

# High-resolution compressive channel estimation for broadband wireless communication systems

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## SUMMARY

Broadband channel is often characterized by a sparse multipath channel where dominant multipath taps are widely separated in time, thereby resulting in a large delay spread. Accurate channel estimation can be done by sampling received signal with analog-to-digital converter (ADC) at Nyquist rate and then estimating all channel taps with high resolution. However, these Nyquist sampling-based methods have two main disadvantages: (i) demand of the high-speed ADC, which already exceeds the capability of current ADC, and (ii) low spectral efficiency. To solve these challenges, compressive channel estimation methods have been proposed. Unfortunately, those channel estimators are vulnerable to low resolution in low-speed ADC sampling systems. In this paper, we propose a high-resolution compressive channel estimation method, which is based on sampling by using multiple low-speed ADCs. Unlike the traditional methods on compressive channel estimation, our proposed method can approximately achieve the performance of lower bound. At the same time, the proposed method can reduce communication cost and improve spectral efficiency. Numerical simulations confirm our proposed method by using low-speed ADC sampling. Copyright © 2012 John Wiley & Sons, Ltd.

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KEY WORDS: high-resolution channel; compressive channel estimation; sub-Nyquist rate sampling; compressive sensing

## 1. INTRODUCTION

With the number of wireless subscribers increasing every day, various wireless devices, for example, smart phones, netbooks, and laptops, generate massive data traffic on the rise. An authoritative industry report predicts that mobile-generated traffic will exceed that from fixed personal computers (PCs) by 2015, underscoring the fact that most information and communication technology services may be expected to migrate to portable mobile devices over the next few years. This trend toward mobile platforms has significant implications for radio technology and wireless networks, which will enable this paradigm shift, as well as for the wide variety of internet applications currently supported by fixed network devices such as PCs and televisions [1]. To satisfy the greedy demand of high-speed data services, high data rate broadband communication is an indispensable technique [2] in the next-generation communication systems.

Broadband signal transmission over multipath channel is often susceptible to frequency-selective fading. Hence, the accurate channel state information is required at the receiver for coherent detection. On the basis of the assumption of rich multipath channel, linear channel estimation methods have been proposed for different advanced systems [3–10]. A typical diagram of high-speed ADC-based channel estimation method is shown in Figure 1. However, these linear methods pose

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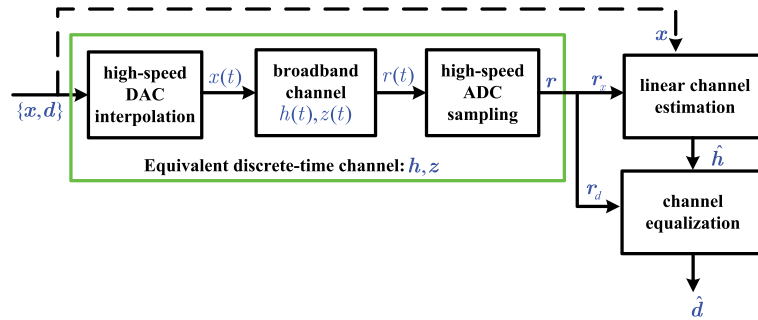


Figure 1. Broadband system model based on the high-speed ADC sampling.

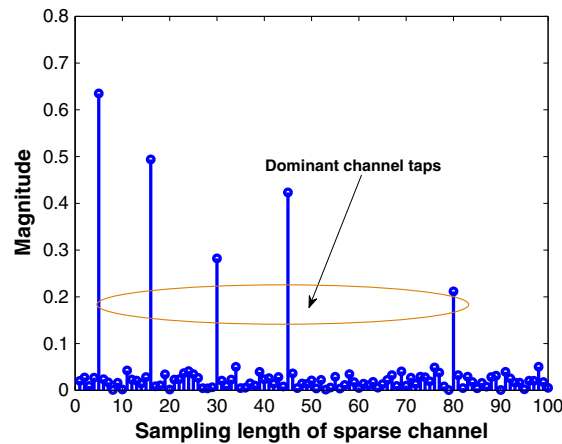


Figure 2. A typical example of sparse multipath channel where the overall sampling length is 100 while the number of dominant channel taps is 5 and most of channel taps are close to the noise level.

two challenges for the next-generation broadband communication. On the one hand, the well-known approach becomes impractical when the bandwidth is too large because it is challenging to build sampling hardware that operates at a sufficient sampling rate. The demands of many modern applications exceed the capabilities of current technology. Even though recent developments in analog-to-digital converter (ADC) technologies have increased the sampling speed, state-of-the-art architectures are not yet adequate for high-dimensional signal processing [11]. Aside from its incapability, high-speed ADC is very expensive in general and cannot be utilized widely. In addition, increasing transmission bandwidth requires much higher ADC sampling rate at the receiver. Equipping low-speed ADC at the receiver is a good candidate to solve the challenge. On the other hand, all of the proposed methods never take advantage of channel sparsity and will then cause the performance to degrade.

Indeed, recent physical experiments have confirmed that a broadband wireless channel easily exhibits sparse structure in various signal spaces [12], for example, time-delay/Doppler spread domain. An typical example of sparse multipath channel is shown in Figure 2. In the sequel, the broadband channel is described by the sparse channel model in which multipath taps are widely separated in time, thereby creating a large delay spread [13]. To relax the strict requirement of high-speed ADC sampling, recently, in [11, 14–16], some pioneering works have been carried out. In these works, different ADCs working at sub-Nyquist sampling rate have been proposed. However, these pioneering works focus on the theoretical analysis of using compressive sensing (CS) [17, 18]. All of the works have not considered their applications on compressive channel estimation in sparse multipath broadband wireless communication systems.

In the sparse multipath communication system, the receiver is equipped with low-speed ADC, which is shown in Figure 3. Compressive channel estimation method using compressive sampling matching pursuit algorithm (CoSaMP) [19] has been proposed in our previous work [20].

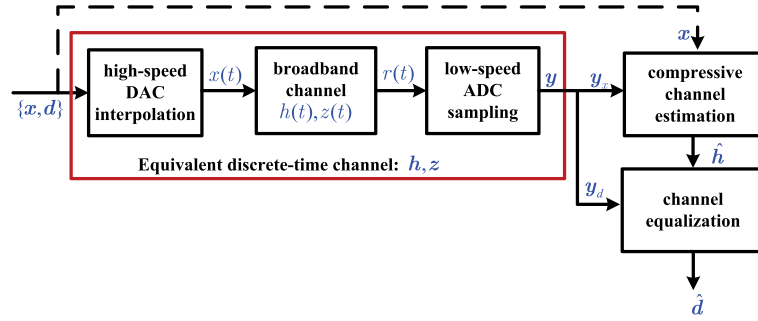


Figure 3. Broadband system model based on the low-speed ADC sampling.

However, low-speed ADC sampling will result in a low-resolution channel estimator by using traditional estimation methods. The worst case is that low-resolution channel estimation often deteriorates equalization at the receiver. To solve the contradiction between high-resolution channel estimation and low communication costs, that is, low-speed ADC sampling, it is necessary to develop high-resolution compressive channel estimation techniques. In this paper, different from the traditional method, we assume that the receiver is equipped with multiple low-speed ADCs as shown in Figure 3. On the basis of the low-speed ADC sampling system model, we propose a high-resolution compressive channel estimation method better than the conventional method in our previous work [20]. Numerical results also confirm the advantage of our proposed method.

Section 2 introduces the system model and problem formulation. Section 3 discusses compressive channel estimation for broadband communication systems under low-speed ADC sampling. In Section 4, various numerical simulation results and discussions on their performance comparison are given. Concluding remarks are presented in Section 5.

### Notations

In this paper, we use boldface lower case letters  $\mathbf{x}$  to denote vectors and boldface capital letters  $\mathbf{X}$  to denote matrices.  $x$  represents the complex Gaussian random variable.  $E[\cdot]$  stands for the expectation operation, and  $\mathbf{X}^T$ ,  $\mathbf{X}^\dagger$  denote the matrix  $\mathbf{X}$  transposition and conjugated transposition operations.  $\|\mathbf{x}\|_0$  accounts the nonzero number of  $\mathbf{x}$ , and  $\|\mathbf{x}\|_2$  is the Euclidean norm of  $\mathbf{x}$ .

## 2. SYSTEM MODEL AND PROBLEM FORMULATION

Assuming a  $W/2$ -bandwidth waveform  $x(t)$  is transmitted over a frequency-selective fading channel  $h(t)$  with additive Gaussian noise  $z(t)$ , the received continuous-time signal waveform is obtained as

$$r(t) = \int_0^{\tau_{\max}} h(\tau)x(t-\tau)d\tau + z(t), \quad (1)$$

where  $\tau_{\max}$  denotes the maximum time-delay spread of channel. According to the Shannon sampling theorem, channel comprises  $N = \lceil W\tau_{\max} \rceil + 1$  sampling taps with the Nyquist rate sampling period  $1/W$ , where  $\lceil W\tau_{\max} \rceil$  is the smallest integer larger than or equal to the  $W\tau_{\max}$ . Hence, the physical channel impulse response  $h(t)$  can be approximated by  $h(t) = \sum_{n=0}^{N-1} h_n \delta(t - n/W)$  [21]. Here, we assume that the  $N$ -length discrete channel vector  $\mathbf{h} = [h_0, h_1, \dots, h_{N-1}]^T$  is supported by only  $K$  dominant channel taps. Such a channel is often termed as  $K$ -sparse multipath channel ( $K \ll N$ ). If we use low-speed ADC with sampling speed  $1/W'$  at the receiver as shown in Figure 3, then the  $m$ th sampling coefficient  $r_m$  is given by

$$r_m = \int_0^T r(t)f(m/W' - t)dt, m = 1, 2, \dots, M, \quad (2)$$

where  $f(t) = \sin \pi t / \pi t$  denotes low-speed ADC sampling function and  $T$  is the signal period. Note that the sub-Nyquist sampling speed  $1/W'$  is much slower than the Nyquist  $1/W$ . Hereby, the equivalent baseband discrete-time system model is described by

$$\mathbf{r} = \mathbf{X}\mathbf{h} + \mathbf{z}, \tag{3}$$

where  $\mathbf{r} = [r_1, r_2, \dots, r_M]^T$  is an  $M$ -dimensional observed signal vector,  $\mathbf{z}$  is an  $M$ -dimensional Gaussian noise samples of zero mean and variance  $\sigma_n^2$ , and equivalent sub-Nyquist sampling-based training matrix  $\mathbf{X}$  is an  $M \times N$  partial Toeplitz matrix of the form

$$\mathbf{X} = \begin{bmatrix} x_{N-1} & x_{N-2} & \cdots & x_1 & x_0 \\ x_{N-2} & x_{N-3} & \cdots & x_2 & x_1 \\ \vdots & \vdots & & \vdots & \vdots \\ x_{N+M-2} & x_{N+M-3} & \cdots & x_M & x_{M-1} \end{bmatrix}. \tag{4}$$

As we know in a conventional signal processing perspective, because we reduce the ADC sampling speed too low to satisfy the Nyquist sampling rate in the system model as shown in Figure 3, the channel estimator is vulnerable to low resolution and poor performance. To make up the inherent incapability of hardware, it is hence necessary to develop a smart estimation method, which can acquire robust high-resolution channel estimator. Unlike the traditional communication system using only one low-speed ADC, we replace it with multiple low-speed ADCs, which are much cheaper than the high-speed ones. Suppose that the receiver is equipped with  $P$  parallel low-speed ADCs as shown in Figure 3. The integration period  $T$  is then split into  $P$  subintervals, and  $\mathbf{y}_m = [y_{m1}, y_{m2}, \dots, y_{mP}]^T, m = 1, 2, \dots, M$  denote the vectors of subsamples collected against the sampling waveform  $\{x_m(t)\}_{m=1}^M$ . The subsample coefficient  $y_{mp}$  is then given by

$$y_{mp} = \int_{(p-1)T/P}^{pT/P} r(t) f(N/P \cdot (m/W') - t) dt, \tag{5}$$

where  $m = 1, 2, \dots, M$ . Then the total number of subsamples collected by all parallel ADCs over all the subperiods is an  $M \times P$  matrix, which is shown in Figure 4. These subsamples can be expressed as

$$\mathbf{Y} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1P} \\ y_{21} & y_{22} & \cdots & y_{2P} \\ \vdots & \vdots & \ddots & \vdots \\ y_{M1} & y_{M2} & \cdots & y_{MP} \end{bmatrix}, \tag{6}$$

where the  $m$ th row contains the subsamples obtained by correlating the measured signal with the waveform  $x_m(t)$  over  $P N/P$ -length subperiods. Compared with ADC-based original  $M$  samples in Equation (3), that is, the sampling matrix  $\mathbf{Y}$  collected at parallel low-speed ADCs over the whole signal duration  $T$  in Equation (6), the relationship between them is given as

$$r_m = \sum_{p=1}^P y_{mp}, m = 1, 2, \dots, M, \tag{7}$$

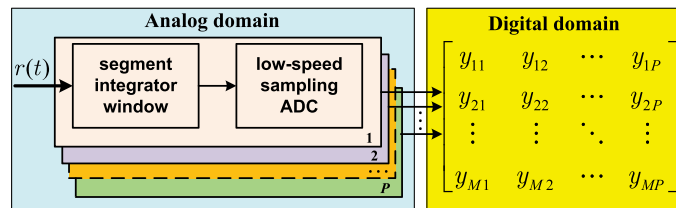


Figure 4. Low-speed ADC working at sub-Nyquist rate sampling.

for the vector  $\mathbf{r}_s = [r_1, r_2, \dots, r_M]^T$ . In the sequence, we can easily find the following relation

$$y_{mp} = \sum_{n=(p-1)N/P}^{pN/P} x_{mn} h_n, \quad (8)$$

where  $p = 1, 2, \dots, P$ ,  $n = 1, 2, \dots, N$ , and  $m = 1, 2, \dots, M$ . Hence, the observation signal  $r_m$  can be rewritten as

$$r_m = \sum_{p=1}^P \sum_{n=(p-1)N/P}^{pN/P} x_{mn} h_n, \quad m = 1, 2, \dots, M. \quad (9)$$

According to Equations (7)–(9), an equivalent extra observation vector  $\mathbf{r}_e = [r_{M+1}, r_{M+2}, \dots, r_{M+M_e}]^T$  can be obtained, where  $M_e$  is the extracted length and its  $m$ th element  $r_{M+m}$  is extracted from observation matrix, given by

$$\begin{aligned} r_{M+m} &= \sum_{p=1}^P y_{\{(m+p-2) \bmod M+1\}p} \\ &= \sum_{p=1}^P \sum_{n=(p-1)N/P}^{pN/P} x_{\{(m+p-2) \bmod M+1\}n} h_n \\ &= \sum_{p=1}^P \sum_{n=(p-1)N/P}^{pN/P} x_{[\beta(m,p)n]} h_n, \end{aligned} \quad (10)$$

for  $m = 1, 2, \dots, M_e$ , where ‘mod’ and  $\beta(m, p) = [(m + p - 2) \bmod M + 1]$  denote modulo operation and permutation function, respectively. According to Equations (7)–(10), we can obtain the following equation

$$\boldsymbol{\phi} = \boldsymbol{\Phi} \mathbf{h} + \boldsymbol{\gamma}, \quad (11)$$

where  $\boldsymbol{\phi} = [\mathbf{r}_s^T, \mathbf{r}_e^T]^T$  denotes equivalent received signal vector,  $\boldsymbol{\Phi} = [\mathbf{X}_s^T, \mathbf{X}_e^T]^T$  denotes overall training matrix, and  $\boldsymbol{\gamma} = [\mathbf{z}_s^T, \mathbf{z}_e^T]^T$  is additive noise. Note that the sub-Nyquist sampling training matrix  $\mathbf{X}_s$  and the extracted training matrix  $\mathbf{X}_e$  can be written equivalently as follows:

$$\mathbf{X}_s = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1(N-1)} & x_{1N} \\ x_{21} & x_{22} & \cdots & x_{2(N-1)} & x_{2N} \\ \vdots & \vdots & & \vdots & \vdots \\ x_{M1} & x_{M2} & \cdots & x_{M(N-1)} & x_{MN} \end{bmatrix}, \quad (12)$$

$$\mathbf{X}_e = \begin{bmatrix} x_{11} & x_{22} & \cdots & x_{(N-1)(N-1)} & x_{NN} \\ \vdots & \vdots & & \vdots & \vdots \\ x_{m,1} & x_{[\beta(m,p)2]} & \cdots & x_{[\beta(m,p)(N-1)]} & x_{[\beta(m,p)N]} \\ \vdots & \vdots & & \vdots & \vdots \\ x_{M_e 1} & x_{[\beta(M_e,p)2]} & \cdots & x_{[\beta(M_e,p)(N-1)]} & x_{[\beta(M_e,p)N]} \end{bmatrix}. \quad (13)$$

### 3. HIGH-RESOLUTION COMPRESSIVE CHANNEL ESTIMATION

We formulate the sub-Nyquist rate sampling-based sparse channel estimation as a CS problem [17, 18]. To ensure the robust high-resolution compressive channel estimation for the system, at first, we analyze the sub-Nyquist sampling matrix  $\mathbf{X}$  that must satisfy restricted isometry property (RIP) [22].

*Definition 1*

Assume that  $X_\Omega$  is a submatrix of  $X$  by extracting its columns with their indexes in the set  $\Omega \subset \{1, 2, \dots, N\}$ . Then the  $K$ -restricted isometry constant (RIC)  $\delta_K \in (0, 1)$  is the smallest number satisfying the following inequality

$$1 - \delta_K \leq \frac{\|X_\Omega \mathbf{h}\|_2^2}{\|\mathbf{h}\|_2^2} \leq 1 + \delta_K, \quad (14)$$

for all subsets  $\Omega$  of support less than or equal to  $K$  (i.e.,  $\text{supp}\{\Omega\} \leq K$ ) and all channel vectors  $\mathbf{h}$  with probability higher than  $1 - C_0 e^{-C_1 M}$ , where  $C_0 = 2(12\delta_K^{-1})^K$  and  $C_1 = \delta_K^2/16 - \delta_K^3/48$ , respectively [23].

For convenience, the preceding definition will be shortened as  $X \in \text{RIP}(K, \delta_K)$ . It is worth noting that the probability of satisfying RIP is decided by the sampling observation length  $M$ . Larger  $M$  can ensure  $X$  to satisfy RIP with higher probability, and vice versa. On the basis of the basic theory on RIP [17], if the training matrix  $X$  satisfies the RIP with higher probability, a more accurate channel estimator can be obtained by using sparse recovery algorithms, for example, CoSaMP [19]. However, increasing  $M$  on channel estimation implies that the ADC sampling speed should be improved. To avoid high-speed ADC sampling, we fix the inherent sampling length  $M$  and extract novel training length  $M_e$  from the initial sub-Nyquist sampling matrix  $X_s$ . On the basis of Definition 1 of the RIP, we derive  $X_s$  and the extracted sub-Nyquist matrix  $\Phi$ , which satisfy RIP with different probabilities. As  $X_s$  is an  $M \times N$  initial sub-Nyquist matrix, the probability of  $X_s$  satisfying RIP is  $P_s = 1 - C_0 e^{-C_1 M}$  for any subset  $\Omega \subset \{1, 2, \dots, N\}$  and  $\text{supp}\{\Omega\} \leq K$ .

*Theorem 1*

If  $X_s \in \text{RIP}(K, \delta_K)$  with the probability  $P_s$ , then  $\Phi \in \text{RIP}(K, \delta_K)$  with probability  $P_e \geq P_s$ .

*Proof*

As the  $\Phi$  is an  $(M + M_e) \times N$  matrix and  $M_e \leq M$ , there exist at least  $\lfloor (M + M_e)/2 \rfloor - M_e$  unused rows, which can be added to the set of rows in  $X_s$ . Hence, we can obtain

$$\begin{aligned} P_e &= \Pr\{\Phi \in \text{RIP}(K, \delta_K)\} \\ &\geq 1 - C_0 e^{\{-C_1 [M + (\lfloor (M + M_e)/2 \rfloor - M_e)]\}} \\ &\geq 1 - C_0 e^{-C_1 M} = P_s, \end{aligned} \quad (15)$$

which proves the theorem.  $\square$

As the equivalent training matrix  $\Phi$  satisfies RIP with high probability, hence CS algorithms (e.g., CoSaMP) can be utilized in a channel estimation problem. Compressive channel estimation methods have been intensively studied in recent years. To make comparison with our previously proposed method [20], a high-resolution compressive channel estimation is implemented by the CoSaMP algorithm [19]. The details of our proposed method are introduced as follows:

Given the received signal vector  $\mathbf{y}$ , equivalent training matrix  $\Phi$ , the number of dominant channel taps is set to  $K$ . The proposed method is composed of four steps:

**Initialization.** Set the dominant taps index set  $\Omega_0 = \emptyset$ , the residual estimation error  $\mathbf{r}_0 = \mathbf{y}$ , and put the initial iteration counter as  $i = 1$ .

**Identification.** Select a column subset  $\Omega_i$  of  $\Phi$  that is most correlated with the residual:

$$\Omega_i = \arg \max |\langle \mathbf{r}_{i-1}, \Phi \rangle|, \text{ and } \Omega_i = \Omega_{i-1} \cup \Omega_i. \quad (16)$$

Using the least-square (LS) method to calculate a channel estimator as  $\Omega_{\text{LS}} = \arg \min \|\mathbf{r} - \Phi \mathbf{h}\|_2$ , select  $K$  maximum dominant taps denoted by  $\mathbf{h}_{\text{LS}}$ . The positions of the selected dominant taps are denoted by  $\Omega_{\text{LS}}$ .

**Merging.** The positions of dominant taps are merged by  $\Omega_i = \Omega_{\text{LS}} \cup \Omega_i$ .

**Estimation.** Compute the best coefficient to approximate the channel vector with chosen columns,

$$\hat{h}_i = \arg \min_{\hat{h}} \left\| \phi - \Phi_{\Omega_i} \hat{h} \right\|_2. \quad (17)$$

**Pruning.** Select the  $\Omega_i$  largest channel taps of  $\mathbf{h}_i$  and set

$$\hat{h}_{\Omega \setminus \Omega_i} = 0. \quad (18)$$

**Iteration.** Update the estimation error:

$$\mathbf{r}_i = \phi - \Phi_{\Omega_i} \hat{\mathbf{h}}_i, \quad (19)$$

increase the iteration counter  $i$ . Repeat (16)–(19) until a stopping criterion is satisfied and then set  $\hat{\mathbf{h}} = \hat{\mathbf{h}}_i$ .

#### 4. NUMERICAL SIMULATIONS

In this section, we will compare the performance of the proposed estimators with 10,000 independent Monte Carlo runs for averaging. The length of sparse multipath channel  $\mathbf{h}$  is set as  $N = 96$ , and its number of dominant channel taps is set as  $K$ . We consider two kinds of distributions on all dominant channel taps that their values are generated from  $1/K$ -uniform distribution and random Gaussian distribution, respectively. The positions of dominant channel taps are randomly allocated within the length of  $\mathbf{h}$  and is subjected to  $\mathbb{E} \{ \|\mathbf{h}\|_2^2 \} = 1$ . The initial  $\mathbf{X}_s$  is an equivalent  $M \times N$  partial Toeplitz matrix. The initial length of training sequence is set as  $M = 32$ , and the extract training length is  $M_e$ . The number of parallel low-speed ADCs is set as  $P = 8$ . The received SNR is defined as  $10 \log (E/\sigma_n^2)$ , where  $E$  is received power. Here, we set the SNR values as 10, 15, and 20 dB in the following numerical simulations.

The estimation performance is evaluated by two criteria: successful recovery of dominant channel taps and average mean square error (Average MSE). The average MSE of channel estimators  $\hat{\mathbf{h}}$  is defined by

$$\text{Average MSE}(\hat{\mathbf{h}}) = \frac{\mathbb{E} \left\{ \|\mathbf{h} - \hat{\mathbf{h}}\|_2^2 \right\}}{L}, \quad (20)$$

where  $\mathbf{h}$  and  $\hat{\mathbf{h}}$  denote the original channel vector and its estimator, respectively. In the numerical results, the lower bound of the channel estimator is based on a high-speed sampling ADC, utilizing different equivalent random partial Toeplitz matrices  $\mathbf{X}$  with training length  $M + M_e$  (long training sequence), which is calculated. Unlike the lower bound, in the traditional method, a low-speed sampling ADC is used, and its equivalent training matrix is  $\mathbf{X}_s$ . The proposed method uses the same matrix  $\mathbf{X}_s$  with training length  $M$  (short training sequence) and its extracted training matrix  $\mathbf{X}_e$ .

In Figures 5 and 6, we compare the successful recovery probability of dominant channel taps as a function of extracted training signal length from 0 to 56. Here, the number of dominant channel taps is set as  $K = 4$ . From the two figures, it is observed that our proposed method is very close to the optimal performance bound, and the successful recovery increases as the extracted training length increases; whereas the traditional method is invariant, and the performance is very poor. In addition, the two figures also showed that the channel taps are related to their distribution, for example, uniform distributed channel taps are reconstructed easier than random distributed Gaussian taps. Furthermore, the average MSE of different methods are also depicted in Figures 7 and 8. From the two figures, it is observed that the estimation performance of our proposed method is better than that of the traditional method and close to the lower bound.

Next, the relationship between the proposed method and channel sparsity is considered, and SNR = 15 dB is used. Here, we consider the uniform and Gaussian distribution of dominant taps

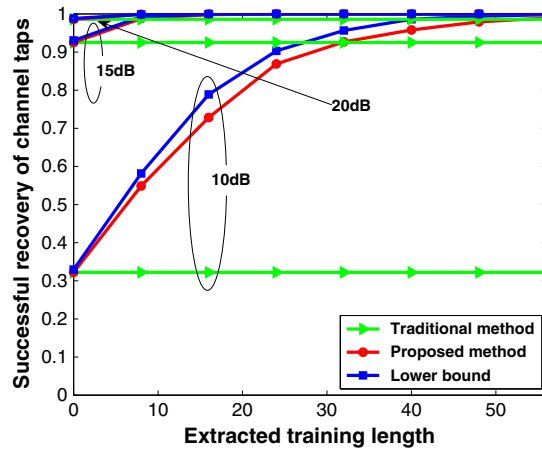


Figure 5. Successful recovery probability of the dominant channel taps versus the extracted training length, where the dominant channel taps satisfy the uniform distribution.

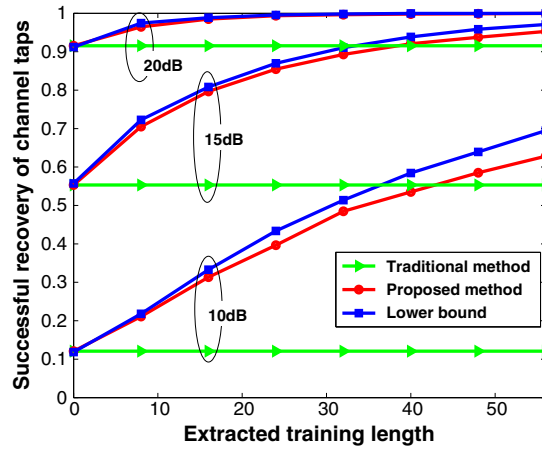


Figure 6. Successful recovery probability of the dominant channel taps versus the extracted training length, where the dominant channel taps satisfy the Gaussian distribution.

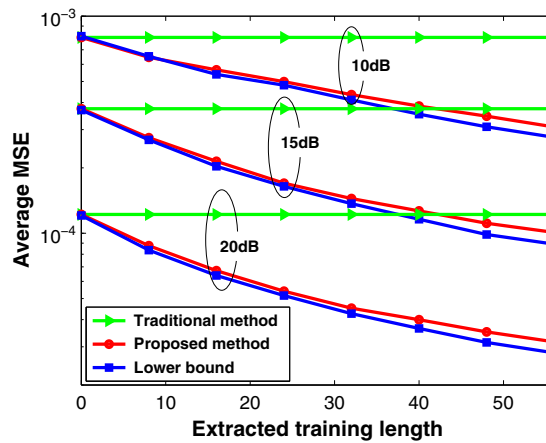


Figure 7. Average MSE performance versus the extracted training length, where the dominant channel taps satisfy the  $1/K$ -uniform distribution.



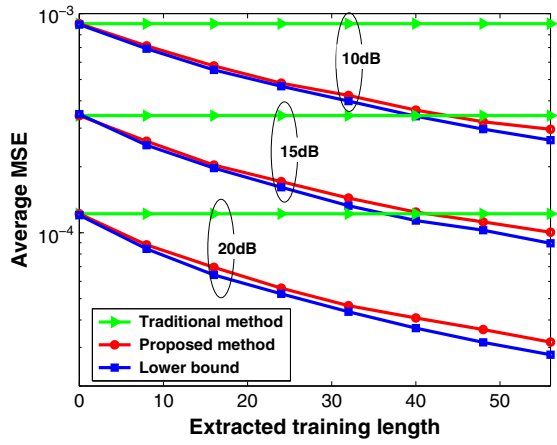


Figure 8. Average MSE performance versus the extracted training length, where the dominant channel taps satisfy the Gaussian distribution.

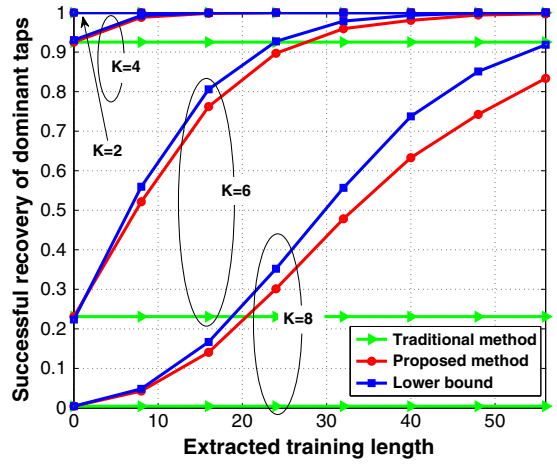


Figure 9. Average MSE performance versus the extracted training length, where the dominant channel taps satisfy the uniform distribution.

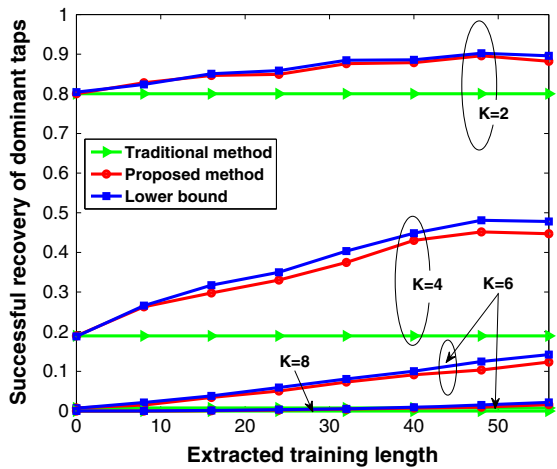


Figure 10. Average MSE performance versus the extracted training length, where the dominant channel taps satisfy the Gaussian distribution.

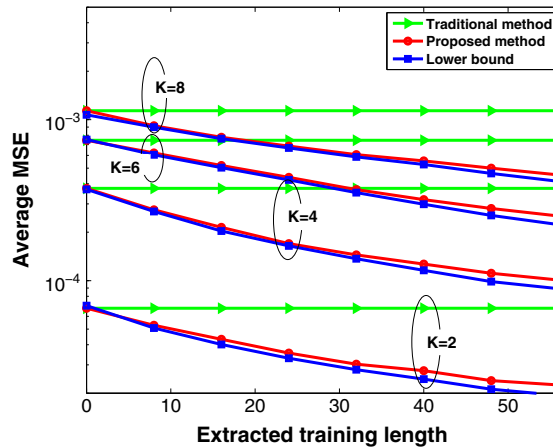


Figure 11. Average MSE performance versus the extracted training length, where the dominant channel taps satisfy the uniform distribution.

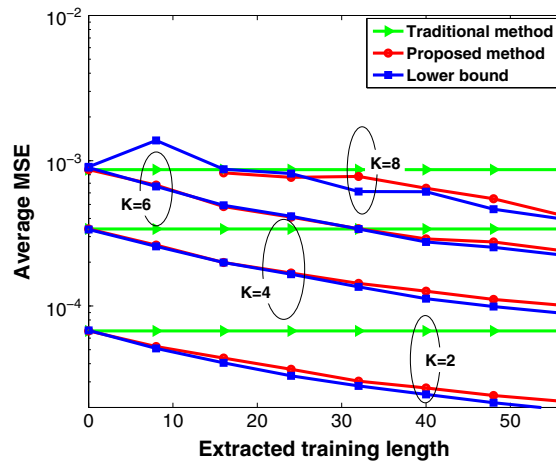


Figure 12. Average MSE performance versus the extracted training length, where the dominant channel taps satisfy the Gaussian distribution.

on a compressive channel estimation. Assume that the number of channel length is the same, while the number of dominant taps are 2, 4, 6, and 8, respectively. Figures 9 and 10 depict their successful recovery probability of dominant channel taps versus different extracted training lengths. From the two figures, we can find that our proposed method can achieve a higher recovery probability than that of the traditional method. As the extracted training length increases, the successful recovery probability of the dominant taps becomes higher by our proposed method without sacrificing spectral resource. Likewise, we also compare their average MSE with different extracted training length as shown in Figures 11 and 12, respectively. From the two figures, it is observed that our proposed method can always approach their lower bounds for any channel sparsity. Hence, the proposed method is stable for the sparse channel with different numbers of dominant channel taps. According to the numerical simulations, the effectiveness of the proposed sub-Nyquist sampling rate of the ADC-based method has been verified.

## 5. CONCLUSION

In this paper, we have investigated a high-resolution compressive channel estimation based on the sub-Nyquist rate sampling ADC. We formulated the channel estimation as a CS problem. In the

sequence, a high-resolution compressive channel estimation method has been proposed for sparse multipath broadband communication systems. A comparison with the traditional sparse channel estimation methods has shown that our proposed method has two advantages: (i) can achieve high-resolution channel estimation and even much better estimation performance and (ii) can save communication cost. In a future work, we will study a multi-antenna high-resolution compressive channel estimation based on a sub-Nyquist sampling ADC.

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