study (Gui *et al*., 2011), Compressed Channel Estimation (CCE) method using Compressive Sampling Matching Pursuit (CoSaMP) algorithm was proposed by using the assumption that TWRC is strictly sparse (The sparsity definition is shown in third section). However, if the strictly sparsity cannot be satisfied, our previous method cannot be applied. In this study, we propose a novel CCE method using bi-sparse constraint that does not require strict sparsity of the channel and compare it to conventional linear LS method estimators by computer simulation.

System model: Considering a typical TWRN where the two terminal users, T_1 and T_2, exchange information with the assistance of an Relay Station (RS), *R*, within two time slots. At the first time slot, T_i, i = 1, 2 transmit N×1 complex data d_i. The received signal at *R* is:

$$y_{\gamma} = H_1 d_1 + H_2 d_2 + n_r, \tag{1}$$

where, H_i, i = 1, 2 is a $N\times N$ (complex) circulant matrix, its first column is $[\ h_i^T,\ 0_{1\times(N-L)}\ ]^T$, here, h_i is an *L*-length

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sparse channel vector and its nonzero coefficients are modeled as a zero-mean complex Gaussian random variable with variance of σ^2_{hi} and $01{\times}(N\text{-}L)$ is a $1{\times}(N\text{-}L_1)$ zero vector. $n_r \epsilon C^N$ is a complex Gaussian noise with zero mean and covariance matrix $E\{n_r n^H_r\} = \sigma^2_r I_N$, where I_N denotes a $N{\times}N$ identity matrix, To minimize the computational burden, R will work under the Amplify-and-Forward (AF) protocol. The received signal in (1) is then amplified by a relay factor given in (2) and then broadcasted by amplify factor:

$$\beta = \sqrt{\frac{P_r}{\sigma^2_{h_1} P_1 + \sigma^2_{h_2} P_2 + \sigma^2_n}} \quad (2)$$

where, P_i, i = 1, 2 and P_r denote the transmitting power of Ti and forward power of R, respectively. Without loss of generality, we will only consider the channel estimation problem at T_1, while the discussion for T_2 can be made correspondingly. The received signal at T_1 is given by:

$$y_1 = \beta H_1 H_1 d_1 + \beta H_1 H_2 d_2 + \hat{n}_1 \quad (3)$$

where, $n_1 = \beta H_1 n_r + n_1$ is a N×1complex Gaussian noise with zero mean and covariance matrix $E\{n_2 n^H_2\} = \sigma^2_n(\beta^2|H_1|^2 + l_N)$ and n_1 is complex Gaussian noise with $CN(0,\sigma^2_n I_N)$. According to the system model (3), the Maximum Likelihood Fata Detection (MLD) is done at the terminal T_1 as (Gao *et al*., 2009):

$$\begin{aligned}\tilde{d}_2 &= \arg\max_{d2} P(y|d_1, d_2) \\ &= \arg\max_{d2} \frac{1}{\pi\sigma^2_n(\alpha^2|H_1|^2 + I_N)} \\ &\times \exp\left\{-\frac{|y - H_1 H_1 d_1 - H_1 H_2 d_2|^2}{\sigma^2_n(\alpha^2|H_1|^2 + I_N)}\right\} \\ &= \arg\max|y_1 - \beta H_1 H_1 d_1 - \beta H_1 H_2 d_2|\end{aligned} \quad (4)$$

From the above, the exact knowledge of H_1 and H_2 are not required for ML data detection. Instead, only composite channel matrices, i.e., H_1H_1 and H_1H_2, are needed. Compared with the channel estimation at RS, where estimated H_1 and H_2 are fed back to T_1, the use of composite channel has the lower estimation error and higher spectrum efficiency (Gao *et al*., 2009).

According to circulant matrix theory, H_i, i = 1, 2 can be decomposed as (Gray, 2006):

$$H_i = W^H \Lambda_i W,\ i = 1, 2 \quad (5)$$

where, $\Lambda_i = diag[H_{i,o},\ldots,H_{i,n},\ldots,H_{i,N-1}] \in C^{N\times N}$ is diagonal matrix of frequency-domain channel coefficients $H_{i,n} = \sum_{l=0}^{L-1} h_i(l) e^{-j2ml/N}, n = 0,\ldots,N-1$; W is the unitary Discrete Fourier Transform (DFT) matrix with $W^{mn} = 1/\sqrt{N.e}^{-j2\pi mn/N}, m, n = 0, 1, \ldots, N-1$.

Therefore, (3) can be rewritten as:

$$y_1 = W^H \beta\Lambda_1\Lambda_1 W d_1 + W^H \beta\Lambda_1\Lambda_2 W d_2 + \hat{n}_1 \quad (6)$$

Given a condition of (2N-1) < N it can be observed that $W^H\ \beta\Lambda_1\Lambda_i W$, i = 1, 2 is the decomposition of a circulant matrix which are constructed from convolution impulse response vectors $h = \beta(h_1 * h_1)$ and $g = \beta(h_1 * h_2)$. The length of h or g is (2L-1). If (6) is left-multiplied by W, then the received signal at T_1 is rewritten as:

$$y = \text{diag}(Wd_1)Fh + \text{diag}(Wd_2)Fg + n \quad (7)$$

where, F represents the first (2L-1) columns of W and $n = \beta\Lambda_1 W n_r + W n_1$ denotes the complex Gaussian random noise vector with zero mean and covariance matrix $E\{nn^H\} = \sigma^2_n(\beta^2|\Lambda_1|^2 + I_N)$. For the purpose of easy application to compressed channel sensing, (7) is rewritten as a simple system model as:

$$y = D_1 h + D_2 g + n \quad (8)$$

where, $D_i = \text{diag}(Wd_i)F$, (i = 1, 2) denotes virutal training signal matrices. Based on the system model (8), compressed channel estimation problem for sparse multipath TWRN will be discussed in the next section.

Compreseed estimation for TWRC: At first, we would like to give a definition to the sparsity of TWRC. Assume that $h = [h_0,\ldots,h_{L-1}]$ is an L-length channel vector. The sparsity measure of channel vector h was measured y mixed-norm as follows (Hoyer, 2001):

$$Sparsity(h) = \frac{\sqrt{L} - \|h\|_1/\|h\|_2}{\sqrt{L} - 1} \quad (9)$$

where, L is the length of h. $\|h\|_1$ denotes ℓ_1 norm given by $\sum_{l=0}^{L-1}|h_l|$ and $\|h\|_2$ denotes ℓ_2 norm given by $\sqrt{\sum_{l=0}^{L-1}|h_l|^2}$ By measuring sparsity of channel vectors, it is determined either vectors or its convolution vectors are sparse. Let the length of channel vector h_i be L_i = 32, i = 1, 2. Hence, the length of its convolution vector h (or g) is L = 63. Assuming that the number of nonzero taps of h_i increasing from 1 to 32, the sparsity of the channel vectors h_1, h_2, h and g are shown in Fig. 1. It is found that the sparsity of h and g correlate with the sparsity h_i. For sparse vectors, its convolution composite vectors remain to be sparse or approximate sparse. Hence, compressed estimation

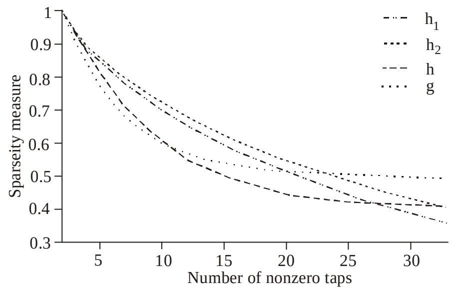

Fig. 1: Channel sparsity versus the number of nonzero channel taps

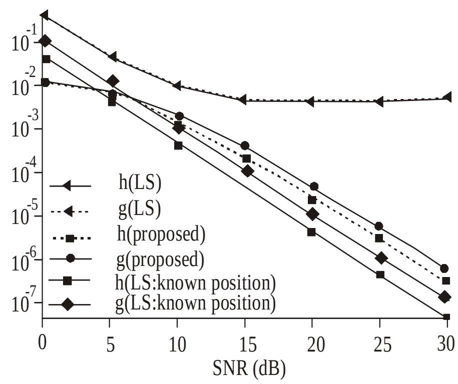

Fig. 2: MSE performance of (b) versus SNR

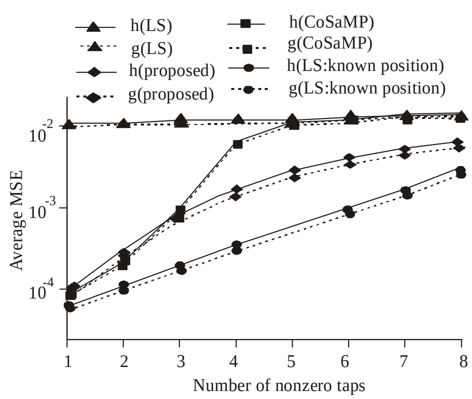

Fig. 3: MSE performance versus the number of nonzero channel taps (SNR = 10 dB)

method can exploit inherent sparsity in multipath TWRC. The composite channel vectors h and g can be estimated by following mixed-norm bi-sparse constraint:

$$\{\hat{h},\hat{g}\} = \arg\min_{\hat{h},\hat{g}} \left\{ \left\| y - D_1 h - D_2 g \right\|_2^2 + \gamma_1 \|h\|_1 + \gamma_2 \|g\|_1 \right\} \qquad (10)$$

where, γ_1 and γ_2 are sparse regularized parameters whose values govern the sparsity of $\hat{h}$ and $\hat{g}$. Large values usually produce sparser channel estimators. However, estimation accuracy may degrade if the parameters are too big (Gui *et al*., 2008). In our study, proper regularized parameters $\gamma_1 = \gamma_2 = \sigma_n \sqrt{2\log N}$ are selected (Tropp and Wright, 2010), where σn denotes noise variance and N is the length of training sequence.

SIMULATION RESULTS

In this section, we will compare the performance of the proposed method with linear LS method and adopt 10000 independent Monte-Carlo runs for average. The length of training sequence is N = 64. All of the nonzero taps of sparse channel vectors h_i, i = 1, 2 are generated following Gaussian distribution and subject to $\|h_i\|_2^2 = 1$. The length of the two channel is $L_i = 32$. The position of nonzero channel taps are randomly generated. The number of nonzero taps of h_1 and h_2 are 2 and 4, respectively. Hence, h sparser than g due to the h_1 sparser than h_2. We set transmit power $P_i = NP_0$ and AF relay power $P_r = 2NP_0$, (where P_0 is unit power) and define the signal to noise ratio (SNR) at T_i as 10 log(P_i / σ_n^2) and SNR at R as 10 log (P_r / σ_n^2).

Simulation result is shown in Fig. 2. Channel estimators are evaluated by Average Mean Square Error (MSE) which is defined by:

$$MSE(\hat{h}) = \frac{\left\| h - \hat{h} \right\|_2^2}{M(2L-1)} \qquad (11)$$

where, h and ĥ denote channel vector and its estimator, respectively, M is the number of Monte Carlo runs and (2L-1) is the overall length of channel vector h . It is found that the proposed method has a better performance than LS-based linear estimation method. Especially, in low SNR situation (less than 10dB), the proposed estimator achieves better performance even when LS estimator has the knowledge of position information of nonzero taps (Fig. 3). It is worth noting that due to the fact that h is sparser than *g*, a more accurate estimation is obtained with mixed-norm bi-sparse constraint.

CONCLUSION

In this study, compressed channel estimation method for sparse multipath TWRN using mixed-norm sparse constraint has been proposed. By using bi-sparse constraint on composite channel vectors, the inherent sparsity can be exploited. Computer simulations have confirmed the performance superiority to the conventional linear LS method of the proposed method and offer a novel standpoint on TWRN channel modeling from compressed sensing perspective.

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