Joint Cooperative-Transmit and Receive FDE for Single-Carrier Incremental Relaying

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***Abstract*—In this paper, we propose joint cooperative-transmit/receive frequency-domain equalization (cooperative-Tx/Rx FDE) with incremental relaying (IR) for broadband single-carrier (SC) transmission under the total and individual transmit power constraint at a source node (S) and a relay node (R). We derive the optimum cooperative-Tx/Rx FDE weights to minimize the mean square error (MSE) after packet combining at a destination node (D). We show that the optimum cooperative-Tx FDE weights allocate the transmit power in the minimum MSE (MMSE) sense in the frequency domain and in the maximal ratio transmission (MRT) sense in the spatial domain. To relax the condition that the complete channel state information (CSI) needs to be shared among all nodes, selection-based suboptimum cooperative-Tx FDE weights are derived. Computer simulation verifies the effectiveness of the proposed schemes and shows that the 5%-outage throughput of the system can be increased by around 30% over the conventional IR with only Rx FDE.**

***Index Terms*—Cooperative relay transmission, frequency-domain equalization (FDE), single-carrier (SC) transmission.**

I. Introduction

A BROADBAND wireless channel is mainly characterized by two components, namely, *frequency-selective fading* and *large-scale fading* such as distance-dependent path loss and log-normally distributed shadowing loss [1]. To overcome the severe frequency-selective fading, various equalization strategies have been proposed for broadband single-carrier (SC) transmission [2]–[4]. One-tap frequency-domain equalization (FDE) has been gaining much attention for its simplicity; affinity for multicarrier transmission, e.g., orthogonal frequency-division multiplexing; and extendibility to more advanced equalization techniques, e.g., turbo FDE [5], [6]. The one-tap FDE can be implemented at either the transmitter or the receiver, and the same performance can be achieved [7], [8]. To further improve the performance, joint transmit/receive (Tx/Rx) FDE has been proposed for point-to-point SC transmission in [9], in which one-tap FDE is implemented at both the transmitter and the receiver by sharing the channel state information (CSI). The Tx/Rx FDE weights are jointly designed based on the minimum mean-square-error (MMSE) criterion under the transmit power constraint. In [10] and [11], an extension of joint Tx/Rx FDE to hybrid automatic repeat request (HARQ) with chase combining [12] was presented; the Tx/Rx FDE weights, which jointly minimize the mean square error (MSE) *after packet combining at the receiver side*, are applied to the packet upon each request of packet retransmission. It was shown that the joint Tx/Rx FDE is effective in obtaining a higher packet combining gain and improves the throughput performance.

In a cellular network, the received signal power fluctuates due to the large-scale fading such as path loss and shadowing loss. Frequency-selective fading may further degrade the system performance. Although the aforementioned joint Tx/Rx FDE can mitigate the effect of frequency-selective fading and exploits it as diversity to improve the transmission performance, it cannot compensate for the power fluctuation due to the large-scale fading. This power fluctuation severely degrades the throughput performance even with HARQ. Recently, cooperative relaying has been extensively studied in [13] and [14] and references therein. A relay node (R) is deployed to assist the transmission from a source node (S) to a destination node (D). R can help to compensate for the aforementioned power fluctuation. Among many relaying protocols, incremental relaying (IR) [13] is effective in achieving high throughput. In IR, R joins the retransmissions only when the packet has been correctly decoded at R but not at D. Use of R provides a spatial diversity gain and, hence, is effective in improving performance.

We are therefore motivated to study the joint Tx/Rx FDE in IR to achieve high throughput even under the severe fading channels. We target to improve the throughput performance by optimizing the Tx FDE weights at S and R in a cooperative manner. Joint cooperative-Tx/Rx FDE (cooperative-Tx/Rx FDE) for broadband SC-IR is proposed. If a packet sent by S has not been correctly decoded at D but at R, retransmission is cooperatively carried out by S and R through cooperative-Tx FDE. The retransmitted packets are combined with earlier received packets in frequency domain using Rx FDE at D. We first derive a set of optimum FDE weights among nodes based on the MMSE criterion with the total and individual transmit power constraint at S and R. The optimum weights jointly minimize the MSE *after packet combining* at D. We show that the optimum cooperative-Tx weights allocate the transmit power in MMSE sense in the frequency domain (across

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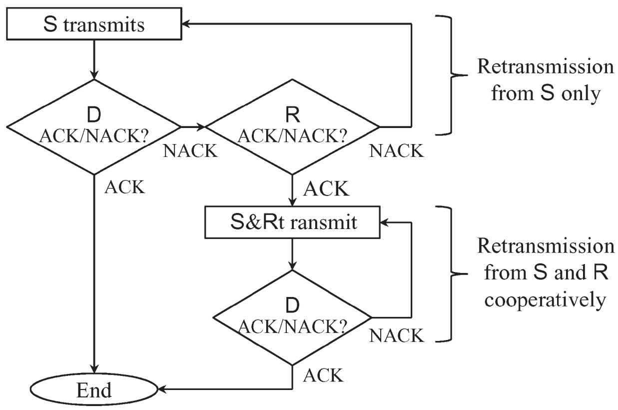

Fig. 1. Flowchart of IR.

the subcarriers) and in maximal ratio transmission (MRT) sense in the spatial domain (across the transmitting nodes). The optimum FDE weights design, however, needs to share the complete CSI of all links among all the nodes. To relax this constraint, we propose a suboptimum cooperative-Tx/Rx FDE weights design. By solving the optimization problem, we show that the suboptimum weights are selection based. Computer simulation shows that the suboptimum selection-based cooperative-Tx/Rx FDE weights provide almost the same performance as the optimum FDE weights. It is shown that the 5%-outage throughput of the system can be increased by around 30% over the conventional IR with Rx FDE only.

The rest of this paper is organized as follows: The principle of SC-IR with joint cooperative-Tx/Rx FDE is described in Section II. In Section III, the optimum FDE weights design is provided. The suboptimum cooperative-Tx FDE weights are derived in Section IV. Computer simulation results are provided in Section V. Section VI concludes this paper.

Notations: $(.)^*$, $(.)^T$, and $(.)^H$ denote conjugate, transpose, and Hermitian transpose operation, respectively. $\mathrm{tr}\{A\}$ denotes the trace operation of matrix A. $\mathrm{diag}\{a_0,\ldots,a_i,\ldots,a_M\}$ expresses an $M \times M$ diagonal matrix whose ith element is given by a_i. $\mathrm{E}[.]$ denotes the ensemble average operation.

II. Single-Carrier Incremental Relaying With Joint Cooperative-Transmit/Receive Frequency-Domain Equalization

A. Basic Concept of IR

Fig. 1 shows the flowchart of IR. S generates a packet from a data sequence. A packet transmitted from S is received by both R and D due to the broadcast nature of radio propagation. If D correctly decodes the packet, it sends an acknowledgment (ACK) signal to S. If D does not decode the packet correctly, it broadcasts a nonacknowledgment (NACK) signal to request for retransmission. If the packet is correctly decoded at R, the retransmission is cooperatively performed by S and R; otherwise, it is done by S only.

In the following, without loss of generality, the signaling process for the qth $(0 \le q \le Q)$ retransmission is considered, where Q is the maximum allowable number of retransmissions. Although $q = 0$ indicates the initial transmission, the terminology "retransmission" is used for simplicity of explanation. Let us denote the retransmission index at which R correctly decodes the packet by $\tilde{q}$ $(0 \le \tilde{q} \le Q)$. For $q \le \tilde{q}$, the retransmission is done by S only. The retransmission is cooperatively done by S and R from the $(\tilde{q}+1)$th retransmission. In this paper, we call the retransmission for $q \le \tilde{q}$ the *broadcast phase* and that for $q > \tilde{q}$ the *cooperation phase*.

B. Transmitted Signal

The maximum delay of the channel impulse response is assumed to be within the cyclic prefix (CP) length, and symbol-spaced discrete-time signal representation is used for the system model.

A binary information bit sequence of length $N_{\rm info}$ bits is first turbo encoded and then data modulated. A data-modulated symbol sequence is divided into $N_{\rm block}$ blocks having N_c symbols each, where N_c is the number of fast Fourier transform (FFT) and inverse FFT (IFFT) points for FDE. Without loss of generality, a signal representation for one N_c-symbol block is considered. The data block is expressed using a vector form as $\mathbf{d} = [d(0),\ldots,d(i),\ldots,d(N_c-1)]^T$. $\mathbf{d}$ is transformed into frequency-domain signal by N_c-point FFT as

$$\mathbf{D} = \mathbf{F}\mathbf{d} \tag{1}$$

where $\mathbf{F}$ is an $N_c \times N_c$ FFT matrix.

At each transmitting node, each frequency component of the signal is multiplied by a cooperative-Tx FDE weight. The weighted frequency-domain signal at node $m (m \in \{\mathsf{S},\mathsf{R}\})$ for the qth retransmission is represented as

$$\mathbf{S}_m^q = \mathbf{U}_m^q \mathbf{D} \tag{2}$$

where $\mathbf{U}_m^q$ is the $N_c \times N_c$ matrix representation of the cooperative-Tx FDE weights for the qth retransmitting packet. $\mathbf{U}_m^q$ is expressed as

$$\mathbf{U}_{\mathsf{S}}^q = \mathrm{diag}\{U_{\mathsf{S}}^q(0),\ldots,U_{\mathsf{S}}^q(k) \\ \ldots,U_{\mathsf{S}}^q(N_c-1)\}, \qquad \text{for } 0 \le q \le Q \tag{3a}$$

$$\mathbf{U}_{\mathsf{R}}^q = \begin{cases} \mathbf{0}, & \text{for } 0 \le q \le \tilde{q} \\ \mathrm{diag}\{U_{\mathsf{R}}^q(0),\ldots,U_{\mathsf{R}}^q(k) \\ \quad \ldots,U_{\mathsf{R}}^q(N_c-1)\} & \\ & \text{for } \tilde{q} < q \le Q. \end{cases} \tag{3b}$$

As will be shown later, the Tx FDE weights are computed under the total and individual transmit power constraint.

The signal is transformed back to the time-domain signal by N_c-point IFFT. The time-domain signal block at node $m (m \in \{\mathsf{S},\mathsf{R}\})$ is represented as

$$\mathbf{s}_m^q = \mathbf{F}^H \mathbf{S}_m^q. \tag{4}$$

After the CP insertion, the signals are transmitted from each node.

C. Received Signal During Broadcast Phase

The packet is retransmitted by S alone during *broadcast phase*, i.e., $q \le \tilde{q}$. At node n $(n \in \{\mathsf{R},\mathsf{D}\})$, the time-domain received signal at the qth retransmission after removing the CP is written as

$$\mathbf{r}_n^q = [r_n^q(0), \ldots, r_n^q(i), \ldots, r_n^q(N_c-1)]^T = \mathbf{h}_{\mathsf{S},n}^q \mathbf{s}_{\mathsf{S}}^q + \boldsymbol{\pi}_n^q \quad (5)$$

where $\mathbf{h}_{m,n}^q$ ($m \in \{\mathsf{S}, \mathsf{R}\}$ and $n \in \{\mathsf{R}, \mathsf{D}\}$) is an $N_c \times N_c$ circulant matrix representing the impulse response of the channel from node m to node n at the qth retransmission [9], and $\boldsymbol{\pi}_n^q = [\pi_n^q(0), \ldots, \pi_n^q(i), \ldots, \pi_n^q(N_c-1)]^T$ is the noise vector at the qth retransmission with each element $\pi_n^q(i)$ being a zero-mean additive white Gaussian noise having variance $2\sigma^2$. $\mathbf{h}_{m,n}^q$ is expressed as

$$\mathbf{h}_{m,n}^q = \underbrace{d_{m,n}^{-\alpha} \times 10^{-\frac{\eta_{m,n}}{10}}}_{\text{large-scale fading}} \times \underbrace{\mathbf{g}_{m,n}^q}_{\text{small-scale fading}} \quad (6)$$

where $d_{m,n}$ and $\eta_{m,n}$ are the distance and the log-normally distributed shadowing loss with the standard deviation of σ_η dB between node m and node n, respectively; α is the path loss exponent; and $\mathbf{g}_{m,n}^q$ is the complex circulant channel matrix.

$\mathbf{r}_n^q$ is transformed into frequency-domain signal $\mathbf{R}_n^q = [R_n^q(0), \ldots, R_n^q(k), \ldots, R_n^q(N_c-1)]^T$ by N_c-point FFT, which is given as

$$\mathbf{R}_n^q = \mathbf{F}\mathbf{r}_n^q = \mathbf{H}_{\mathsf{S},n}^q \mathbf{S}_{\mathsf{S}}^q + \boldsymbol{\Pi}_n^q \quad (7)$$

with $\mathbf{H}_{m,n}^q = \mathbf{F}\mathbf{h}_{m,n}^q \mathbf{F}^H$ and $\boldsymbol{\Pi}_n^q = \mathbf{F}\boldsymbol{\pi}_n^q$. Due to the circulant property of $\mathbf{h}_{m,n}^q$, the channel gain matrix $\mathbf{H}_{m,n}^q$ of size $N_c \times N_c$ becomes diagonal and can be expressed as

$$\mathbf{H}_{m,n}^q = \mathrm{diag}\{H_{m,n}^q(0), \ldots, H_{m,n}^q(k), \ldots, H_{m,n}^q(N_c-1)\}. \quad (8)$$

After receiving the qth retransmitted packet, node n ($n \in \{\mathsf{R}, \mathsf{D}\}$) has $(q+1)$ received packets, i.e., during the initial transmission and the q retransmissions, in its buffer to recover the packet. They are combined in frequency domain as [15]

$$\hat{\mathbf{D}}_n = \left[\hat{D}_n(0), \ldots, \hat{D}_n(k), \ldots, \hat{D}_n(N_c-1)\right]^T = \sum_{q'=0}^{q} \mathbf{V}_n^{q'} \mathbf{R}_n^{q'} \quad (9)$$

where $\mathbf{V}_n^{q'} = \mathrm{diag}\{V_n^{q'}(0), \ldots, V_n^{q'}(k), \ldots, V_n^{q'}(N_c-1)\}$ $(0 \leq q' \leq q)$ are the Rx FDE weights for packet combining at node n.

After combining all the packets in frequency domain, the signal is transformed back to the time-domain signal by N_c-point IFFT as

$$\hat{\mathbf{d}}_n = \left[\hat{d}_n(0), \ldots, \hat{d}_n(i), \ldots, \hat{d}_n(N_c-1)\right]^t = \mathbf{F}^H \hat{\mathbf{D}}_n. \quad (10)$$

Then, the log-likelihood ratio (LLR) is calculated for channel decoding. If D correctly decodes the packet, ACK signal is broadcasted.

D. Joint Tx/Rx FDE Weights During Broadcast Phase

During broadcast phase, the retransmission from S to node n ($n \in \{\mathsf{R}, \mathsf{D}\}$) is the same as the joint Tx/Rx FDE with *given* Tx FDE weights, i.e., $\mathbf{U}_{\mathsf{S}}^q$. Hence, the optimum Rx FDE weights at node n ($n \in \{\mathsf{R}, \mathsf{D}\}$) are given by [10]

$$V_{n,\mathrm{opt}}^{q'}(k) = \frac{\left\{H_{\mathsf{S},n}^{q'}(k) U_{\mathsf{S}}^{q'}(k)\right\}^*}{\sum_{q'=0}^{q} \left|H_{\mathsf{S},n}^{q'}(k) U_{\mathsf{S}}^{q'}(k)\right|^2 + 2\sigma^2}. \quad (11)$$

However, the design criterion of the Tx FDE weights at S, i.e., $\mathbf{U}_{\mathsf{S}}^q$, during broadcast phase depends not only on the $\mathsf{S} \to \mathsf{D}$ link but on the $\mathsf{S} \to \mathsf{R}$ link as well as S broadcasts the signal to both R and D. If Tx FDE weights are derived to minimize the MSE at R, the possibility of cooperation increases. However, the probability that the packet is correctly decoded at D cannot be improved. On the other hand, if Tx FDE weights are derived to minimize the MSE at D (this is the optimum in the case of direct transmission), the probability that R can decode the packet does not improve, and R cannot join the retransmissions. Accordingly, the benefit of IR cannot be enjoyed.

Hence, in addition to the case with no Tx FDE, two types of Tx FDE weights are considered at S during broadcast phase.

- No Tx FDE: The total power is uniformly allocated to the whole frequency band (or to all the subcarriers), and the weights become

$$\left|U_{\mathsf{S}}^q(k)\right|^2 = 2P \qquad \forall k. \quad (12)$$

- Type I (denoted as "type-I Tx FDE"): Tx FDE weights are derived such that the MSE after packet combining at D is minimized, and the weights are given as [10]

$$\left|U_{\mathsf{S}}^q(k)\right|^2 = \max\left[\frac{1}{\sqrt{\mu}}\sqrt{\frac{\sigma^2}{\left|H_{\mathsf{S},\mathsf{D}}^q(k)\right|^2}} - \frac{\Xi_{\mathsf{D}}^q(k)}{\left|H_{\mathsf{S},\mathsf{D}}^q(k)\right|^2}, 0\right] \quad \forall k. \quad (13)$$

- Type II (denoted as "type-II Tx FDE"): Tx FDE weights are derived such that the MSE after packet combining at R is minimized, and the weights are given as [10]

$$\left|U_{\mathsf{S}}^q(k)\right|^2 = \max\left[\frac{1}{\sqrt{\mu}}\sqrt{\frac{\sigma^2}{\left|H_{\mathsf{S},\mathsf{R}}^q(k)\right|^2}} - \frac{\Xi_{\mathsf{R}}^q(k)}{\left|H_{\mathsf{S},\mathsf{R}}^q(k)\right|^2}, 0\right] \quad \forall k. \quad (14)$$

In (13) and (14), $\max[a,b]$ returns the larger value of a and b, μ is determined to guarantee the total transmit power constraint [10], and $\Xi_n^q(k)$ ($n \in \{\mathsf{R}, \mathsf{D}\}$) is defined as

$$\Xi_n^q(k) \triangleq \sigma^2 + \sum_{q'=0}^{q-1} \left|H_{\mathsf{S},n}^{q'}(k) U_{\mathsf{S}}^{q'}(k)\right|^2. \quad (15)$$

E. Received Signal at D *During Cooperation Phase*

Once the packet is correctly decoded at R, the packet is retransmitted by S and R cooperatively for $q > \tilde{q}$. At D, the time-domain received signal at the qth retransmission after removing the CP is written as

$$\mathbf{r}_{\mathsf{D}}^q = \mathbf{h}_{\mathsf{S},\mathsf{D}}^q \mathbf{s}_{\mathsf{S}}^q + \mathbf{h}_{\mathsf{R},\mathsf{D}}^q \mathbf{s}_{\mathsf{R}}^q + \boldsymbol{\pi}_{\mathsf{D}}^q \quad (16)$$

where $\mathbf{h}_{\mathsf{S,D}}^q$ and $\mathbf{h}_{\mathsf{R,D}}^q$ are both circulant matrices. $\mathbf{r}_{\mathsf{D}}^q$ is transformed into frequency-domain signal $\mathbf{R}_{\mathsf{D}}^q$ by N_c-point FFT, which is given as

$$\mathbf{R}_{\mathsf{D}}^q = \mathbf{F}\mathbf{r}_{\mathsf{D}}^q = \mathbf{H}_{\mathsf{S,D}}^q \mathbf{S}_{\mathsf{S}}^q + \mathbf{H}_{\mathsf{R,D}}^q \mathbf{S}_{\mathsf{R}}^q + \mathbf{\Pi}_{\mathsf{D}}^q. \tag{17}$$

After receiving the qth retransmitted packet, D has $(q+1)$ received packets. They are combined in frequency domain as [15]

$$\hat{\mathbf{D}}_{\mathsf{D}} = \sum_{q'=0}^{q} \mathbf{V}_{\mathsf{D}}^{q'} \mathbf{R}_{\mathsf{D}}^{q'}. \tag{18}$$

After combining all the packets in frequency domain, the signal is transformed back to the time-domain signal by N_c-point IFFT as

$$\hat{\mathbf{d}}_{\mathsf{D}} = \mathbf{F}^H \hat{\mathbf{D}}_{\mathsf{D}}. \tag{19}$$

Then, the LLR is calculated for channel decoding. If there is residual error after channel decoding, NACK signal is broadcasted. Otherwise, ACK signal is broadcasted.

III. Optimum Joint Cooperative-Transmit/Receive Frequency-Domain Equalization Weights During Cooperation Phase

Here, we develop the optimum cooperative-Tx/Rx FDE weights design for the cooperation phase, i.e., $q > \tilde{q}$.

A. MSE and Optimization Problem

Let us define the error vector between the transmitted and the equalized data blocks as $\mathbf{e} \triangleq \mathbf{d} - \hat{\mathbf{d}}_{\mathsf{D}}$. Then, the MSE is given as

$$\begin{aligned} e &= \mathrm{tr}\left[\mathrm{E}\{\mathbf{e}\mathbf{e}^H\}\right] \\ &= \mathrm{tr}\left[\mathrm{E}\left\{(\mathbf{d}-\hat{\mathbf{d}}_{\mathsf{D}})(\mathbf{d}-\hat{\mathbf{d}}_{\mathsf{D}})^H\right\}\right] \\ &= \mathrm{E}\left[\sum_{k=0}^{N_c-1}\left|D(k)-\hat{D}_{\mathsf{D}}(k)\right|^2\right]. \end{aligned} \tag{20}$$

From (2), (17), and (18), e can be rewritten as

$$\begin{aligned} e = {} & 2\sigma^2 \sum_{k=0}^{N_c-1}\sum_{q'=0}^{q}\left|V_{\mathsf{D}}^{q'}(k)\right|^2 \\ & + \sum_{k=0}^{N_c-1}\left|1 - \underbrace{\sum_{q'=0}^{\tilde{q}} V_{\mathsf{D}}^{q'}(k) H_{\mathsf{S}}^{q'}(k) U_{\mathsf{S}}^{q'}(k)}_{\text{Retransmission in broadcast phase}} \right. \\ & \left. - \underbrace{\sum_{q'=\tilde{q}+1}^{q} V_{\mathsf{D}}^{q'}(k)\left\{H_{\mathsf{S}}^{q'}(k)U_{\mathsf{S}}^{q'}(k) + H_{\mathsf{R}}^{q'}(k)U_{\mathsf{R}}^{q'}(k)\right\}}_{\text{Retransmission in cooperation phase}}\right|^2. \end{aligned} \tag{21}$$

In this paper, the transmit signal power of each retransmission is assumed to be limited by $2P$. Therefore, an optimization problem to derive the FDE weights for the qth retransmission in the cooperation phase is formulated as

$$\min_{\mathbf{U}_{\mathsf{S}}^q, \mathbf{U}_{\mathsf{R}}^q, \left\{\mathbf{V}_{\mathsf{D}}^{q'}; 0 \le q' \le q\right\}} e \tag{22a}$$

$$\text{s.t.} \quad \sum_{k=0}^{N_c-1}\left|U_{\mathsf{S}}^q(k)\right|^2 + \sum_{k=0}^{N_c-1}\left|U_{\mathsf{R}}^q(k)\right|^2 \le 2P \tag{22b}$$

$$-\left|U_{\mathsf{S}}^q(k)\right|^2 \le 0, -\left|U_{\mathsf{R}}^q(k)\right|^2 \le 0 \; \forall k. \tag{22c}$$

To obtain the optimum cooperative-Tx/Rx FDE weights, there are two different approaches.

1) The *optimum* Rx weights are derived for *arbitrarily given* cooperative-Tx FDE weights. Then, the *optimum* cooperative-Tx FDE weights are derived based on the obtained *optimum* Rx FDE weights.
2) The *optimum* cooperative-Tx FDE weights are derived for *arbitrarily given* Rx FDE weights. Then, the *optimum* Rx FDE weights are derived based on the obtained *optimum* cooperative-Tx FDE weights.

In the following sections, we take the first approach, i.e., the optimum Rx FDE weights $\{\mathbf{V}_{\mathsf{D},\mathrm{opt}}^{q'}; 0 \le q' \le q\}$ at D for packet combining is derived for arbitrarily given cooperative-Tx FDE weights at S and R for the qth retransmission, i.e., $\mathbf{U}_{\mathsf{S}}^q$ and $\mathbf{U}_{\mathsf{R}}^q$. Then, for the derived $\{\mathbf{V}_{\mathsf{D},\mathrm{opt}}^{q'}; 0 \le q' \le q\}$, the optimum $\mathbf{U}_{\mathsf{S},\mathrm{opt}}^q$ and $\mathbf{U}_{\mathsf{R},\mathrm{opt}}^q$ are obtained. In the Appendix, the weight derivation for the second approach is given. It is shown that the derived cooperative-Tx/Rx FDE weights are the same.

B. Optimum Rx FDE Weights

The set of the optimum Rx FDE weights for packet combining at D should satisfy

$$\frac{\partial e}{\partial V_{\mathsf{D}}^{q'}(k)} = 0 \qquad \forall q' \text{ and } \forall k. \tag{23}$$

These optimum weights can be easily derived using (21), which gives (24), shown at the bottom of the page. Since the optimum Rx FDE weight is a function of the cooperative-Tx FDE

$$V_{\mathsf{D},\mathrm{opt}}^{q'}(k) = \frac{\left\{H_{\mathsf{S,D}}^{q'}(k)U_{\mathsf{S}}^{q'}(k) + H_{\mathsf{R,D}}^{q'}(k)U_{\mathsf{R}}^{q'}(k)\right\}^*}{\left(\sum_{q''=0}^{\tilde{q}}\left|H_{\mathsf{S,D}}^{q''}(k)U_{\mathsf{S}}^{q''}(k)\right|^2 + \sum_{q''=\tilde{q}+1}^{q}\left|H_{\mathsf{S,D}}^{q''}(k)U_{\mathsf{S}}^{q''}(k) + H_{\mathsf{R,D}}^{q''}(k)U_{\mathsf{R}}^{q''}(k)\right|^2 + 2\sigma^2\right)} \tag{24}$$

weights and the current and past channels, D needs to update the Rx FDE weights as it receives the retransmitted packet.

Substituting (24) into (22a), the optimization problem (22) can be rewritten as

$$\min_{\{\mathbf{U}_{\mathsf{S}}^q,\mathbf{U}_{\mathsf{R}}^q\}} e=\sum_{k=0}^{N_c-1}\frac{2\sigma^2}{\Omega^q(k)+\left|H_{\mathsf{S,D}}^q(k)U_{\mathsf{S}}^q(k)+H_{\mathsf{R,D}}^q(k)U_{\mathsf{R}}^q(k)\right|^2} \tag{25a}$$

$$\text{s.t.}\ \sum_{k=0}^{N_c-1}\left|U_{\mathsf{S}}^q(k)\right|^2+\sum_{k=0}^{N_c-1}\left|U_{\mathsf{R}}^q(k)\right|^2-2P\le 0 \tag{25b}$$

$$-\left|U_{\mathsf{S}}^q(k)\right|^2\le 0, -\left|U_{\mathsf{R}}^q(k)\right|^2\le 0\ \forall\, k \tag{25c}$$

where

$$\Omega^q(k)\triangleq\sum_{q'=0}^{\tilde{q}}\left|H_{\mathsf{S,D}}^{q'}(k)U_{\mathsf{S}}^{q'}(k)\right|^2 + \sum_{q'=\tilde{q}+1}^{q-1}\left|H_{\mathsf{S,D}}^{q'}(k)U_{\mathsf{S}}^{q'}(k)+H_{\mathsf{R,D}}^{q'}(k)U_{\mathsf{R}}^{q'}(k)\right|^2+2\sigma^2. \tag{26}$$

From (26), $\Omega^q(k)$ only depends on the previous retransmissions. Hence, it can be considered as a constant value at the qth retransmission.

C. Optimum Cooperative-Tx FDE Weights

We proceed to derive the optimum cooperative-Tx FDE weights at S and R for the qth retransmission. By applying Cauchy–Schwarz inequality to the denominator on the right-hand side of (25a), we obtain (27), shown at the bottom of the page, where equality holds, i.e., MSE e is minimized, if and only if the following condition is satisfied:

$$\frac{\{U_{\mathsf{S}}^q(k)\}^*}{H_{\mathsf{S,D}}^q(k)}=\frac{\{U_{\mathsf{R}}^q(k)\}^*}{H_{\mathsf{R,D}}^q(k)}=\Theta\in\Re\qquad\forall\, k. \tag{28}$$

Note that $\Re$ represents a set of real numbers.

By taking (28) as an additional constraint, the optimization problem can be rewritten as

$$\min_{P_{\mathsf{S}}^q(k),P_{\mathsf{R}}^q(k)} e = \sum_{k=0}^{N_c-1}\frac{\sigma^2}{\left(\tilde{\Omega}^q(k)+\left\{\left|H_{\mathsf{S,D}}^q(k)\right|^2+\left|H_{\mathsf{R,D}}^q(k)\right|^2\right\}\{P_{\mathsf{S}}^q(k)+P_{\mathsf{R}}^q(k)\}\right)} \tag{29a}$$

$$\text{s.t.}\ \sum_{k=0}^{N_c-1}\{P_{\mathsf{S}}^q(k)+P_{\mathsf{R}}^q(k)\}-P\le 0 \tag{29b}$$

$$-P_{\mathsf{S}}^q(k)\le 0 \text{ and } -P_{\mathsf{R}}^q(k)\le 0\quad\forall\, k \tag{29c}$$

where $\tilde{\Omega}^q(k)\triangleq(\Omega^q(k)/2)$, $P_{\mathsf{S}}^q(k)\triangleq(|U_{\mathsf{S}}^q(k)|^2/2)$, and $P_{\mathsf{R}}^q(k)\triangleq(|U_{\mathsf{R}}^q(k)|^2/2)$.

To solve (29), a Lagrange function is defined as

$$J\triangleq e+\mu\left\{\sum_{k=0}^{N_c-1}\{P_{\mathsf{S}}^q(k)+P_{\mathsf{R}}^q(k)\}-P\right\} - \sum_{k=0}^{N_c-1}\{\eta_1^q(k)P_{\mathsf{S}}^q(k)+\eta_2^q(k)P_{\mathsf{R}}^q(k)\} \tag{30}$$

where μ and $\{\eta_1^q(k),\eta_2^q(k);0\le k<N_c\}$ are nonnegative Lagrange multipliers. Since (29) is a convex optimization problem [16], the solution satisfies the Karush–Kuhn–Tucker (KKT) condition [17], [18], which is expressed as

$$\begin{cases}\frac{\partial J}{\partial P_{\mathsf{S}}^q(k)}=0, \frac{\partial J}{\partial P_{\mathsf{R}}^q(k)}=0, P_{\mathsf{S}}^q(k)\ge 0, P_{\mathsf{R}}^q(k)\ge 0\\ \sum_{k=0}^{N_c-1}\{P_{\mathsf{S}}^q(k)+P_{\mathsf{R}}^q(k)\}-P\le 0,\mu\ge 0\\ \eta_1^q(k)\ge 0,\eta_2^q(k)\ge 0,\eta_1^q(k)P_{\mathsf{S}}^q(k)=0,\eta_2^q(k)P_{\mathsf{R}}^q(k)=0\\ \mu\left\{\sum_{k=0}^{N_c-1}\{P_{\mathsf{S}}^q(k)+P_{\mathsf{R}}^q(k)\}-P\right\}=0.\end{cases} \tag{31}$$

Solving the given conditions, we have

$$P_{\mathsf{S,opt}}^q(k)+P_{\mathsf{R,opt}}^q(k)=\max\left[\frac{1}{\sqrt{\mu}}\sqrt{\frac{\sigma^2}{\left|H_{\mathsf{S,D}}^q(k)\right|^2+\left|H_{\mathsf{R,D}}^q(k)\right|^2}} - \frac{\tilde{\Omega}^q(k)}{|H_{\mathsf{S}}^q(k)|^2+|H_{\mathsf{R}}^q(k)|^2},0\right] \tag{32}$$

where μ is determined to satisfy

$$\sum_{k=0}^{N_c-1}\left\{P_{\mathsf{S,opt}}^q(k)+P_{\mathsf{R,opt}}^q(k)\right\}=P. \tag{33}$$

Finally, from (28) and (32), the optimum cooperative-Tx FDE weights for node m ($m\in\{\mathsf{S},\mathsf{R}\}$) become

$$U_{m,\text{opt.}}^q(k)=\frac{\left\{H_{m,\mathsf{D}}^q(k)\right\}^*}{\sqrt{\left|H_{\mathsf{S,D}}^q(k)\right|^2+\left|H_{\mathsf{R,D}}^q(k)\right|^2}} \times\sqrt{P_{\mathsf{S,opt}}^q(k)+P_{\mathsf{R,opt}}^q(k)}. \tag{34}$$

$$e\ge\sum_{k=0}^{N_c-1}\frac{2\sigma^2}{\left(\Omega^q(k)+\left\{\left|H_{\mathsf{S,D}}^q(k)\right|^2+\left|H_{\mathsf{R,D}}^q(k)\right|^2\right\}\times\left\{|U_{\mathsf{S}}^q(k)|^2+|U_{\mathsf{R}}^q(k)|^2\right\}\right)} \tag{27}$$

From (32) and (34), the optimum cooperative-Tx FDE weights can be interpreted as 2-D weights; one dimension is frequency, and the other is space, i.e., transmitting node. Taking $\sqrt{|H^q_{\mathsf{S,D}}(k)|^2 + |H^q_{\mathsf{R,D}}(k)|^2}$ as an equivalent channel gain, we allocate the frequency-domain power based on MMSE as (32). Then, the allocated power at each frequency is then distributed to S and R, where the phases of the weights are decided based on (34). At each frequency, the transmit FDE weights are MRT weights between S and R [19]. By substituting the obtained optimum cooperative-Tx FDE weights $U^q_{m,\mathrm{opt}}(k)$ $(m \in \{\mathsf{S}, \mathsf{R}\})$ into (24), the optimum Rx FDE weight, i.e., $\{V^{q'}_{\mathsf{D},\mathrm{opt}}(k); 0 \leq q' \leq q\}$, is obtained.

D. Optimum Cooperative-Tx/Rx FDE Weights for Incremental Redundancy

In the case of chase combining, every packet contains the same data symbol block. Hence, all the packets are combined in the frequency domain (18), and the cooperative-Tx/Rx FDE weights are derived by taking into account all the retransmitted packets, as shown in (24) and (26). On the other hand, the same data symbol block is retransmitted every P packets in the case of incremental redundancy, where P denotes the number of puncturing patterns. Hence, when incremental redundancy is used for the HARQ strategy, only the packets that carry the same data block are taken into account to derive the cooperative-Tx/Rx FDE weights, and only those packets are combined in frequency domain.

IV. Suboptimum Cooperative-Transmit Frequency-Domain Equalization Weights During Cooperation Phase

To obtain the set of optimum cooperative-Tx FDE weights, a complete set of CSI needs to be shared among all the nodes. To reduce the overhead to share the CSI, we consider a suboptimum Tx FDE weights design method.

Let us revisit the MSE to be minimized, which is expressed as (35), shown at the bottom of the page, where $\Omega^q(k)$ is given in (26), and

$$\Psi(k) \triangleq 2\mathrm{Re}\left[H^q_{\mathsf{S,D}}(k)U^q_{\mathsf{S}}(k)\left\{H^q_{\mathsf{R,D}}(k)U^q_{\mathsf{R}}(k)\right\}^*\right]. \quad (36)$$

To guarantee the spatial diversity gain, it is easy to see that $\Psi(k)$ should be greater than or equal to 0. To make $\Psi(k) \geq 0$, we have following approaches:

1) to let either of the weights be 0 such that $\Psi(k) = 0$;
2) to use the Tx FDE weights that compensate the phase rotation of the channel such that $\Psi(k) \geq 0$.

The first approach is a selection-based approach. In this approach, the MRT-based cooperative-Tx FDE is replaced by the selection-based cooperative-Tx FDE, which is similar to antenna selection diversity. The second approach is an equal-gain (EG) combining approach. Only its own channel to D is necessary for the selection-based approach. Furthermore, the phase information of its own channel to D is unnecessary as MMSE-FDE is used at D, i.e., the phase rotation caused by the channel can be compensated at D. Hence, the requirement for weight derivation can be relaxed. In the following sections, the detail of each suboptimum Tx FDE weight is given.

A. Subcarrier-Selection (SCS) Weights and Transmitting-Node-Selection (TNS) Weights

Two selection-based cooperative-Tx FDE weights are considered, namely, SCS and TNS. In SCS, different transmitting nodes may be selected for different subcarriers. In TNS, either S or R is selected for retransmission across all the subcarriers. The optimization problem for the selection-based approach is formulated as (37a)–(37c), shown at the bottom of the page. The given problem is again a standard convex optimization problem. Hence, the solution is obtained by solving the KKT conditions

$$e = \sum_{k=0}^{N_c-1} \frac{2\sigma^2}{\left(\Omega^q(k) + \left|H^q_{\mathsf{S,D}}(k)\right|^2 \left|U^q_{\mathsf{S}}(k)\right|^2 + \left|H^q_{\mathsf{R,D}}(k)\right|^2 \left|U^q_{\mathsf{R}}(k)\right|^2 + \Psi(k)\right)} \quad (35)$$

$$\min_{\{\mathbf{U}^q_{\mathsf{S}}, \mathbf{U}^q_{\mathsf{R}}\}} e = \sum_{k=0}^{N_c-1} \frac{2\sigma^2}{\left(\Omega^q(k) + \left|H^q_{\mathsf{S,D}}(k)\right|^2 \left|U^q_{\mathsf{S}}(k)\right|^2 + \left|H^q_{\mathsf{R,D}}(k)\right|^2 \left|U^q_{\mathsf{R}}(k)\right|^2\right)} \quad (37a)$$

$$\text{s.t.} \sum_{k=0}^{N_c-1} \left|U^q_{\mathsf{S}}(k)\right|^2 + \sum_{k=0}^{N_c-1} \left|U^q_{\mathsf{R}}(k)\right|^2 - 2P \leq 0 \quad (37b)$$

$$-\left|U^q_{\mathsf{S}}(k)\right|^2 \leq 0, -\left|U^q_{\mathsf{R}}(k)\right|^2 \leq 0 \quad \forall k \quad (37c)$$

given in (31). The following suboptimum cooperative-Tx FDE weights are obtained.

- SCS-based suboptimum cooperative-Tx FDE weights are given by (38), shown at the bottom of the page.
- TNS-based suboptimum cooperative-Tx FDE weights are given by (39), shown at the bottom of the page.

B. EG Weights

For EG-based suboptimum Tx FDE weights, the cooperative-Tx FDE weights are chosen to satisfy

$$H_{\mathsf{S}}^{q}(k)U_{\mathsf{S}}^{q}(k)\left\{H_{\mathsf{S}}^{q}(k)U_{\mathsf{S}}^{q}(k)\right\}^{*}\propto\left|H_{\mathsf{S,D}}^{q}(k)\right|^{2}\left|H_{\mathsf{R,D}}^{q}(k)\right|^{2} \quad (40)$$

$$\sum_{k=0}^{N_c-1}\left|U_{\mathsf{S}}^{q}(k)\right|^{2}+\sum_{k=0}^{N_c-1}\left|U_{\mathsf{R}}^{q}(k)\right|^{2}-2P\le 0. \quad (41)$$

Hence, we can choose

$$U_{m,\mathrm{EG}}^{q}(k)=\sqrt{P}\times\frac{\left\{H_{m,\mathsf{D}}^{q}(k)\right\}^{*}}{\left|H_{m,\mathsf{D}}^{q}(k)\right|}. \quad (42)$$

From (42), the available transmit power is equally distributed to the two nodes, and hence, the amplitude information of each channel is not required at transmitting nodes. Thus, the required system complexity can be reduced.

V. Performance Evaluation

A. Simulation Parameters

Here, we present the simulated performance of the proposed schemes. The simulation parameters are summarized in Table I. Fig. 2 shows the deployment of S, R, and D. We consider a linear network model. The distances between two nodes are *normalized* such that the distance between R and D becomes 1.0. The normalized distance between S and D is denoted by d and is varied from 1.2 to 2.1. For the propagation channel, the path loss exponent is set to $\alpha = 3.5$, log-normally distributed shadowing loss with standard deviation of $\sigma = 8$ dB, and L-path frequency-selective block Rayleigh fading having a uniform power delay profile are considered. Except in Fig. 5(a), L is set to 16. The channel is constant during one packet

TABLE I
Simulation Parameters

Channel Parameters	
Normalized distance	$d = 1.2 \sim 2.1$
Path-loss exponent	$\alpha = 3.5$
Shadowing standard deviation	$\sigma = 8$ dB
Number of resolvable paths	$L = 1, 2, 4, 8, 16$
Power delay profile	Uniform
Normalized SNR	$SNR_{ref} = 0.0 \sim 9.0$ dB
Signaling Parameters	
Coding / Decoding	Turbo coding with $(13, 15)_8$ two RSC encoders / Log-MAP decoders with 8 iterations
Coding rate	$R = 1/3,\ 2/3$
Data modulation	QPSK
Number of FFT/IFFT points	$N_c = 256$
Number of GI samples	$N_g = 32$
Type of HARQ	Chase combining Incremental redundancy
Maximum number of retransmissions	$Q = 10$
Channel estimation and error detection	Ideal
Feedback delay	None
Correlation between channel gains for retransmission	Independent

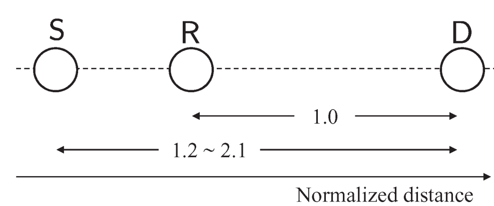

Fig. 2. Node deployment.

but is independent from one packet to another. The transmit power, i.e., P, is determined to make the received signal-to-noise power ratio (SNR) at the distance 1.0 to be $\mathrm{SNR}_{\mathrm{ref}}$ (this is referred to as reference SNR in this paper). Hence, we have $P = d^{\alpha}\cdot\mathrm{SNR}_{\mathrm{ref}}$. Each data symbol block is turbo coded with a code rate of R and a generator polynomial of $(13, 15)_8$. The information bit length is set to $N_{\mathrm{info}} = 1018$ bits. The

$$\begin{cases}\left|U_{m,\mathrm{SCS}}^{q}(k)\right|^{2}=\max\left[\frac{1}{\sqrt{\mu}}\sqrt{\frac{\sigma^{2}}{\left|H_{m,\mathsf{D}}^{q}(k)\right|^{2}}}-\frac{\tilde{\Omega}^{q}(k)}{\left|H_{m,\mathsf{D}}^{q}(k)\right|^{2}},0\right], & \text{if } m=\arg\max_{m'}\left|H_{m',\mathsf{D}}^{q}(k)\right|^{2}\\ \left|U_{m,\mathrm{SCS}}^{q}(k)\right|^{2}=0, & \text{otherwise}\end{cases} \quad (38)$$

$$\begin{cases}\left|U_{m,\mathrm{TNS}}^{q}(k)\right|^{2}=\max\left[\frac{1}{\sqrt{\mu}}\sqrt{\frac{\sigma^{2}}{\left|H_{m,\mathsf{D}}^{q}(k)\right|^{2}}}-\frac{\tilde{\Omega}^{q}(k)}{\left|H_{m,\mathsf{D}}^{q}(k)\right|^{2}},0\right], & \text{if } m=\arg\max_{m'}\sum_{k'=0}^{N_c-1}\left|H_{m',\mathsf{D}}^{q}(k')\right|^{2}\\ \left|U_{m,\mathrm{TNS}}^{q}(k)\right|^{2}=0, & \text{otherwise}\end{cases} \quad (39)$$

TABLE II
PUNCTURING PATTERNS

Chase combining		
$R = 1/3$		$R = 2/3$
$\begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$		$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$
Incremental redundancy		
$q \mod 3 = 0$	$q \mod 3 = 1$	$q \mod 3 = 2$
$\begin{pmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$

output sequences from the encoder are punctured according to the puncturing patterns in Table II to generate a sequence with code rates of $R = 1/3$ and 2/3. One packet is composed of $N_{\text{block}} = 6$ and $N_{\text{block}} = 4$ blocks for $R = 1/3$ and 2/3, respectively. For a soft-input–soft-output decoder at R and D, Log-MAP decoding with eight iterations is used. Quadrature phase-shift keying (QPSK) data modulation is used. Ideal channel estimation is assumed, and required CSI is assumed to be available at each node to calculate FDE weights. Chase combining [12] and incremental redundancy [20] are used for the HARQ strategy. Except in Fig. 6, chase combining is used. The maximum allowable number of retransmissions is set to $Q = 10$.

The throughput is defined as

$$T = \frac{N_{\text{info}}}{T_c \times (N_c + N_g) \times N_{\text{block}}} \times \frac{1}{(1/T_c)} \times \frac{P_{\text{rx}}}{\sum_{p=1}^{P_{\text{tx}}} (q_p^{\text{req}} + 1)} \quad \text{(b/s/Hz)} \quad (43)$$

where P_{tx} and P_{rx} are the number of packets transmitted and that of successfully received, respectively. The symbol duration is denoted by T_c, and the frequency bandwidth of the signal becomes $(1/T_c)$ accordingly. The required number of retransmissions for successful reception of the pth packet is denoted by q_p^{req}. In the following simulations, we set $P_{\text{tx}} = 41$.

We use the following notations. The performance of IR with the optimum joint cooperative-Tx/Rx FDE is denoted by "JC-MRT," and those of the suboptimum joint cooperative-Tx/Rx FDE are denoted by "JC-SCS," "JC-TNS," and "JC-EG," respectively. The performance of joint Tx/Rx FDE without IR [10] is denoted by "JFDE," and that of IR with Rx FDE at D is only denoted by "IR." In the case of IR, once R correctly decodes the packet, the retransmission is performed by node m $(m \in \{\mathsf{S}, \mathsf{R}\})$, whose average channel gain over all frequencies is larger, i.e., $m = \arg\max_{m'} \sum_{k=0}^{N_c-1} |H_{m',\mathsf{D}}^{q}(k)|^2$.

B. Selection of Transmit Weights for Broadcast Phase

First, the impact of the Tx FDE weights selection at S during the broadcast phase on the throughput performance is evaluated. The joint cooperative-Tx/Rx FDE weights during the cooperation phase are derived based on the optimum design, i.e., (24) and (34). The normalized distance between S and D is set to $d = 1.2$, 1.5, and 1.8. The cumulative distribution function (cdf) of the throughput T (in bits per second per Hertz) is shown in Fig. 3. In the figure, type-I Tx FDE provides higher throughput than type-II Tx FDE for $d = 1.2$. On the other hand, type-II Tx FDE provides much higher throughput performance than type-I Tx FDE when $d = 1.5$ and 1.8. The reason for these phenomena can be explained as follows: When d is small, it is better to increase the possibility of correct decoding at D by matching the Tx FDE weight to the S $\rightarrow$\{D\} channel rather than to increase the possibility of correct decoding at R. On the other hand, when d is large, i.e., the transmission performance between S and D is poor due to the high path loss, it is better to make R join the retransmission as much as possible by matching the Tx FDE weights to the S $\rightarrow$ R channel. Surprisingly, no Tx FDE achieves the best performance except for a low-throughput region with $d = 1.8$ as it requires the help of R due to the poor S $\rightarrow$ D channel condition. This is because it can achieve a compromise between type-I Tx FDE and type-II Tx FDE. Hence, in the following, no Tx FDE is applied at S during the broadcast phase.

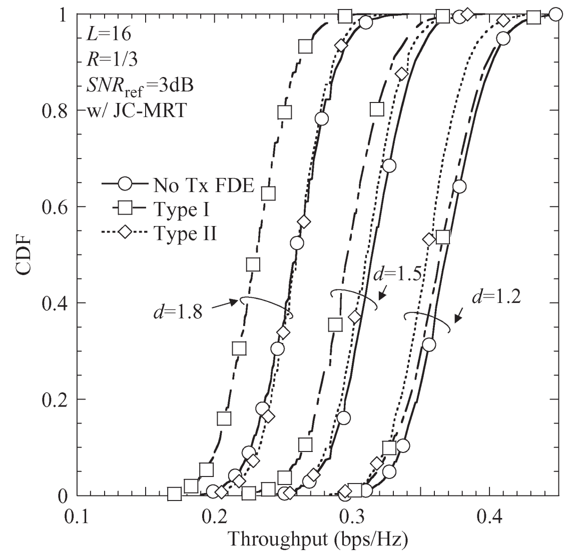

Fig. 3. Impact of Tx FDE weights at S during broadcast phase.

C. Optimum Cooperative-Tx/Rx FDE and Suboptimum Cooperative-Tx/Rx FDE

Here, the performances of the system with the optimum cooperative-Tx/Rx FDE weights and that with the suboptimum weights are shown. In Fig. 4, the complementary cdf (ccdf) of the number of required retransmissions q_p^{req}, that of packet loss rate $P_{\text{lost}} \triangleq (P_{\text{tx}} - P_{\text{rx}}/P_{\text{tx}})$, and the cdf of the throughput T (b/s/Hz) are shown. The normalized distance between S and D is set to $d = 1.5$, and the normalized SNR is set to $\text{SNR}_{\text{ref}} = 3$ dB.

In Fig. 4(a), the conventional JFDE [10] requires a large number of retransmissions due to the large path loss between S and D. Accordingly, the packet loss rate of the conventional JFDE is significantly high as it requires a large number of retransmissions, as shown in Fig. 4(b). By introducing IR,

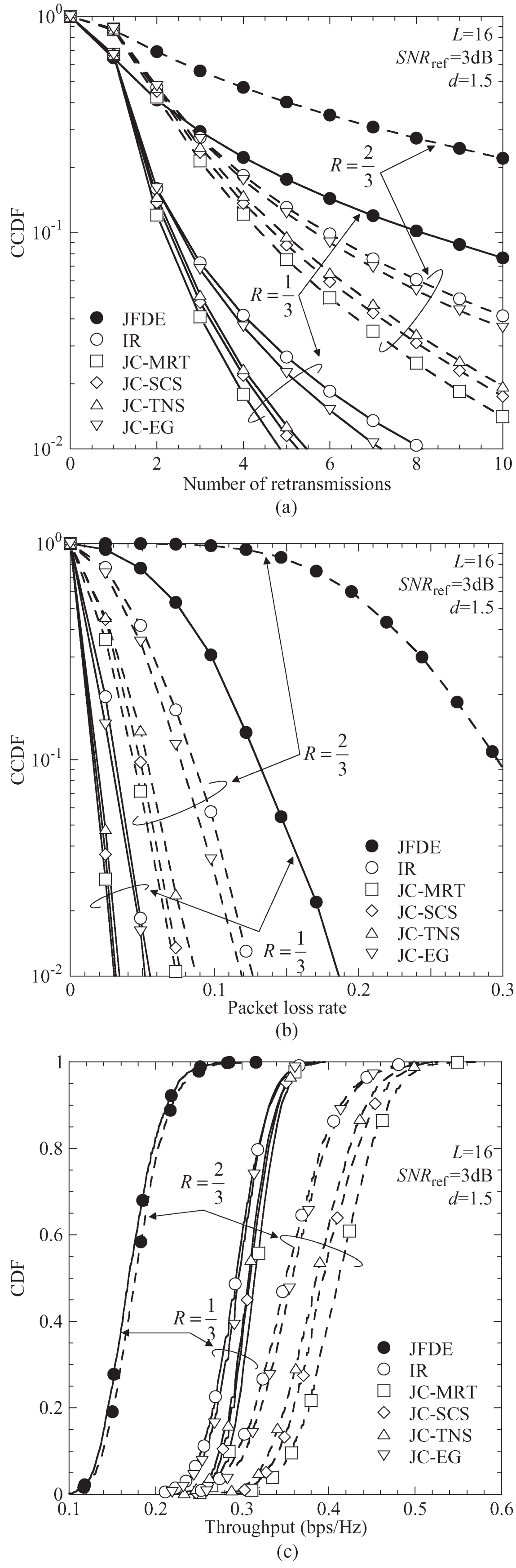

Fig. 4. Performance improvement obtained by the proposed joint cooperative-Tx/Rx FDE. (a) Required number of retransmissions per packet for correct decoding. (b) Packet loss rate. (c) Throughput (in bits per second per Hertz).

the performance is significantly improved due to the spatial diversity gain and the smaller path loss between R and D. The performance improvement is further enhanced by the proposed joint cooperative-Tx/Rx FDE approaches. The performance improvement provided by the proposed joint cooperative-Tx/Rx FDE over the Rx FDE only becomes more significant when the coding rate is $R = 2/3$. Fig. 4(c) clearly shows that significant throughput enhancement can be achieved by the proposed joint cooperative-Tx/Rx FDE. In particular, JC-MRT is effective in increasing the throughput performance of the users who are experiencing bad channel conditions, i.e., the lower throughput users. The joint cooperative-Tx/Rx FDE is more effective when the code rate becomes higher. For example, the 1%(5%)-outage throughput of the system is increased by 20%(15%) for $R = 1/3$, whereas it is increased by 24%(21%) for $R = 2/3$ compared with IR with Rx FDE only. Here, p%-outage throughput is defined as the throughput that $(1 - p)$% users of the system achieve. This is because the achievable spatial and frequency diversity gains by the joint cooperative-Tx/Rx FDE become more dominant when the coding gain becomes smaller. As a consequence, the performance improvement over the IR becomes smaller in the case of JC-EG when the coding rate becomes higher, i.e., $R = 2/3$, because it cannot fully exploit the spatial and frequency diversity gains.

As shown in the figures, the throughput performance with the suboptimum approaches, i.e., JC-SCS and JC-TNS, degrades compared with the optimum JC-MRT. However, the performance degradation is slight and can still provide the throughput enhancement compared with the conventional JFDE and IR with Rx FDE only.

D. Impact of Channel Model

Here, the impact of the channel model on the throughput performance is shown.

Fig. 5(a) shows the 5%-outage throughput of the system as a function of the number of multipaths L. In the figure, the throughput performance increases as the number of multipaths L increases due to the larger frequency diversity gain. The figure also shows that the proposed joint cooperative-Tx/Rx FDE can enhance the throughput performance even under the small number of multipaths, e.g., $L = 2$ or 4. The 5%-outage throughput is plotted in Fig. 5(b) as a function of the normalized SNR, $\mathrm{SNR_{ref}}$ with $d = 1.5$. To achieve a certain target throughput, the required SNR is reduced by about 3 dB when the joint cooperative-Tx/Rx FDE is used compared with IR. Fig. 5(c) shows the 5% outage throughput as a function of the normalized distance between S and D, i.e., d, with $\mathrm{SNR_{ref}} = 3$ dB. In the figure, the throughput performance degrades as the normalized distance d becomes larger. When $d = 2.1$, the performance of the proposed joint cooperative-Tx/Rx FDE is almost the same as that of the conventional IR. The reason is explained as follows: Since the distance between S and R is large when d is large, it requires a large number of retransmissions for the correct decoding of the packet at R. Furthermore, the advantage of participation of S into the retransmission is negligible due to the large path loss. Hence, all the joint cooperative-Tx/Rx FDE approaches degenerate into JC-TNS, i.e., the selection

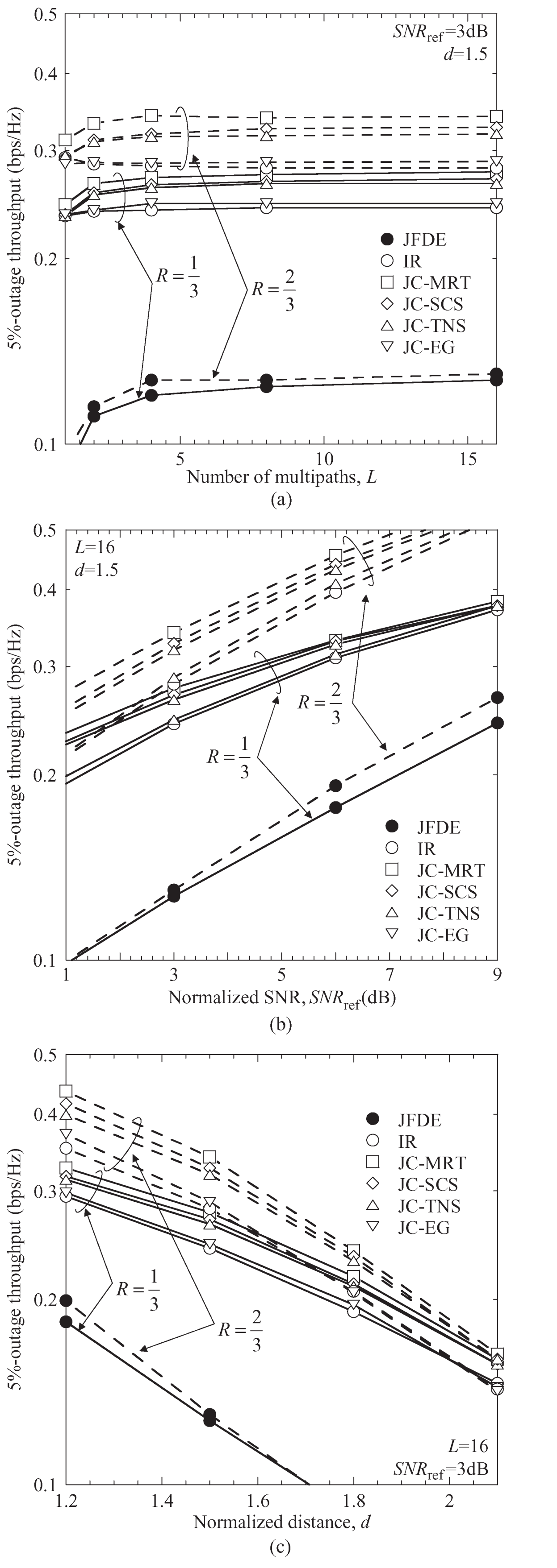

Fig. 5. Impact of the channel model. (a) Number of multipaths L. (b) Normalized SNR $\mathrm{SNR}_{\mathrm{ref}}$ (in decibels). (c) Normalized distance d.

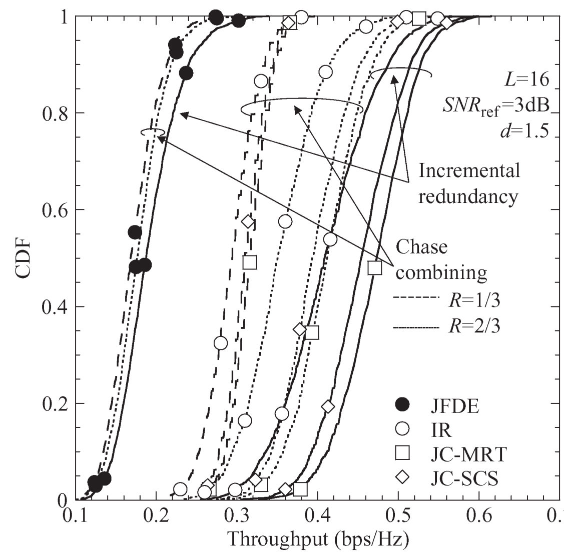

Fig. 6. Throughput performance with different HARQ strategies.

of the transmitting node, and almost always, R is selected for retransmission. Thus, the performance difference between the joint cooperative-Tx/Rx FDE and IRx becomes smaller as d becomes larger.

E. Chase Combining and Incremental Redundancy

Finally, the throughput performance of the proposed system with different HARQ strategies is shown in Fig. 6. As shown in the figure, the proposed joint cooperative-Tx/Rx FDE is also effective when incremental redundancy is used. By the use of the optimum joint cooperative-Tx/Rx FDE (suboptimum cooperative-Tx/Rx FDE), the 5% throughput can be increased by 23% (19%) compared with the conventional IR with Rx FDE only.

VI. Conclusion

In this paper, we proposed an SC incremental relay system using joint cooperative-Tx/Rx FDE. The packet retransmission is cooperatively carried out by S and R, and the packet combining is done by D using FDE. We first derived the optimum cooperative-Tx/Rx FDE weights, which jointly minimize the MSE after packet combining at D. We then proposed the suboptimum cooperative-Tx FDE weights to reduce the amount of CSI shared among all the nodes. We showed that the optimum cooperative-Tx FDE weights allocate the transmission power in MMSE sense across the subcarriers and in MRT sense across the transmitting nodes. The suboptimum selection-based methods, i.e., SCS and TNS, can reduce the amount of overhead to share CSI while providing almost the same performance as the optimum cooperative-Tx FDE weights.

Computer simulation showed that the required number of retransmissions for successful reception of the packet at D can be significantly reduced compared with the conventional methods. Although they are suboptimum, SCS and TNS can provide almost the same performance as the optimum weights.

APPENDIX
DERIVATION OF SET OF COOPERATIVE-TRANSMIT/RECEIVE FREQUENCY-DOMAIN EQUALIZATION WEIGHTS

Here, we derive the set of optimum cooperative-Tx/Rx FDE weights as follows: The *optimum* cooperative-Tx FDE weights are derived for *arbitrarily given* Rx FDE weights. Then, the *optimum* Rx FDE weights are derived based on the obtained *optimum* cooperative-Tx FDE weights.

Let us redefine the Lagrange function as

$$J = \sum_{k=0}^{N_c-1} \left| 1 - \sum_{q'=0}^{\tilde{q}} V_{\mathsf{D}}^{q'}(k) H_{\mathsf{S,D}}^{q'}(k) U_{\mathsf{S}}^{q'} - \sum_{q'=\tilde{q}+1}^{q} V_{\mathsf{D}}^{q'}(k) \left\{ H_{\mathsf{S,D}}^{q'}(k) U_{\mathsf{S}}^{q'}(k) + H_{\mathsf{R,D}}^{q'}(k) U_{\mathsf{R}}^{q'}(k) \right\} \right|^2 + 2\sigma^2 \sum_{k=0}^{N_c-1} \sum_{q'=0}^{q} \left| V_{\mathsf{D}}^{q'}(k) \right|^2 + \mu \left\{ \sum_{k=0}^{N_c-1} |U_{\mathsf{S}}^q(k)|^2 + \sum_{k=0}^{N_c-1} |U_{\mathsf{R}}^q(k)|^2 - 2P \right\}. \quad (44)$$

Since the set of the optimum cooperative-Tx FDE weights should satisfy

$$\frac{\partial J}{\partial U_{\mathsf{S}}^q(k)} = 0 \quad \text{and} \quad \frac{\partial J}{\partial U_{\mathsf{R}}^q(k)} = 0 \qquad \forall k \quad (45)$$

we have the following optimum cooperative-Tx FDE weights at node m ($m \in \{\mathsf{S}, \mathsf{R}\}$):

$$U_m^q(k) = \frac{\left\{ V_{\mathsf{D}}^q(k) H_{m,\mathsf{D}}^q(k) \right\}^*}{|V_{\mathsf{D}}^q(k)|^2 \left\{ \left| H_{\mathsf{S,D}}^q(k) \right|^2 + \left| H_{\mathsf{R,D}}^q(k) \right|^2 \right\} + \mu} \times \Phi^q(k) \quad (46)$$

with

$$\Phi^q(k) \triangleq 1 - \sum_{q'=0}^{\tilde{q}} V_{\mathsf{D}}^{q'}(k) H_{\mathsf{S,D}}^{q'}(k) - \sum_{q'=\tilde{q}+1}^{q-1} V_{\mathsf{D}}^{q'}(k) \left\{ H_{\mathsf{S,D}}^{q'}(k) U_{\mathsf{S}}^{q'}(k) + H_{\mathsf{R,D}}^{q'}(k) U_{\mathsf{R}}^{q'}(k) \right\}. \quad (47)$$

By substituting (46) into (44), the Lagrange function is rewritten as

$$J = \sum_{k=0}^{N_c-1} \frac{\mu \times |\Phi^q(k)|^2}{|V_{\mathsf{D}}^q(k)|^2 \left\{ \left| H_{\mathsf{S,D}}^q(k) \right|^2 + \left| H_{\mathsf{R,D}}^q(k) \right|^2 \right\} + \mu} + 2\sigma^2 \sum_{k=0}^{N_c-1} \sum_{q'=0}^{q} \left| V_{\mathsf{D}}^{q'}(k) \right|^2 - \mu \times 2P. \quad (48)$$

The optimum Rx FDE weights should satisfy

$$\frac{\partial J}{\partial V_{\mathsf{D}}^{q'}(k)} = 0 \qquad \text{for} \quad 0 \le q' < q \quad \text{and} \quad \forall k \quad (49a)$$

$$\frac{\partial J}{\partial |V_{\mathsf{D}}^q(k)|^2} = 0. \quad (49b)$$

From (49a), we have

$$V_{\mathsf{D}}^{q'}(k) = \begin{cases} \dfrac{\left\{ H_{\mathsf{S,D}}^{q'}(k) U_{\mathsf{S}}^{q'}(k) \right\}^*}{\Omega^q(k) + \frac{2\sigma^2}{\mu} \left\{ \left| V_{\mathsf{D}}^q(k) \right|^2 \left(\left| H_{\mathsf{S,D}}^q(k) \right|^2 + \left| H_{\mathsf{R,D}}^q(k) \right|^2 \right) \right\}} \\ \quad \text{for } 0 \le q' \le \tilde{q} \\ \dfrac{\left\{ H_{\mathsf{S,D}}^{q'}(k) U_{\mathsf{S}}^{q'}(k) + H_{\mathsf{R,D}}^{q'}(k) U_{\mathsf{R}}^{q'}(k) \right\}^*}{\Omega^q(k) + \frac{2\sigma^2}{\mu} \left\{ \left| V_{\mathsf{D}}^q(k) \right|^2 \left(\left| H_{\mathsf{S,D}}^q(k) \right|^2 + \left| H_{\mathsf{R,D}}^q(k) \right|^2 \right) \right\}} \\ \quad \text{for } \tilde{q} < q' < q. \end{cases} \quad (50)$$

We can further modify the Lagrange function by substituting the obtained $V_{\mathsf{D}}^{q'}(k)$ into (48) as

$$J = \sum_{k=0}^{N_c-1} \frac{2\sigma^2}{\Omega^q(k) + \frac{2\sigma^2}{\mu} \left\{ |V_{\mathsf{D}}^q(k)|^2 \left\{ \left| H_{\mathsf{S,D}}^q(k) \right|^2 + \left| H_{\mathsf{R,D}}^q(k) \right|^2 \right\} \right\}} + 2\sigma^2 \times \sum_{k=0}^{N_c-1} |V_{\mathsf{D}}^q(k)|^2 - \mu \times 2P. \quad (51)$$

By solving KKT conditions, we obtain

$$|V_{\mathsf{D}}^q(k)|^2 = \max \left[\sqrt{\frac{\mu}{2\sigma^2} \frac{1}{\left| H_{\mathsf{S,D}}^q(k) \right|^2 + \left| H_{\mathsf{R,D}}^q(k) \right|^2}} - \frac{\mu}{2\sigma^2} \frac{2\Omega^q(k)}{\left| H_{\mathsf{S,D}}^q(k) \right|^2 + \left| H_{\mathsf{R,D}}^q(k) \right|^2}, 0 \right]. \quad (52)$$

From (47) and (50), we have

$$\Phi^q(k) = \frac{\frac{2\sigma^2}{\mu} \times \left\{ |V_{\mathsf{D}}^q(k)|^2 \left\{ \left| H_{\mathsf{S,D}}^q(k) \right|^2 + \left| H_{\mathsf{R,D}}^q(k) \right|^2 \right\} + \mu \right\}}{2\Omega^q(k) + \frac{2\sigma^2}{\mu} |V_{\mathsf{D}}^q(k)|^2 \left\{ \left| H_{\mathsf{S,D}}^q(k) \right|^2 + \left| H_{\mathsf{R,D}}^q(k) \right|^2 \right\}}. \quad (53)$$

Then, (46) becomes

$$U_m^q(k) = \frac{\frac{2\sigma^2}{\mu} \times \left\{ V_{\mathsf{D}}^q(k) H_{m,\mathsf{D}}^q(k) \right\}^*}{2\Omega^q(k) + \frac{2\sigma^2}{\mu} |V_{\mathsf{D}}^q(k)|^2 \left\{ \left| H_{\mathsf{S,D}}^q(k) \right|^2 + \left| H_{\mathsf{R,D}}^q(k) \right|^2 \right\}}. \quad (54)$$

By defining $P_m^q(k) \triangleq (|U_m^q(k)|^2/2)$ ($m \in \{\mathsf{S}, \mathsf{R}\}$) and substituting (52) into (54), we have

$$P_{\mathsf{S}}^q(k) + P_{\mathsf{R}}^q(k) = \max \left[\frac{1}{\sqrt{\mu'}} \sqrt{\frac{\sigma^2}{\left| H_{\mathsf{S,D}}^q(k) \right|^2 + \left| H_{\mathsf{R,D}}^q(k) \right|^2}} - \frac{\Omega^q(k)}{\left| H_{\mathsf{S,D}}^q(k) \right|^2 + \left| H_{\mathsf{R,D}}^q(k) \right|^2}, 0 \right] \quad (55)$$

with $\mu' = 2\mu$. Equation (55) is exactly the same as (32).

Furthermore, the following relationship is derived from (52) and (55):

$$\left|V_{\mathsf{D}}^{q}(k)\right|^2 = \frac{\mu}{2\sigma^2}\left\{\left|U_{\mathsf{S}}^{q}(k)\right|^2 + \left|U_{\mathsf{R}}^{q}(k)\right|^2\right\}. \tag{56}$$

Then, we can rewrite the second term of the denominator in (54) as

$$\begin{aligned}
&\frac{2\sigma^2}{\mu}\left|V_{\mathsf{D}}^{q}(k)\right|^2\left\{\left|H_{\mathsf{S,D}}^{q}(k)\right|^2 + \left|H_{\mathsf{R,D}}^{q}(k)\right|^2\right\}\\
&\quad\overset{(a)}{=}\left\{\left|U_{\mathsf{S}}^{q}(k)\right|^2 + \left|U_{\mathsf{R}}^{q}(k)\right|^2\right\}\times\left\{\left|H_{\mathsf{S,D}}^{q}(k)\right|^2 + \left|H_{\mathsf{R,D}}^{q}(k)\right|^2\right\}\\
&\quad\overset{(b)}{=}\left|U_{\mathsf{S}}^{q}(k)H_{\mathsf{S,D}}^{q}(k) + U_{\mathsf{R}}^{q}(k)H_{\mathsf{R,D}}^{q}(k)\right|^2
\end{aligned} \tag{57}$$

where (a) is from (56), and (b) is due to the fact that (54) satisfies the equality condition for Cauchy–Schwarz inequality.

Hence, by plugging (54) with (57) into (50) and (56), the Rx FDE weights become

$$\begin{aligned}
V^{q'}(k) &= \frac{\left\{H_{\mathsf{S,D}}^{q'}(k)U_{\mathsf{S}}^{q'}(k) + H_{\mathsf{R,D}}^{q'}(k)U_{\mathsf{R}}^{q'}(k)\right\}^*}{\Omega^q(k) + \left|H_{\mathsf{S,D}}^{q}(k)U_{\mathsf{S}}^{q}(k) + H_{\mathsf{R,D}}^{q}(k)U_{\mathsf{R}}^{q}(k)\right|^2}\\
&\qquad\qquad \text{for}\quad 0 \le q' < q\\
\left|V_{\mathsf{D}}^{q}(k)\right| &= \sqrt{\frac{\mu}{2\sigma^2}\left\{\left|U_{\mathsf{S}}^{q}(k)\right|^2 + \left|U_{\mathsf{R}}^{q}(k)\right|^2\right\}}\\
&= \frac{\left|H_{\mathsf{S,D}}^{q}(k)U_{\mathsf{S}}^{q}(k) + H_{\mathsf{R,D}}^{q}(k)U_{\mathsf{R}}^{q}(k)\right|}{\Omega^q(k) + \left|H_{\mathsf{S,D}}^{q}(k)U_{\mathsf{S}}^{q}(k) + H_{\mathsf{R,D}}^{q}(k)U_{\mathsf{R}}^{q}(k)\right|^2}.
\end{aligned} \tag{58}$$

These optimum Rx FDE weights are exactly the same as (24). Hence, we have proved that the direction of derivation of the optimum cooperative-Tx/Rx FDE weights does not affect the results.

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