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Adaptive system identification using robust LMS/F algorithm

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ABSTRACT

Adaptive system identification (ASI) problems have attracted both academic and industrial attentions for a long time. As one of the classical approaches for ASI, performance of least mean square (LMS) is unstable in low signal-to-noise ratio (SNR) region. On the contrary, least mean fourth (LMF) algorithm is difficult to implement in practical system because of its high computational complexity in high SNR region, and hence it is usually neglected by researchers. In this paper, we propose an effective approach to identify unknown system adaptively by using combined LMS and LMF algorithms in different SNR regions. Experiment-based parameter selection is established to optimize the performance as well as to keep the low computational complexity. Copyright © 2013 John Wiley & Sons, Ltd.

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KEY WORDS: LMS; LMF; LMS/F; adaptive system identification (ASI)

1. INTRODUCTION

Adaptive system identification (ASI) includes many applications such as interference cancelation [1–3], spectral subtraction [4, 5], wireless localization [6–9], channel equalization [10–13], and adaptive beamforming [14]. One of most popular algorithms is least mean square (LMS), which is proposed by Widrow *et al*. [15]. Its popularity comes from the fact that it is very simple to implement. As a consequence, the LMS algorithm was widely used in many applications [16–18]. Many researchers have proposed different methods for improving steady-state performance of LMS without dramatically increasing the computational complexity. However, LMS cannot achieve good steady-state performance in low signal-to-noise ratio (SNR) region, for example, $SNR < 5$ dB.

In LMS family, the least mean fourth (LMF) algorithm was first developed by Walach and Widrow [19]. The algorithm applied the fourth-order power optimization criterion instead of the square power used for LMS. This idea came from the fact that higher-order power filters can mitigate noise interference effectively, especial in low SNR region [20]. However, the computational complexity of LMF is very high, which is caused by higher-order power optimization in its updating equation.

To fully take the advantages of both LMS and LMF, a combined LMS/F algorithm has been proposed by Lim and Harris [21] as a method to improve the performance of LMS adaptive filter without sacrificing its simplicity and stability. However, the proposed method only considered its updating equation, and it neglected specific applications in different SNR regions. In this paper, we introduce the combined LMS/F algorithm to ASI by considering the tradeoff between convergence speed and steady-state performance. The cost function of LMS/F is constructed for adaptive filter updating. In

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different SNR regions, experiment-based parameter selection is established to optimize the performance while keeping its computational complexity low.

In Section 2, both standard LMS and LMF algorithms are introduced. In Section 3, the combined LMS/F algorithm is presented, and its cost function is constructed. In Section 4, we propose the parameter selection method by using computer simulations. Concluding remarks are presented in Section 5.

2. LMS AND LMF ALGORITHMS

Assume an unknown system as shown in Figure 1, input signal is $x(n)$ at time n and coefficients-vector of an N-length finite impulse response (FIR) filter is $\mathbf{w}=[w_1,w_2,\ldots,w_N]^{\mathrm{T}}$, then output signal $y(n)$ is given by

$$y(n) = \mathbf{w}^T\mathbf{x}(n) + z(n), \tag{1}$$

where $\mathbf{x}(n)=[x(n), x(n-1),\ldots, x(n-N+1)]^{\mathrm{T}}$ is input signal vector and $z(n)$ is the observed noise, which is assumed to be independent with $\mathbf{x}(n)$. The objective of LMS-type filters is to identify the unknown coefficients-vector adaptively by using the input signal $\mathbf{x}(n)$ and the output $y(n)$ in an iterative way. Let $\mathbf{w}(n)$ be the estimated coefficients-vector at nth iteration, then instantaneous estimation error is defined as $e(n)=y(n)-\mathbf{w}^{\mathrm{T}}(n)\mathbf{x}(n)$.

In the standard LMS algorithm [11], its cost function $L_1(n)$ was constructed by

$$L_1(n) = \frac{1}{2}e^2(n). \tag{2}$$

The corresponding updating equation of LMS is given by

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu_1\frac{\partial L_1(n)}{\partial \mathbf{w}(n)} = \mathbf{w}(n) + \mu_1 e(n)\mathbf{x}(n), \tag{3}$$

where μ_1 is the step size, which determines the stability and the speed of convergence. The necessary condition of reliable LMS-based ASI is $0<\mu_1<2/\gamma_{\max}$, where parameter $\gamma_{\max}$ is the maximum eigenvalue of the covariance matrix $R=E\{x(n)x^{\mathrm{T}}(n)\}$ of $\mathbf{x}(n)$.

In the standard LMF algorithm, its cost function $L_2(n)$ is written as

$$L_2(n) = \frac{1}{2}e^4(n). \tag{4}$$

The filter coefficients-vector $\mathbf{w}(n)$ is then updated by

$$w(n+1) = w(n) - \mu_2\frac{\partial L_2(n)}{\partial w_2(n)} = w(n) + \mu_2 e^3(n)x(n), \tag{5}$$

where μ_2 is the step size, which determines the stability and the speed of convergence. The necessary condition of reliable LMF-based ASI is $0<\mu_2<1/(N\gamma_{\max})$ [15]. Let $\mathbf{v}(n)$ denote the weight error vector, which is defined as

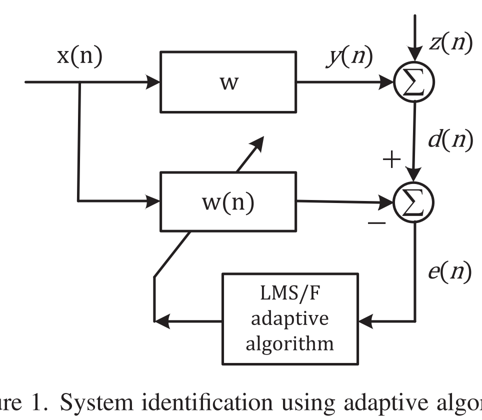

Figure 1. System identification using adaptive algorithm.

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$$\mathbf{v}(n) = \mathbf{w}(n) - \mathbf{w}_{\mathrm{opt}}, \tag{6}$$

where $\mathbf{w}_{\mathrm{opt}}$ is the optimal weight vector, which minimizes the mean square error and is given by

$$\mathbf{w}_{\mathrm{opt}} = \mathbf{R}^{-1}\mathbf{p}, \tag{7}$$

where $\mathbf{p} = E\{\mathbf{x}(n)y(n)\}$ is expectation of the cross-correlation between the input vector $\mathbf{x}(n)$ and the output vector $y(n)$. It is assumed that the weight error vector $\mathbf{v}(n)$ and the input vector $\mathbf{x}(n)$ are statistically independent. The optimal mean square estimation (MSE) error $e_{\min}$ is given by

$$e_{\min} = y(n) - \mathbf{w}_{\mathrm{opt}}^T\mathbf{x}(n). \tag{8}$$

Combining (3), (5), and (6), we can obtain

$$e(n) = e_{\min} - \mathbf{v}^T(n)\mathbf{x}(n). \tag{9}$$

Hence, under the independence assumption, the steady-state excess MSE is given by

$$P_{\mathrm{ex}}(\infty) = \lim_{n\to\infty} E[e^2(n)] = \frac{\mathrm{Tr}\left(\mathbf{R}(\mathbf{I} - \mu\mathbf{R})^{-1}\right)}{2 - \mathrm{Tr}\left(\mathbf{R}(\mathbf{I} - \mu\mathbf{R})^{-1}\right)} E\left[\mathbf{v}^2(n)\right]. \tag{10}$$

3. COMBINED LMS/F ALGORITHM

According to the cost functions of LMS in Equation (2) and LMF in Equation (4), the cost function of standard LMS/F can be constructed as

$$L_3(n) = \frac{1}{2}e^2(n) - \frac{1}{2}\varepsilon\ln\left(e^2(n) + \varepsilon\right), \tag{11}$$

where the threshold parameter $\varepsilon > 0$ is used to trade off between convergence speed and performance. Then the update equation of LMS/F can be given by

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_3\frac{\partial L_3(n)}{\partial \mathbf{w}(n)} = \mathbf{w}(n) + \mu_3\frac{e^3(n)}{e^2(n) + \varepsilon}\mathbf{x}(n), \tag{12}$$

where the gradient step-size μ_3 is used to control the convergence and performance. However, in practical FIR filter system in different SNR regions, selecting a reasonable parameter ε for LMS/F is very important, because the parameter ε can well trade off steady-state performance and convergence speed. In this paper, by virtue of computer simulations discussed in the next section, we recommend a parameter selection method in different SNR regions. In addition, we can also find that the adaptive updating equation in Equation (12), when $\varepsilon \gg e^2(n)$, behaves like the standard LMF algorithm with a step size of μ_3/ε; when $\varepsilon \ll e^2(n)$, it reduces to the standard LMS algorithm with a step size of μ_3.

4. EXPERIMENTAL RESULTS

In this section, the proposed method is compared with LMS-based and LMF-based filters. The performance is evaluated using the mean square deviation (MSD) standard, which is defined

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$$MSE\{w(n)\} = E\left\{\|w(n) - w\|_2^2\right\}, \tag{13}$$

where $\mathbf{w}(n)$ denotes nth adaptive updating filter coefficients-vector. Experiments have been carried out to compare their convergence speed and steady-state performance with different SNRs, that is, $SNR=$ 5–25 dB. The input signal and the observed noise are white Gaussian random sequences with variance of 1 and 0.001, respectively. One thousand Monte Carlo runs are adopted for average. Assume that the three filters have the same length, that is, $N=16$. The gradient descend step size of LMS is set as $\mu_1=0.05$, which is recommend by El-Mahdy [18]. However, two step-size parameters of LMF and LMS/F have not presented specific N-length filters, for example, $N=16$. Consider the two SNR region, that is, $SNR=5$ dB and 15 dB; in computer simulations as shown in Figures 2 and 3, step-size parameters are suggested as $\mu_3=0.002$ to achieve good steady-state performance and to keep fast convergence speed. We can find that as the step size becomes smaller, better MSD performance can be achieved. Unfortunately, the convergence speed becomes slower at the same time. Hence, we

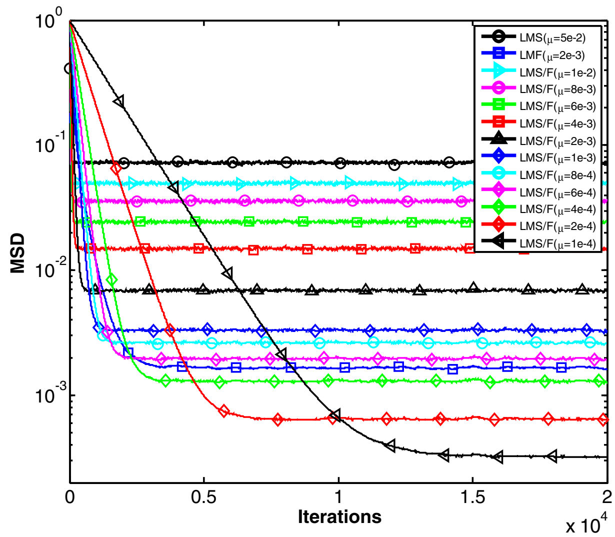

Figure 2. Performance evaluation versus step-sizes μ ($SNR=5$ dB).

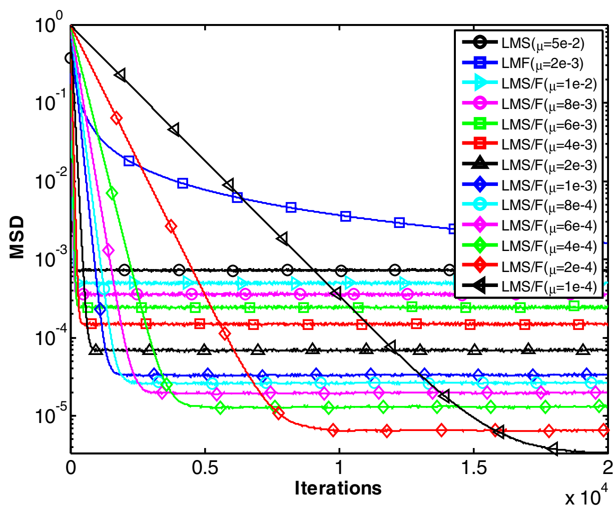

Figure 3. Performance evaluation versus step-sizes μ ($SNR=15$ dB).

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recommend that the step size of LMS/F should be selected in the range [0.01, 0.001]. Similarly, a smaller step size of LMF can also obtain better performance but at the cost of low convergence speed.

To trade off convergence speed and performance, we also select a step-size parameter of LMF as $\mu_2 = 0.002$ in Figures 4–7, where *SNR* changes from 5 to 25 dB. As shown in Figure 4, when the SNR is low, that is, $SNR = 5$ dB, both LMS and LMS/F algorithms yield faster convergence than LMF. However, LMF can achieve better steady-state identification performance than LMS and LMS/F with a cost of slow convergence (over 3000 iterations). In practical system, it is necessary to balance steady-state performance and convergence speed. In low SNR region, we recommend that the parameter threshold ε be chosen in the range $\varepsilon = [0.3,1)$, because the steady-state performance of LMS/F is much better than that of LMS when its iteration times are less than 1000. In the medium SNR region, for example, $SNR = 10$ and 15 dB, with simulation results as shown in Figures 5 and 6, parameter ε is recommended to be selected in the range $\varepsilon = (0,0.3]$; the LMS/F algorithm can keep a fast convergence speed, and its steady-state performance is better than that in the LMS algorithm. In high SNR region, for example, $SNR = 25$ dB as shown in Figure 7, both

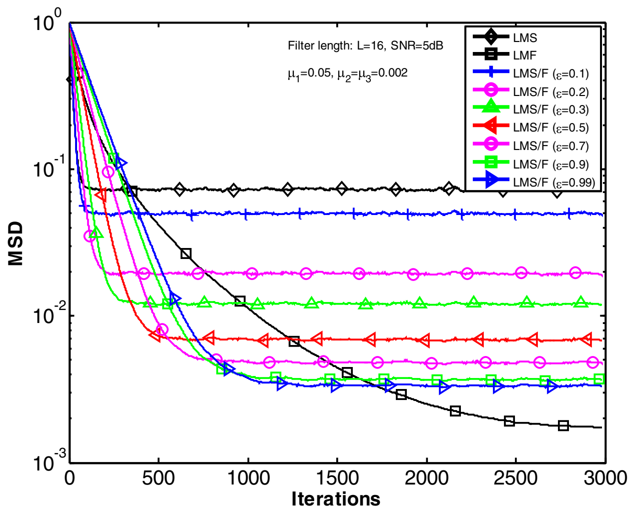

Figure 4. Performance evaluation versus parameter threshold ε ($SNR = 5$ dB).

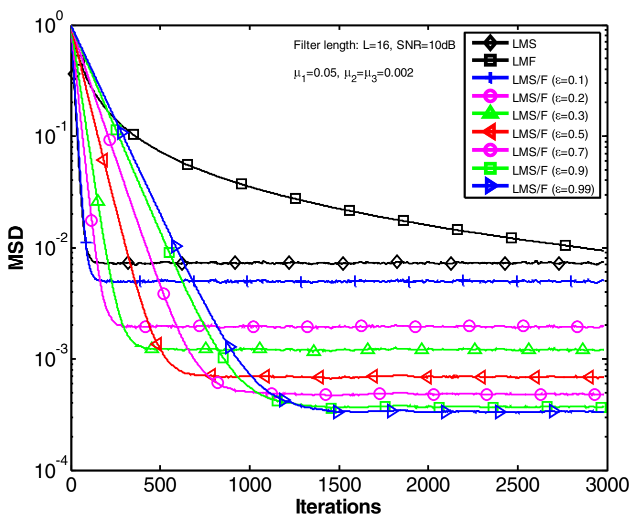

Figure 5. Performance evaluation versus parameter threshold ε ($SNR = 10$ dB).

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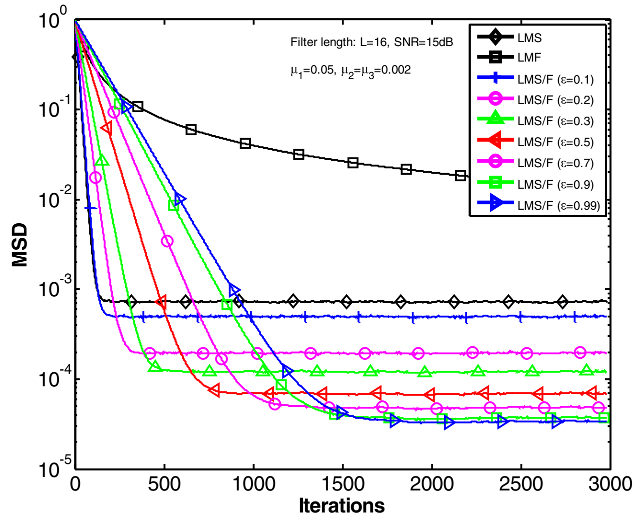

Figure 6. Performance evaluation versus parameter threshold ε ($SNR = 15$ dB).

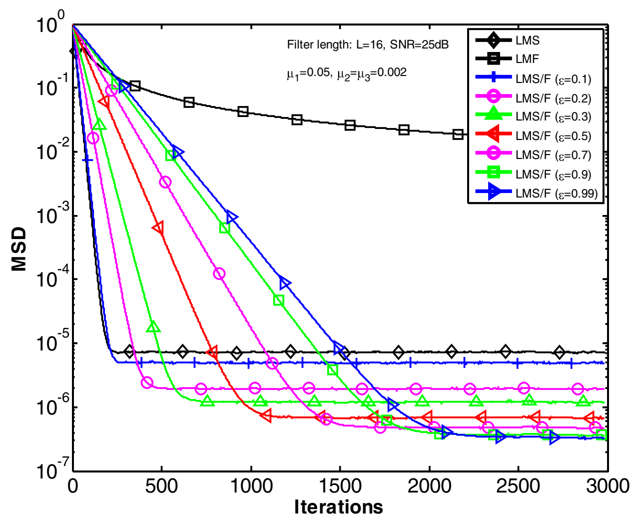

Figure 7. Performance evaluation versus parameter threshold ε ($SNR = 25$ dB).

LMS and LMS/F can achieve good steady-state performance (MSD is less than 10^{-5}). Although LMS/F can achieve better performance than LMS, it is not recommended in practical ASI when low calculation complexity is required.

5. CONCLUSION

In this paper, we have investigated LMS/F algorithm with its application to ASI. Experiment-based parameter selection method was proposed for both step size of gradient descend and threshold of LMS/F algorithm. According to computer simulations, we found that LMS/F can achieve much better steady-state performance than LMS in low SNR region, and its convergence speed is faster than LMF. In high SNR region, LMS/F can also achieve better performance than LMS; however, it is not recommended in practical ASI when low convergence speed is considered. In one work, the LMS/F algorithm is a good choice for ASI in low SNR region.

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