Performance Analysis of Fountain Codes in Multihop Relay Networks

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***Abstract*—Fountain codes have been extensively employed in *delay-tolerant networks (DTNs)* due to their near-capacity performance with very low encoding/decoding complexity. A decode-and-forward-based relaying strategy is ideally suited for fountain codes in such networks due to its ability to recover the source message from any subset of encoded packets with sufficient mutual information. However, the unreliable nature of the channel may lead to the starvation of some subsequent nodes with good channel conditions. By cooperation among the forwarding nodes, the overall latency of such networks can be alleviated. This paper analytically quantifies the latency of both *cooperative* and *conventional* fountain-coded delay-tolerant multihop networks by deriving the exact closed-form equations for the channel usage. The overall latency suffered by such networks forces conservation of the end-to-end delay, particularly for real-time applications. However, by constraining the total delay (the number of encoded transmissions), the performance of fountain codes deteriorates due to the lack of encoded packets for retrieving the entire source message. This degradation can be gauged by the average packet loss experienced with partial decoding of fountain codes. The exact closed-form equation for the average packet loss based on the channel usage for such *delay-constrained networks (DCNs)* is derived in this paper. The tradeoff between average delay and the channel usage required for successful decoding is also analyzed. It is observed that the average packet loss can be minimized by optimizing the total delay based on the performance across each link. Finally, the pros and cons of using *DCNs* and *DTNs* employing fountain codes are evaluated, and theoretical grounding to the simulated results is provided.**

***Index Terms*—Conventional multihop networks, cooperative multihop networks, delay-constrained transmission, fountain/rateless codes.**

I. Introduction

IN WIRELESS networks, research is mainly focused on efforts to improve power efficiency with better quality of service (QoS). In such networks, the resource at a premium is the available energy, owing to the limited capacity of batteries powering the wireless nodes. The imposed power constraints limit the overall transmission range of the nodes in the network. This restricts the permissible distance between the source and destination nodes for successful direct transmission. As the distance increases, the energy expenditure and transmission delay also increase. Efficient utilization of transmitted signal energy and usage of shorter transmission distances result in lower power levels throughout the system [1]–[3].

Such constraints on link budget make multihop transmission a potential technique for reliably delivering the data over large distances. Along with much less path loss, multihop transmission provides increased channel capacity, decreased interference levels, reduced terminal radiation, extended battery life, and improved connectivity [4], [5]. This favors the use of such multihop transmission schemes, which allows distant nodes to communicate with each other more efficiently. In such networks, messages are relayed over sequential point-to-point links. Routing protocols determine the intermediate nodes that are selected as the forwarding nodes in such multihop networks. The broadcast nature of the wireless channel provides multiple opportunities to the intermediate nodes in the route to reliably receive the packets transmitted by a source node. It then forwards the received information toward the final destination. The number of hops and the route has a significant impact on energy efficiency and latency performances. Long-hop routes demand substantial transmission power but minimize the cost for reception, computation, etc. Meanwhile, minimizing the number of hops reduces the end-to-end delay. Several advantages for long-hop routing have been proposed by Haenggi in [6] and Haenggi and Puccinelli in [7]. It has been found to be a competitive strategy in every aspect, particularly high energy efficiency, in many networks. The optimal one-hop length is analyzed in [6], where the objective is to minimize the total energy consumption by the network. Thus, choosing an efficient routing scheme is a multiobjective optimization problem. The intermediate relays and the number of hops or the route are determined based on the requirements of the system.

In traditional multihop networks, each intermediate node in the network solely relies on the information transmitted by its immediate predecessor node in the previous hop, and the destination simply listens to the last node in the route. In this paper, such relaying is referred to as *conventional multihop transmission*, as is known from ad hoc networking systems. By exploiting the nonlinearity of attenuation as a function of distance, conventional transmission benefits from a

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reduction in end-to-end path loss. A further reduction in energy consumption can be obtained by taking advantage of cooperative relaying, which exploits the spatial diversity offered by multihop transmission. Such diversity is referred to as *multihop diversity*. In such cases, diversity is achieved by receiving signals that have been transmitted by multiple previous relay nodes along the route [8]. Such relaying is referred to as *cooperative multihop transmission*. In cooperative multihop transmission, the number of cooperating nodes around each node should be optimized based on the permissible energy consumption of the system.

In multihop transmission, the overheard information at the intermediate nodes is processed based on the transmission technique employed. Of the forwarding strategies employed at the intermediate nodes, decode-and-forward (DF)-based relaying mitigates the effect of noise amplification and, thereby, offers an improved performance. Fountain codes, which are also known as rateless codes, introduced in [9], are ideally suited to DF-based relaying, where the intermediate nodes monitor the transmitting node virtually indefinitely until it has enough mutual information to decode. In such cases, outage is virtually never experienced as the transmissions could be indefinitely long. Rateless codes are erasure coding schemes where the source, oblivious of the channel state information (CSI), generates as many encoding symbols as needed for correct decoding at the destination node (accumulates enough mutual information for proper decoding) [10]. However, rateless codes have been shown to offer good performance in other channels such as binary symmetric channel, additive white Gaussian noise (AWGN), and fading channels [11]–[13]. In addition, very low encoding/decoding complexity favors the use of rateless codes. These aspects encourage the extension of the concept of mutual information accumulation using rateless codes into multihop networks and to analyze its impacts.

Rateless coding is particularly advantageous for a network with multiple nodes, as the source data can be reconstructed from any subset of encoded packets with sufficient mutual information. The relays along the route accumulate the mutual information, and successful relays in such schemes assist the source in transmission. The performance of such schemes has been analyzed in [10] and [14]. In [15], a throughput optimal rateless coding scheme is proposed to relay Luby Transform codes across multiple nodes. Most of the recent works on rateless codes are focused on delay-tolerant networks (DTNs), which are designed for efficient transmission but can tolerate long latency [16]–[18]. In such systems, each of the intermediate nodes acts as an independent fountain encoder/decoder, where it decodes the entire source message before transmitting the reencoded information. This process continues until the message reaches the destination node after several hops. In spite of the capability to improve system capacity and wireless coverage in fading environments, DTNs suffer from high latency. This is due to the delay encountered during the total recovery and reencoding of information at the intermediate nodes. In addition, the performance of DTNs depends on the intermediate link quality, and the worst link becomes the bottleneck of the entire system as it affects the reliability of information delivered across multiple hops.

Due to the explosive amount of delay-sensitive traffic in future wireless applications, mitigating latency becomes an important issue for multihop networks [19]–[21]. Further, waiting for the entire message to be decoded can introduce long delays in rateless coded networks. This motivates the analysis of delay-constrained multihop transmission employing rateless codes. In such delay-constrained networks (DCNs), the maximum duration for total transmission is fixed. This, in turn, causes the number of encoded packets that are transmitted per hop to be fixed. In such cases, *partial* or *total* recovery of the source packets can be performed at the intermediate nodes depending on the channel conditions [22]. In partial decoding, the decoder attempts to retrieve the maximum source packets with insufficient number of encoded packets. Those packets that are correctly received within the interval are transmitted in the subsequent hops to enable real-time communication over multihop networks. The number of packets lost will be a function of elapsed time, which, in turn, is a factor of the QoS requirement of the system.

The main contribution of this paper is to provide insight on the throughput performance with delay constraints on multihop systems employing rateless codes. Such an analysis, which compares DTNs and DCNs employing rateless codes, is the first of its kind to the best of the authors' knowledge. This paper further provides analytical formulations to compute the required number of transmissions, which, in turn, reflects the delay incurred in DTNs. As opposed to DTNs, DCNs have strict delay constraints, which cause some packets to be dropped across the hops. The number of packets that are reliably delivered per hop in DCNs is analyzed in this paper. Theoretical formulations are done for conventional and cooperative multihop scenarios. This paper highlights the tradeoffs existing between throughput and delay for DTNs and DCNs.

The organization of this paper is as follows. The system model used for analyzing DTNs and DCNs is discussed in Section II and the concept of data recovery using rateless codes is also described. The theoretical performance evaluations of both DTN and DCN transmission schemes are done in Section III. Details of the simulation studies and results are discussed in Section IV. Finally, concluding remarks and future extensions of this work discussed in Section V wrap up this paper.

II. Analysis Model

In this paper, a wireless multihop network, as shown in Fig. 1, is considered, where source $\mathcal{S}$, which generates a continuous stream of information bits, communicates with destination $\mathcal{D}$ by hopping through the $w \times n$ intermediate relays ($\mathcal{R}_{ij}$). All nodes are equipped with a single omnidirectional antenna and are constrained by half-duplex transmission.

A. Received Signal and Channel Model

The continuous stream of information bits ($\mathbf{u}$) generated by $\mathcal{S}$ is first grouped into blocks of size $p_k \times b$, where p_k is the number of packets per encoding set, and b is the bit size of each data packet. At any instant t, the information block fed to the rateless encoder can be represented as $\mathbf{u}(t) = [u_1 \; u_2 \cdots u_{p_k}]^T$.

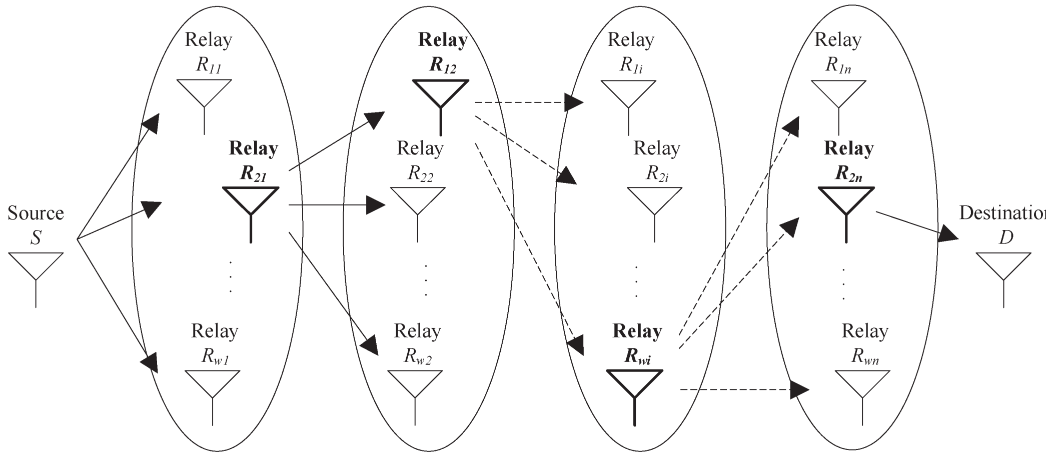

Fig. 1. System configuration with a source node ($\mathcal{S}$), a destination node ($\mathcal{D}$), and $w \times n$ relay nodes ($\mathcal{R}_{ij}$'s) with only one relay node forwarding per hop.

The encoder continuously generates encoded packets c_i's of size $1 \times b$ until either sufficient mutual information is received at the decoder for proper decoding or a maximum predetermined number of coded packets (L) are transmitted (for delay-constrained transmission). The channel between any pair of nodes is assumed to be frequency flat, where the channel gain remains quasi-static for a fading block of one coded packet (i.e., up to b bits) and independently varies across packets. The corresponding received signal over a quasi-static fading channel can be expressed as [23]

$$\mathbf{Y}_q = d_{pq}^{-\alpha/2} H_{pq} \mathbf{X}_p + \mathbf{N}_q \tag{1}$$

where $p \in \{\mathcal{S}, \mathcal{R}_{ij}\}$ and $q \in \{\mathcal{R}_{ij}, \mathcal{D}\}$, $\mathbf{X}_p$ is the transmitted encoded packet, α captures the path loss exponent, d_{pq} is the distance between transmitting node p and receiving node q (without loss of generality, shadowing is not considered in this paper), $H_{pq} \in \mathbb{C}^{1\times 1}$ is an element of channel transfer matrix H between nodes p and q, $\mathbf{N}_q \in \mathbb{C}^{1\times b}$ is the noise matrix that corresponds to AWGN with zero mean and variance N_0, and $\mathbf{Y}_q \in \mathbb{C}^{1\times b}$ is the received signal matrix. The entries of channel matrix H are assumed to be independent and identically Nakagami-m distributed. Each node is assumed to transmit under an equal power constraint, and hence, $E[|\mathbf{X}_p|^2] = \epsilon \forall p$, where ϵ is the average energy per symbol during transmission. Equalization is performed on the received signal in (1) using conventional channel equalization methods such as zero forcing by assuming the availability of CSI at the receiver. However, the transmitter does not know the CSI *a priori* and generates encoded packets that are bounded by the transmission technique employed.

For reliable transmission, a layered coding approach is considered, where error correction coding is applied to each rateless coded packet [24]. Assuming capacity achieving Gaussian codebooks[1] and in accordance with Shannon's theorem, the encoded packets transmitted through a channel can be decoded with vanishing error probability when the error correction code-rate R_c is less than the channel capacity C. The channel is considered to be in outage whenever this constraint is violated [26]. In an outage event, there is no guarantee that the transmitted encoded packet can be decoded without error, and such packets are considered erased. Hence, the quasi-static wireless channel can be treated as an erasure channel with erasure probability, i.e.,

$$P_e^{pq} = \Pr[C < R_c] = \Pr\left\{ |H_{pq}|^2 < \frac{2^{R_c} - 1}{\gamma} \right\} \tag{2}$$

where $\gamma = \epsilon/(d_{pq}^{\alpha} N_0)$. Hence, erasure probability P_e^{pq} across nodes p and q is simply the probability density function of $|H_{pq}|^2$; for Nakagami-m fading channels, this is given as [27]

$$P_e^{pq} = 1 - \sum_{k=0}^{m-1} \frac{1}{k!} (m\sigma)^k \exp(-m\sigma) \tag{3}$$

where $\sigma = (2^{R_c} - 1)/\gamma$. When $m = 1$, the Nakagami channel reduces to a Rayleigh fading channel. The wireless channel can hence be treated as an equivalent erasure channel, and the analysis for the rateless coded system can be performed using the same approach as in an erasure channel. For simplicity of exposition, binary phase-shift keying (BPSK) modulation with coherent detection is used. The stream of recovered bits is then fed to the decoder.

B. Background on Rateless and Random Linear Codes

Rateless codes are modern flexible FEC codes, which do not impose a fixed coding rate. An infinite number of encoded packets are generated through the rateless encoding process by linearly combining d randomly selected packets (chosen according to some degree distribution Ω) from the message block $\mathbf{u}$ [25, Ch. 50]. Each rateless code is completely defined by the degree distribution Ω, which is a discrete probability distribution over $[1, p_k]$. For simplicity of analysis, a class of efficient rateless codes, random linear codes, is considered in this paper. Each rateless coded packet is further encoded with error-correction code and modulated before sequentially broadcasting over the channel, i.e., the transmitter serves as a perpetual fountain.

[1]This assumption is not very limiting, as many capacity achieving codes such as turbo and low-density parity-check codes have been reported [25].

The receivers tune into the on-going broadcast transmission at any arbitrary time, as rateless codes have the unique ability to recover the source message from any subset of encoded packets with sufficient mutual information. The received unerased encoded packets are then fed to the decoder. Each received encoded packet generates a column of generator matrix G, and the entire source packets can be recovered when the received generator matrix is full rank, i.e., $Rank(G) = p_k$. The decoder determines this by attempting to invert G using Gaussian elimination. With delay-constrained transmissions, the received encoded packets may not be sufficient to recover all of the source packets (depending on the channel conditions), and the maximum number of source packets are retrieved by performing partial decoding [28]. The source packets that emerge from the decoding process are utilized for subsequent transmission.

C. Routing and Network Configuration

The layered multihop network in Fig. 1 is assumed to be stable and has a predefined route from $\mathcal{S}$ to $\mathcal{D}$ via n hops [29]. During transmission, the nodes will be organized into n hops and w nodes per hop, and only the best relay node per hop is selected for transmission. The best relay is selected with proper coordination among the nodes based on the forward link and on the quality of information at the relay (determined by the number of source packets recovered at the relay). As this paper investigates the impact of employing rateless codes in DTNs and DCNs, routing and medium access control for transmission is not considered within the scope of this paper. As transmissions are broadcast by nature, the rateless coded packets are received by all nodes within the transmission range of the transmitter. In this paper, two different types of multihop transmission schemes, namely, *conventional* and *cooperative* multihop transmission, are considered for analysis. More details on these transmission schemes are described in Section III.

For DTNs, the transmission is completed when all of the source packets are correctly decoded at $\mathcal{D}$. This approach is therefore very inefficient and time consuming as the poor quality links between nodes across hops introduce long delays. This starves the subsequent hops of good channel quality. Hence, in DCNs, the message is transmitted from $\mathcal{S}$ to $\mathcal{D}$ via n hops with a hard delay deadline. The number of encoded packets that are transmitted depends on the delay. To constrain the overall delay, the channel usage per hop (number of encoded packets transmitted) is fixed to a maximum value (L). The hard delay deadline causes some packets to be dropped/lost across the hop, and only the successfully recovered packets are transmitted in the subsequent hop. To recover the lost packets in such cases, each node can send a negative acknowledgement (NACK) indicating the packets lost during each hop. Such selective repeat approaches inform the transmitter of the lost packets. These packets are then fountain coded and transmitted to reconstruct all the original data packets at the receiver. In short, system reliability is guaranteed hop-by-hop by using selective repeat approaches but at the cost of spectral efficiency. The average packets lost for DCNs and the required number of transmissions per hop for DTNs are derived in the following section.

III. Performance Analysis

One of the main impediments to multihop transmission is the overall latency encountered to deliver the information to the destination. The overall delay encountered for rateless coded DTNs will be high as the packets need to be retrieved and reencoded at each hop until the information is reliably delivered to the destination. In wireless networks, the transmission environment (channel) is another constraining factor that affects the total delay. The total delay for such delay-tolerant systems characterized by the number of transmissions (encoded packets) required in delivering the information reliably to the destination is analyzed here.

Most real-time applications will demand not only a high bandwidth but also a predictable QoS, which cannot be guaranteed by the best-effort delivery networks, such as DTNs. In this context, rateless coded DCNs can be employed to have a finite latency with a specified QoS. With finite delay, these networks cannot guarantee an arbitrarily small error probability as the received mutual information (dependent on the channel characteristics) may not be sufficient to recover the entire source packets. Rateless coded DCNs are thereby characterized by the reliability of information delivered to the destination. This can be determined by the packets lost across the hops due to the finite delay. The performance of DCNs based on the average packet loss with finite number of transmissions is analytically studied here.

Analysis for DTNs and DCNs is performed for both conventional and cooperative multihop transmissions. For simplicity of analysis, a generic two-node network where the source directly communicates with the destination is considered for conventional transmission. For cooperative transmission, a three-node network where a relay cooperates with the source in transmissions is analyzed. Fountain coding is performed on p_k source packets by employing a uniform degree distribution, i.e., Ω. The present analysis is independent of the employed degree distribution, and therefore, the same technique can be extended to other (more practical) degree distributions, such as Robust Soliton distribution [9].

A. DTNs

In DTNs, the main objective is to deliver the source packets reliably over multiple hops without any constraint imposed on the system. In such networks, the forwarding node transmits fountain-coded packets until any of the nodes in the next hop correctly decodes. The details of the transmission and reception schemes for DTNs based on the system configuration in Fig. 1 are explained in Section II. The intermediate nodes receive the information from the node in the previous hop or a subset of the previous relaying nodes based on the transmission scheme employed. The latency of such a network can be computed by evaluating the channel usage per hop or, in other words, by computing the average number of transmissions $(\overline{N})$ required for successful decoding. $\overline{N}$ can be computed as

$$\mathbb{E}[N] = \sum_{n_i=p_k}^{\infty} n_i \Pr(N = n_i) \tag{4}$$

where N represents the number of transmissions required for the receiver to successfully decode, and $\Pr(N = n_i)$ is the corresponding probability mass function (PMF). The lower bound on the required number of transmissions is p_k, as at least p_k transmissions are required to decode all p_k source packets from as many encoded packets. The upper bound goes to infinity owing to the rateless property of the code, which stipulates the transmission to proceed until successful decoding at the receiver. For a rateless coded system, the probability of successfully recovering the p_k source packets is the probability of the $p_k \times i$ received generator matrix to be of rank p_k, where i is the number of received unerased encoded packets. From the cumulative mass function for successful decoding [30], the corresponding PMF for an equiprobable binary generator matrix to be of rank p_k after receiving i unerased packets can be expressed as

$$P_{\text{rank}}^{p_k}(i) = \begin{cases} 0, & \text{if } i < p_k \\ \prod_{j=0}^{p_k-1}(1-2^{j-i}), & \text{if } i = p_k \\ \frac{2^{-i}(2^{p_k}-1)}{1-2^{p_k-i}} \prod_{j=0}^{p_k-1}(1-2^{j-i+1}), & \text{if } i > p_k \end{cases}. \tag{5}$$

An analytical evaluation of DTNs for conventional and cooperative multihop transmissions is done by calculating the corresponding PMFs, as given below.

$\overline{N}$ for Conventional Multihop Transmission: With conventional multihop transmission, the encoded packets are available only to those nodes in the subsequent hop. In such DTNs, the transmission terminates when all the source packets are reliably decoded by the receiver. When transmissions are performed over a wireless erasure channel with erasure probability P_e^{pq} across nodes p and q, the number of encoded packets received unerased is random and is related to the number of transmissions, i.e., T_r. Consider exactly p_k packets are to be received in T_r transmissions, which implies the unerased reception of the final transmission with probability $(1 - P_e^{pq})$. Then, the probability of receiving the p_kth unerased encoded packet on the T_rth transmission can be computed as

$$P_{p_k}^{T_r}\left(P_e^{pq}\right) = (1-P_e^{pq}) \binom{T_r-1}{p_k-1} \left(P_e^{pq}\right)^{T_r-p_k} \left(1 - P_e^{pq}\right)^{(p_k-1)}. \tag{6}$$

Consider that node q decodes after T_r transmissions (the received generator matrix is full rank); the probability of receiving a full-rank generator matrix at node q after T_r transmissions can be computed using (5) and (6) as

$$\Pr(N = T_r) = \Pr_{p_k}\left(T_r, P_e^{pq}\right) = \sum_{i=p_k}^{T_r} P_{rank}^{p_k}(i) P_i^{T_r}\left(P_e^{pq}\right). \tag{7}$$

The summation is for the number of unerased packets received from T_r transmissions. The terms in (7) correspond to the probability of receiving a full-rank generator matrix from the unerased encoded packets received from corresponding source transmissions. The average number of transmissions $(\overline{N})$ required for conventional multihop DTNs can be computed by evaluating (4) using (7).

$\overline{N}$ for Cooperative Multihop Transmission: In cooperative multihop transmission, the receiver combines the information from all or a subset of previous nodes in the network. Cooperative fusion of information from a subset of nodes in the network improves the system performance [8]. To ensure the robustness of the present analysis, nodes are assumed to combine the information from only one cooperating relay node in the route. As the best relay from each hop transmits the information, the transmitting node is considered the source node, and the nodes in the next hop are considered the relay nodes. Although the present results can be extended to cases where nodes combine information from multiple nodes, the complexity of analysis exponentially increases with the number of cooperating nodes.

In cooperative transmission, the information broadcast by $\mathcal{S}$ is monitored by relay nodes and $\mathcal{D}$, as explained in Section II. As soon as either any of the relay nodes or $\mathcal{D}$ has accumulated sufficient mutual information, an ACK is transmitted to $\mathcal{S}$. If $\mathcal{R}$ decodes first, then $\mathcal{S}$ ceases transmission, and $\mathcal{R}$ transmits the fountain-coded packets ($\mathcal{R}$ switches from reception to transmission) until $\mathcal{D}$ is able to correctly decode. Thus, cooperation between nodes opportunistically selects the transmitter between $\mathcal{S}$ and $\mathcal{R}$ based on the relay message quality and the instantaneous channel. The probability that $\mathcal{R}$ has received p_k linearly independent encoded packets after T_r transmission (T_r basically captures the instant when the generator matrix at $\mathcal{R}$ has reached full rank) can be computed similar to (7) with $P_e^{pq} = P_e^{SR}$, where P_e^{SR} is the erasure probability across $\mathcal{S}$ and $\mathcal{R}$. Then, the probability that $\mathcal{R}$ enters into cooperation mode within T_r transmissions can be computed using (7) as

$$\Pr_{RCoop}\left(T_r, P_e^{SR}\right) = \sum_{i=p_k}^{T_r} \Pr_{p_k}\left(i, P_e^{SR}\right). \tag{8}$$

The PMF of the number of transmissions required to have a full-rank generator matrix at $\mathcal{D}$ depends on two cases. For the first case, $\mathcal{R}$ decodes before $\mathcal{D}$. In such cases, $\mathcal{R}$ transmits the encoded packets in the next slot (cooperation mode). For this case, consider the scenario where out of the total T_r transmissions, l packets are received unerased at $\mathcal{D}$. Of the l unerased encoded packets, $\mathcal{D}$ receives j packets from i transmissions by $\mathcal{S}$. The remaining $(l - j)$ packets are received at $\mathcal{D}$ from $(T_r - i)$ transmissions by $\mathcal{R}$, provided $\mathcal{R}$ has decoded p_k source packets from i transmissions by $\mathcal{S}$. By the total probability theorem over all disjoint sets that partition the sample space, the PMF of the number of transmissions required for cooperation can be evaluated as

$$\Pr_{N-Coop}(T_r) = \sum_{l=p_k}^{T_r} P_{rank}^{p_k}(l) \sum_{i=p_k}^{T_r-1} \sum_{j=\max(0,l-T_r+i)}^{\min((l-1),i)} \binom{i}{j} \times \left(1-P_e^{SD}\right)^j \left(P_e^{SD}\right)^{i-j} \times P_{l-j}^{T_r-i}\left(P_e^{RD}\right) \times \Pr_{p_k}\left(i, P_e^{SR}\right) \tag{9}$$

where P_e^{SD} and P_e^{RD} are the erasure probabilities across $\mathcal{S}$ and $\mathcal{D}$ and $\mathcal{R}$ and $\mathcal{D}$, respectively. The first summation represents the reception of l encoded packets, which can vary from the minimum number of p_k to receiving all packets from T_r transmissions. The second and third summation represents the reception of j unerased encoded packets from i source transmissions, where the limits are obtained by considering all probable combinations. Equation (9) can be interpreted as the probability of the cooperative reception of p_k linearly independent packets from the l encoded packets obtained from a total of T_r transmissions by $\mathcal{S}$ and $\mathcal{R}$.

For the next case, $\mathcal{R}$ failed to decode the source message even after proper decoding at $\mathcal{D}$, i.e., the system operated in noncooperation mode (direct transmission). For this case, all the packets at $\mathcal{D}$ are received from $\mathcal{S}$. The corresponding PMF of the number of transmissions required for noncooperative transmission can be evaluated as

$$\Pr_{N-Direct}(T_r) = \sum_{l=p_k}^{T_r} P_{rank}^{p_k}(l) \times P_l^{T_r}\left(P_e^{SD}\right) \times \left(1 - \Pr_{RCoop}\left(T_r - 1, P_e^{SR}\right)\right). \quad (10)$$

Therefore, the PMF of receiving a full-rank generator matrix at $\mathcal{D}$ after T_r transmissions can be given as

$$\Pr(N = T_r) = \Pr_{N-Direct}(T_r) + \Pr_{N-Coop}(T_r). \quad (11)$$

Evaluating (4) using the PMF computed in (11), the average number of slots required for cooperative multihop transmission can be estimated.

For the case when the probability of packet erasures is available for all n hops, the total number of transmissions (T_{tot}) required to have a full-rank matrix at $\mathcal{D}$ can be calculated by considering all possible combinations for each hop. This gives an estimate of the maximum total delay that will be encountered by the information packets to reach $\mathcal{D}$.

B. DCNs

Although DTNs assure the reliable delivery of information, the total delay encountered is boundless. The performance of such systems is dependent on the link quality, and similar to networking systems, the worst link becomes the bottleneck of the entire system. However, most real-time communication systems are delay intolerant, where the transmission needs to be completed within a specified duration. In such DCNs, the overall transmission delay is fixed. To constrain the total delay in rateless coded multihop networks and to mitigate the impact of worst links, the maximum number of transmissions at each hop is limited to a specified value "L." An ideal fountain code can reconstruct the entire source message reliably from any p_k received encoded packets [9]. However, ideal fountain codes have not been found so far. Practical codes such as random linear codes require a slightly increased number of encoded packets for successful decoding; hence, the maximum number of transmissions required per hop is larger than the number of source packets. By constraining the maximum number of transmissions per hop to a predefined value of L, the maximum total number of transmissions for the n-hop network configuration in Fig. 1 is

$$T_{tot} = n \times L. \quad (12)$$

DCNs do not guarantee the delivery of all the p_k packets across the n hops due to the fixed delay. The source packets that are correctly decoded are functions of the channel characteristics and the channel usage. Thereby, partial or total recovery of source packets can be performed at the intermediate nodes. Only a subset of source packets that was correctly decoded is forwarded in the subsequent hops to conform to the total delay. The average number of packets that can be recovered $(\overline{P}_k)$ is a function of the rank of the received generator matrix and can be computed by evaluating its expectation over the entire range, i.e.,

$$\mathbb{E}[P_k] = \sum_{p_i} p_i \Pr(P_k = p_i) \quad (13)$$

where P_k represents the number of packets correctly decoded, and $\Pr(P_k = p_i)$ is the corresponding PMF. For analytical simplicity, the number of packets correctly decoded is assumed to be equal to the rank of the received generator matrix. In reality, the number of first-degree encoded packets obtained by the rateless decoding process will be less than or equal to the rank due to the linear combination of transmitted packets.

An analytical evaluation of DCNs based on the number of packets recovered for conventional and cooperative multihop systems are done by computing the corresponding PMF, as given below.

$\overline{P}_k$ for Conventional Multihop Transmission: To compute the PMF of the number of packets recovered for conventional DCNs, there are two cases to be considered. For the first case, the receiver is able to correctly decode the entire source packets, i.e., the received generator matrix is full rank with L or less number of channel uses. Then, the corresponding PMF of decoding p_k packets transmitted along a channel with erasure probability P_e^{pq} across transmitting node p and receiving node q by the total probability theorem can be given as

$$\Pr\left[Rank(G) = p_k\right] = \sum_{i=p_k}^{L} \Pr_{p_k}\left(i, P_e^{pq}\right). \quad (14)$$

Equation (14) corresponds to the probability of receiving a full-rank generator matrix from the unerased encoded packets received from equivalent source transmissions.

For the second case, the receiver is unable to recover the entire transmitted packets within the specified duration. The receiver thereby decodes only a subset of p_k source packets within the permissible maximum number of transmissions (L). By the total probability theorem, the PMF of decoding only $p_i(< p_k)$ source packets in L transmissions can be given as

$$\Pr\left[Rank(G) = p_i\right] = \sum_{j=p_i}^{L} P_{lr}(p_i, j, p_k) \times \binom{L}{j} \times \left(1 - P_e^{pq}\right)^j \left(P_e^{pq}\right)^{L-j} \quad (15)$$

where $p_i \in \{0 \text{ to } (p_k - 1)\}$, and $P_{lr}(p_i, j, p_k)$ is the PMF of receiving a low-rank submatrix p_i from a higher dimension matrix of order $p_k \times j$. The detailed steps to evaluate $P_{lr}(p_i, j, p_k)$ are shown in the Appendix. Equation (15) signifies the reception of a generator matrix of rank p_i from the j unerased encoded packets received from L source transmissions, where the number of received packets (j) varies from the minimum limit of p_i (at least p_i packets are required to have a generator matrix of rank p_i) to receiving all packets from L source transmissions.

$\overline{P}_k$ *for Cooperative Multihop Transmission:* The detailed transmission process for such a cooperative multihop transmission scheme was explained in the previous section. With a maximum of L rateless coded transmissions by each node, there are two cases to be considered. For the first case, $\mathcal{D}$ is able to decode all the p_k source packets. Within this case, the entire encoded packets to decode the p_k packets can be received from $\mathcal{S}$ (provided $\mathcal{R}$ is unable to decode with corresponding source transmissions), and hence, no relaying gain is achieved. By the total probability theorem, the probability of receiving all the encoded packets by direct transmission from L or less number of transmissions can be computed using (10) as

$$\Pr_{Direct}\left(L, P_e^{SR}, P_e^{SD}\right) = \sum_{T_s=p_k}^{L} \Pr_{N-Direct}(T_s). \quad (16)$$

Equation (16) signifies the reception of a full-rank matrix at $\mathcal{D}$ from source transmissions, provided $\mathcal{R}$ is unable to decode with the corresponding source transmissions.

Once the relay is able to decode, $\mathcal{R}$ cooperates with $\mathcal{S}$ in transmitting the rateless coded packets. By the total probability theorem, the probability of receiving the encoded packets through cooperation can be computed as

$$\begin{aligned}\Pr_{Coop}&\left(L, P_e^{SR}, P_e^{SD}, P_e^{RD}\right)\\ &= \sum_{l=p_k}^{2L} \sum_{T_s=(\max((l-L),p_k))}^{L} \sum_{i=\max(0,(l-L))}^{\min((l-1),T_s)} \sum_{T_r=l-i}^{L} P_{rank}^{p_k}(l)\\ &\times \binom{T_s}{i} \left(1-P_e^{SD}\right)^i \left(P_e^{SD}\right)^{T_s-i} \times P_{l-i}^{T_r}\left(P_e^{RD}\right)\\ &\times \Pr_{p_k}\left(T_s, P_e^{SR}\right). \end{aligned} \quad (17)$$

The first summation is for the reception of l packets. The second and third summation represents the reception of i unerased encoded packets from T_s source transmissions, where the limits are obtained by considering all probable combinations. The final summation represents the number of relay transmissions required for receiving the remaining $(l-i)$ unerased encoded packets. Equation (17) can be interpreted as the probability of the cooperative reception of p_k linearly independent packets from l encoded packets obtained from T_s and T_r, i.e., source and relay transmissions, respectively.

The corresponding PMF of the number of packets required for receiving a full-rank matrix at $\mathcal{D}$ with cooperation for a maximum of L transmissions can be given as

$$\begin{aligned}\Pr\left[Rank(G)=p_k\right] &= \Pr_{Direct}\left(L, P_e^{SR}, P_e^{SD}\right)\\ &\quad + \Pr_{Coop}\left(L, P_e^{SR}, P_e^{SD}, P_e^{RD}\right). \end{aligned} \quad (18)$$

For the second case, $\mathcal{D}$ is able to correctly decode only a subset $p_i(< p_k)$ of source packets. The encoded packets for decoding p_i packets can be received from $\mathcal{S}$, $\mathcal{R}$, or a combination of both. All the encoded packets for decoding the p_i packets can be received from $\mathcal{S}$ when either the SR channel is unreliable ($P_e^{SR} \gg P_e^{SD}$; hence, $\mathcal{R}$ is unable to decode more than p_i packets from L source transmissions) or all of the transmissions from relay are erased due to an unreliable RD link. The probability of decoding exactly p_i packets at $\mathcal{R}$ from L source transmissions can be computed as

$$\begin{aligned}\Pr_{RDec}\left(p_i, L, P_e^{SR}\right) &= \sum_{j=p_i}^{L} P_{lr}(p_i, j, p_k)\\ &\times \binom{L}{j} \left(1-P_e^{SR}\right)^j \left(P_e^{SR}\right)^{L-j}. \end{aligned} \quad (19)$$

Then, the probability of $\mathcal{R}$ decoding more than p_i packets across the SR link in L transmissions by $\mathcal{S}$ can be evaluated as

$$\begin{aligned}\Pr_{RDec>p_i}\left(L, P_e^{SR}\right) &= \sum_{i=p_i}^{p_k-1} \Pr_{RDec}\left(p_i, L, P_e^{SR}\right)\\ &\quad + \Pr_{RCoop}\left(L, P_e^{SR}\right) \end{aligned} \quad (20)$$

where the first term represents the recovery of p_i or more source packets (but less than the entire source packets) from L source transmissions, and the second term represents the successful decoding of all the source packets at $\mathcal{R}$ (receiving a full-rank generator matrix at $\mathcal{R}$). The probability of receiving all the p_i packets at $\mathcal{D}$ by direct transmission using the total probability theorem can be evaluated using (20) as

$$\begin{aligned}\Pr_{Direct}&\left(p_i, L, P_e^{SR}, P_e^{SD}, P_e^{RD}\right)\\ &= \sum_{l=p_i}^{L} P_{lr}(p_i, l, p_k) \binom{L}{l} \left(1-P_e^{SD}\right)^l \left(P_e^{SD}\right)^{L-l}\\ &\times \left(1 - \Pr_{RDec>p_i}\left(L, P_e^{SR}\right)\left(1-\left(P_e^{RD}\right)^L\right)\right). \end{aligned} \quad (21)$$

The cooperative scenario where $\mathcal{D}$ has received encoded packets from both $\mathcal{S}$ and $\mathcal{R}$ need to be addressed in two contexts. In the first event, $\mathcal{R}$ has recovered more than (p_i+1) packets from L transmissions by $\mathcal{S}$, which can be computed similar to (20). The second event is the decoding of exactly p_i packets at $\mathcal{R}$ from p_i or more packets received from L source transmissions, as computed in (19). Then, the corresponding probability for the cooperative scenario can be computed using (19) and (20) as

$$\begin{aligned}\Pr_{Coop}&\left(p_i, L, P_e^{SR}, P_e^{SD}, P_e^{RD}\right)\\ &= \sum_{l=p_i}^{2L} P_{lr}(p_i, l, p_k) \sum_{j=\max(0,l-L)}^{\min(l-1,L)} \binom{L}{j} \left(1-P_e^{SD}\right)^j \left(P_e^{SD}\right)^{L-j}\\ &\times \Bigg(\binom{L}{l-j} \left(1-P_e^{RD}\right)^{l-j} \left(P_e^{RD}\right)^{L-l+j} \Pr_{RDec>(p_i+1)}\left(L, P_e^{SR}\right)\\ &\quad + \Pr_{RDec}\left(p_i, L, P_e^{SR}\right)\\ &\quad \times \sum_{k=(l-j)}^{L} P_{l-j}^{L}\left(P_e^{RD}\right)\Bigg) \end{aligned} \quad (22)$$

where the first summation represents the reception of l unerased encoded packets from L transmissions by $\mathcal{S}$ and $\mathcal{R}$. The second summation signifies the reception of j packets out of the l unerased encoded packets at $\mathcal{D}$ from L source transmissions. The remaining $(l-j)$ unerased packets are received from $\mathcal{R}$. Relay $\mathcal{R}$ cooperates with transmission only when it has successfully decoded p_i or more source packets. The $(l-j)$ packets are received from L transmissions by $\mathcal{R}$ only when it has decoded more than p_i source packets. Further, the remaining $(l-j)$ packets can be obtained from $(l-j)$ or more transmissions by $\mathcal{R}$ when it has exactly decoded p_i source packets. The given equation can be interpreted as the probability of receiving p_i linearly independent packets at $\mathcal{D}$. The PMF of the number of packets required for correctly decoding p_i packets at $\mathcal{D}$ with the number of transmissions by $\mathcal{S}$ and $\mathcal{R}$ constrained to L can be given from (21) and (22) as

$$\Pr\left[Rank(G)=p_i\right] = \Pr_{Direct}\left(p_i, L, P_e^{SR}, P_e^{SD}, P_e^{RD}\right) + \Pr_{Coop}\left(p_i, L, P_e^{SR}, P_e^{SD}, P_e^{RD}\right). \quad (23)$$

Therefore, the PMF of recovering p_i packets when the channel usage per node is constrained to maximum L transmissions can be evaluated using (14), (15), (18), and (23) for conventional and cooperative relaying cases as

$$\Pr(P_k = p_i) = \begin{cases} \Pr\left[Rank(G)=p_k\right], & \text{if } p_i = p_k \\ \Pr\left[Rank(G)=p_i\right], & \text{else.} \end{cases} \quad (24)$$

By evaluating (13) using (24), $\overline{P}_k$, which is the average number of packets that can be recovered for DCNs, can be computed.

IV. Numerical Results and Discussion

Here, the analytical and empirical results on the performance of rateless coded DTNs and DCNs discussed in this paper are analyzed. For computational tractability, the multihop network shown in Fig. 1 with n hops and w nodes per hop clustered together is analyzed. Two nodes per cluster $(w=2)$ is considered for simulations with the best relay node forwarding the information. The channel between nodes is modeled as independent Nakagami-m distributed, as explained in Section II. The typical path loss exponent (α) for mobile networks is in the range of 3–5, and the value of 3.5 is chosen to model an urban environment.

Initially, the theoretical results in Section III are presented in two simulations that demonstrate the high level of accuracy of the derived closed-form equations. Fig. 2 shows the expected and simulated number of transmissions required for cooperative and conventional DTNs. It is observed that the theoretical values of the average number of transmissions (shown by the markers) required for successful decoding closely matches the simulated values (shown by the solid line). Cooperation among nodes results in reasonable reduction in the required number of transmissions as it selects the better source or relay nodes for transmissions based on the relay message quality and the instantaneous channel. At high SNRs, both transmission schemes require few additional encoded packets, as rateless codes require slightly more encoded packets than the p_k information packets for successful decoding.

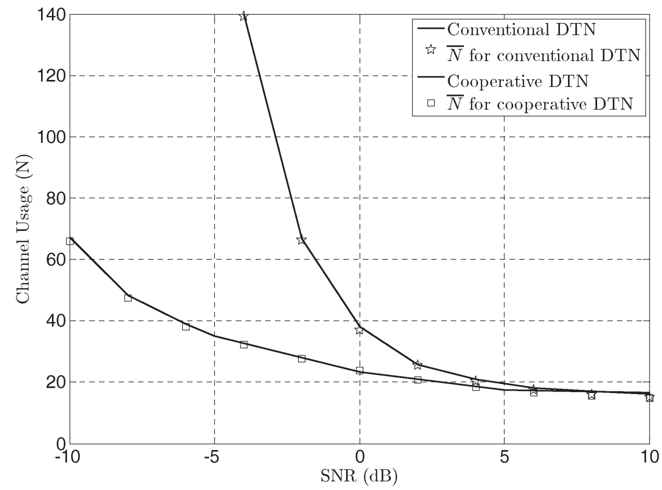

Fig. 2. (Solid line) Simulated and (markers) theoretical number of slots required for transmitting 12 information packets (p_k) using rateless codes for different SNRs in a Rayleigh $(m=1)$ fading channel.

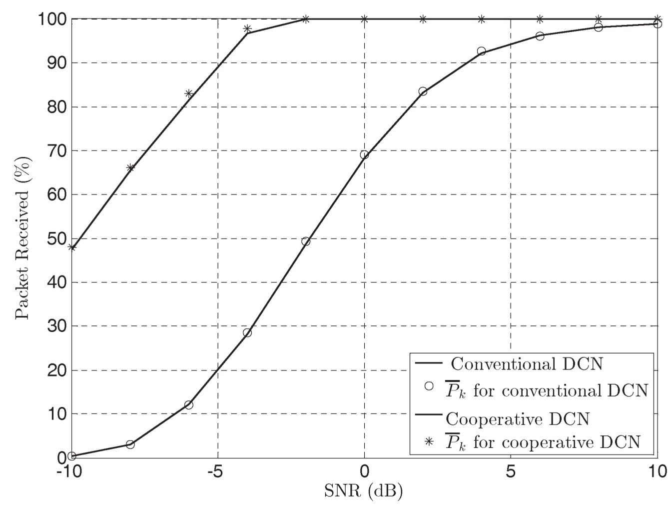

Fig. 3. (Solid line) Simulated and (markers) theoretical percentage of packets received for varying SNRs with an information packet size (p_k) of 12 and maximum number of transmission (L) limited to 20 in a Rayleigh $(m=1)$ fading channel.

The expected and simulated percentage of packets received for cooperative and conventional DCNs with varying SNRs is shown in Fig. 3. When the SNR is low or when the links are least reliable, the average packet loss is very large. However, packet loss diminishes with an increase in the reliability of links as more encoded packets are delivered within the specified number of slots. However, a significant variation in packet reception is observed with cooperation as the probability of unreliable links decreases with the number of links available for transmission. Thereby, cooperation among nodes results in better reliability. The simulated percentage of packets received at the destination (shown by the solid line) conforms to the closed-form equations for the average percentage of received packets (shown by the markers).

A. Simulation Settings

To compare the performance of DTNs and DCNs for the network configuration in Fig. 1, the intermediate terminals are

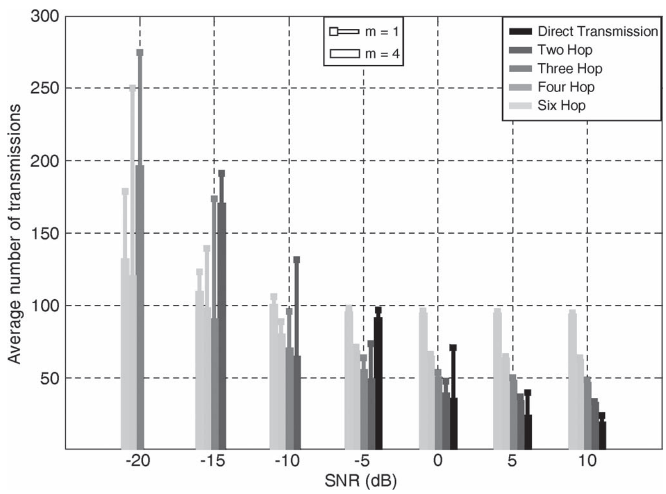

Fig. 4. Average number of transmissions required for conventional DTNs in Rayleigh ($m = 1$) and Nakagami-4 fading channels.

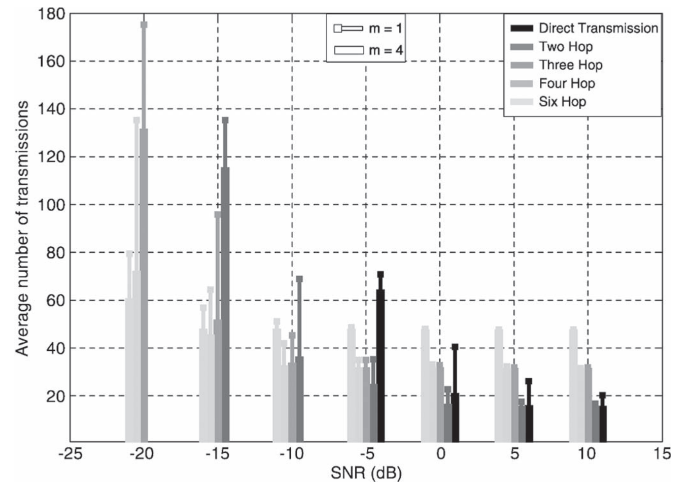

Fig. 5. Average number of transmissions required for cooperative DTNs in Rayleigh ($m = 1$) and Nakagami-4 fading channels.

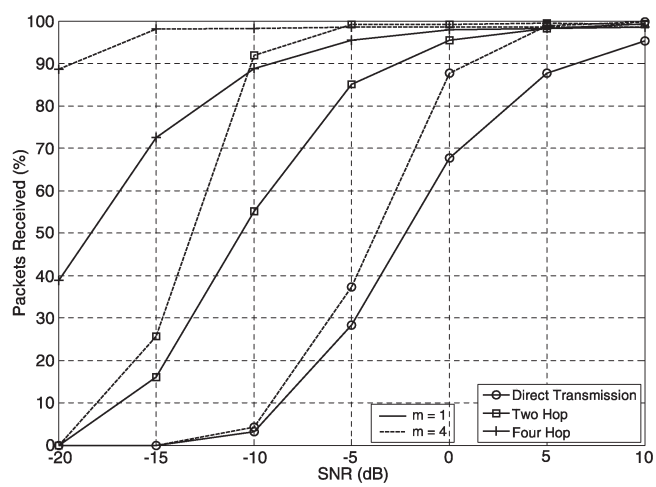

Fig. 6. Comparison of percentage of packets received with an SNR for conventional DCNs for varying number of hops in Rayleigh ($m = 1$) and Nakagami-4 fading channels.

considered to be fixed and collinear across the n hops. The intermediate nodes thus divide the direct path between the source and destination nodes into equal-length segments. Such a configuration serves to validate the analytical results and to illustrate the benefits that can be realized with an optimal placement of the nodes. A uniform power constraint, as explained in Section II, is also employed to comply with the total power constraint.

A Monte Carlo simulation exercise based on the realistic communication environment discussed in Section II is performed. Noiseless ACK/NACK is assumed for simplicity. Without loss of generality, the transmitter and the receiver are supposed to use a deterministic random generator for rateless coding. Thus, the receiver easily synchronizes with the transmitter. The number of source packets (p_k) is taken as 12. Further, the packet length (b) is set to 256 bits. For analytical simplicity, BPSK with coherent detection for transmission and reception is used. The packets received through the channel are considered erased based on the erasure technique presented in Section II. The unerased received encoded packets are retained in the buffer until the receiver accumulates sufficient mutual information for reliable decoding. Decoding is deemed successful once the received generator matrix is of full rank. The performance of the n-hop network is quantified with respect to the average received SNR variation over the direct (SD) link.

B. Performance of DTNs

The average number of transmissions required for conventional DTNs with varying number of hops in a Nakagami-m fading channel is shown in Fig. 4. It is clear in Fig. 4 that, at low SNRs, multihopping is the best method of communication. At these SNRs, the source can communicate with the destination only with the aid of relay nodes. However, the optimal number of hops is dependent on the channel fading conditions. Initially, in a Rayleigh fading channel $(m = 1)$, transmission with the maximum number of hops is optimal. However, as the SNR improves, the optimal number of hops for transmission decreases, and it also decreases with $m = 4$, which implies less severe channel fading conditions. At higher SNRs, direct transmission becomes the optimal means of transmission. It stems from the fact that the number of linear packet combinations required for rateless decoding is slightly larger than p_k when the channel is reliable. The number of transmissions thereby has a cumulative effect with the number of hops when the channel becomes reliable for conventional DTNs.

Fig. 5 shows the performance of cooperative DTNs in terms of the average number of transmissions required for successful decoding in Nakagami-m fading channels. As compared with conventional DTNs, it is observed that cooperative DTNs require much less number of slots. Further, it is observed that even at high SNRs, cooperation among nodes results in mitigating the required number of transmissions. Hence, cooperation is always beneficial to reduce the required number of transmissions.

C. Performance of DCNs

Fig. 6 shows the variation of percentage of packets received with an SNR for conventional DCNs in Nakagami-m fading

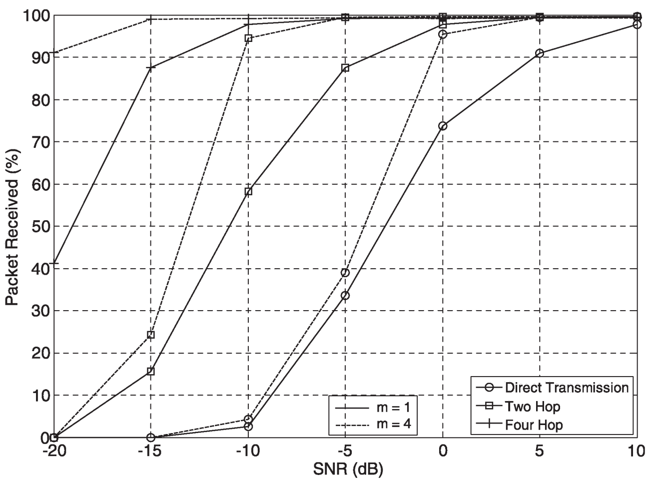

Fig. 7. Comparison of percentage of packets received with an SNR for cooperative DCNs for varying number of hops in Rayleigh $(m = 1)$ and Nakagami-4 fading channels.

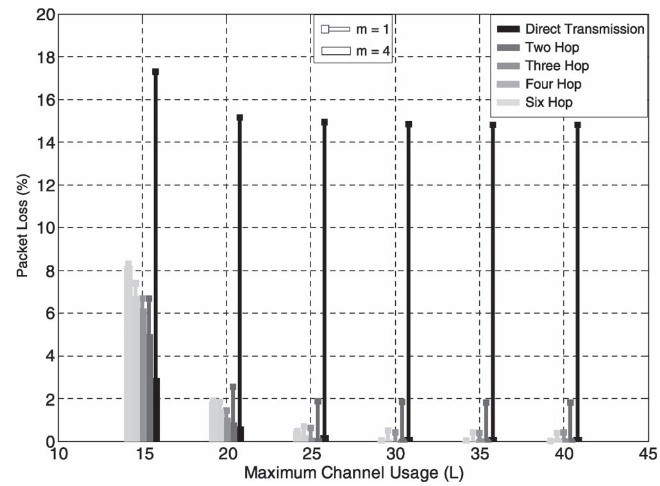

Fig. 8. Variation of packet loss with channel usage per hop for conventional DCNs at an average SNR of 4 dB in Rayleigh $(m = 1)$ and Nakagami-4 fading channels.

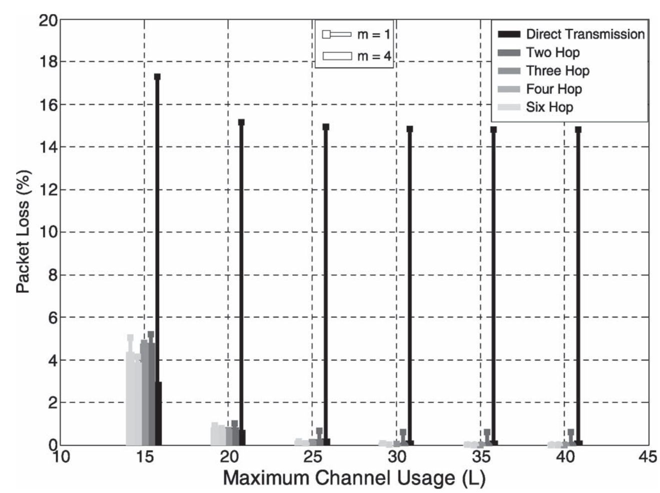

Fig. 9. Variation of packet loss with channel usage per hop for cooperative DCNs at an average SNR of 4 dB in Rayleigh $(m = 1)$ and Nakagami-4 fading channels.

channels, where the maximum number of transmissions (L) is 20. It is visible that packet reception improves with reliability of the link (as SNR increases). Fig. 6 reveals that a less severe fading environment $(m = 4)$ results in better reliability. More number of relay nodes forwarding the information results in an improvement in link reliability due to the collinear placement of nodes. This results in an improvement in the performance of conventional DCNs with the number of hops, as shown in Fig. 6.

Fig. 7 shows the variation of packets received with an SNR for cooperative DCNs in Nakagami-m fading channels. For DCNs, only the correctly decoded packets are transmitted during each subsequent hop. However, as channel usage per hop remains constant across the number of hops, the code-rate effectively decreases with the number of hops. Thereby, performance degradation with the number of hops is limited. In addition, as the number of hops increases, the percentage of packets received alleviates due to the collinear intermediate node arrangement. By comparing Figs. 6 and 7, it can be observed that the packet reception improves by cooperation. In Figs. 6 and 7, it is clearly illustrated that the reliability of DCNs improves with link quality and favorable channel conditions (Nakagami-4 channel). It is also evident that the number of hops required to have a specified QoS reduces with cooperation among nodes. However, with delay constraints, the total delay is reduced when the links are unreliable for transmission. These results further portray the tradeoff existing for successful packet reception with varying SNRs when the maximum number of rateless coded transmissions (L) is fixed.

Figs. 8 and 9 show the variation of packet loss in Nakagami-m fading channels with a limit on the maximum channel usage for conventional and cooperative DCNs, respectively, at an average SNR of 4 dB. It is observed that the number of packets that can be correctly decoded is a factor of the reliability of the links irrespective of channel usage. Although there is a reduction in packet loss with an increase in the reliability of links, there is a limit on the number of successfully decoded packets for fixed channel usage. Thus, the maximum channel usage (L) should be optimized based on the packet loss, which is a function of the reliability of links and channel quality. While for Rayleigh fading $(m = 1)$ the packet loss remains high, for Nakagami-m fading $(m = 4)$, the packet loss is lower. It is clear that optimizing the number of channel uses mitigates the infinite delay and delivers the number of packets based on the reliability of the links. It is also evident in Figs. 8 and 9 that the packet loss will decrease with cooperation among the nodes, and lesser number of hops is required to achieve the same QoS as compared with conventional DCNs.

D. *Performance Comparison of DTNs and DCNs*

Fig. 10 shows the comparison of the bit error rate (BER) for conventional DTNs and DCNs with varying number of hops in a Rayleigh $(m = 1)$ fading channel. Fixed intermediate nodes at collinear locations decrease the effective path loss, and thereby, a performance improvement can be observed with an increase in the number of hops. For DCNs, as only the correctly decoded packets are transmitted in subsequent hops, there is an error floor at high SNRs for a fixed maximum number of

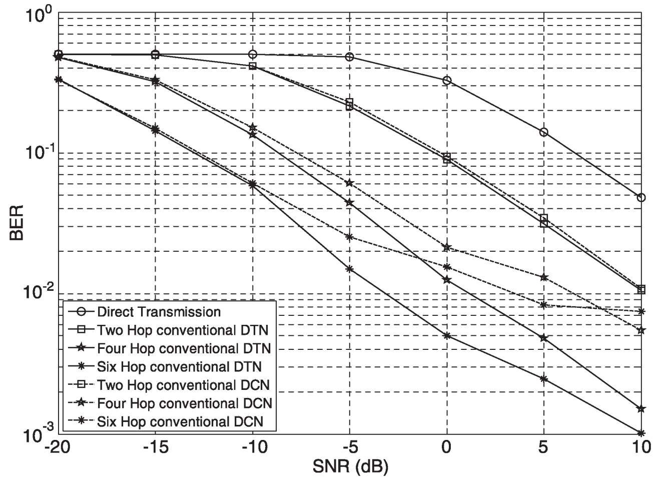

Fig. 10. BER performance comparison for conventional DTNs and DCNs in a Rayleigh $(m = 1)$ fading channel.

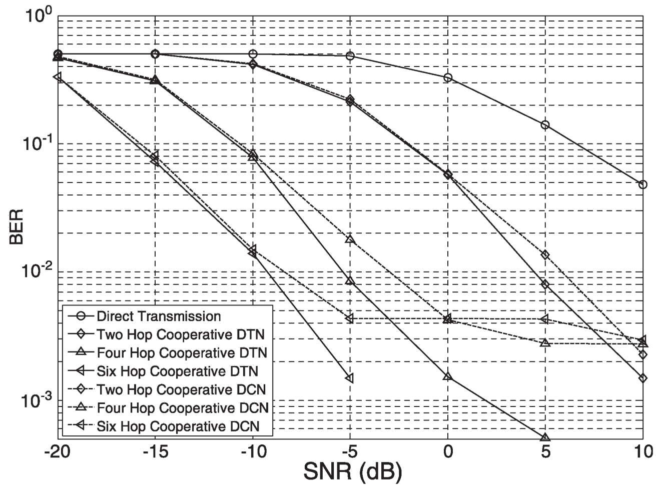

Fig. 11. BER performance comparison for cooperative DTNs and DCNs in a Rayleigh $(m = 1)$ fading channel.

channel uses. For simulations, an outer code is not utilized, and hence, the rate (R_c) is taken as 1 for the erasure technique presented in Section II.

The comparison of the BER for cooperative DCNs and DTNs in a Rayleigh $(m = 1)$ fading channel is presented in Fig. 11. The performance improves with cooperation among the nodes. The performance tradeoff with delay is visible in Figs. 10 and 11. Further, it is evident that the best performance can be achieved without any delay constraints, whereas the performance degrades with more restrictive delay constraints. In addition, with fewer numbers of hops, DCNs perform comparably with DTNs. However, as the number of hops increases, the performance of DCNs degrades, stemming from the fact that in DCNs, only the correctly decoded packets are transmitted in the subsequent hops. This has an accumulative effect on the BER performance, which accounts for an error floor at high SNRs for DCNs.

V. Conclusion

In this paper, the impact of delay constraints on multihop wireless relay networks employing rateless codes has been analyzed. The possible tradeoffs existing between average throughput and average delay are emphasized. The performance of DTNs is determined by the link quality, and similar to networking systems, the worst link becomes the bottleneck of the entire system. Thus, the more reliable links are underutilized, which restrict the overall throughput of the network. The overall delay encountered by the network can be limited by constraining the number of transmissions per hop, as in DCNs. However, in DCNs, increasing the number of transmissions does not necessarily cause an improvement in performance. Hence, the channel usage should be optimized based on the QoS requirement of the network and the channel quality. Further, increasing the number of hops beyond a certain limit does not necessarily improve the performance of rateless codes. The number of transmissions per hop and the number of hops should be appropriately selected as the number of encoded packets required for rateless decoding has an accumulative effect. This is to minimize the total delay and packet loss. In extreme cases of high SNRs, both DCNs and DTNs give comparable performance.

Appendix
Probability Mass Function of Receiving a Low-Rank Submatrix From a Higher Dimension Matrix

Let $GF(2)$ be a Galois Field of size 2. Each new fountain-coded packet is associated with an encoding vector over $GF(2)$ of dimension p_k, where each packet is obtained by the linear combination of p_k source packets, as explained in Section II. The p_k source packets can be recovered if p_k linearly independent packets are received, i.e., $Rank(G)$ is p_k, where G is the received generator matrix.

As an all-zero encoding vector does not contain any information, it is assumed that an all-zero column is not generated by the encoder. Then, it is certain to receive a rank 1 matrix from a nonzero encoding set with one column. With the next encoding set, there are two columns that are dependent among them, and then, the probability of having two linearly independent columns can be obtained as $(1 - 1/2^p_k)$ [31]. By extending this property for R_p received vectors, the probability of receiving l independent columns with $l = p_k = R_p$ is

$$P_{lr}(l, R_p, p_k) = \prod_{i=0}^{l-1} \left(1 - \frac{2^i - 1}{2^{p_k}}\right). \tag{25}$$

Then, as R_p increases, there are $2^{(R_p - l)}$ columns that are dependent among them. Then, the probability of having l linearly independent columns from R_p received vectors is

$$P_{lr}(l, R_p, p_k) = \Gamma \times 2^{(R_p - l) - p_k} \times \prod_{i=0}^{l-1} \left(1 - (2^i - 1)2^{-p_k}\right) \tag{26}$$

where Γ is a constant. By evaluation, constant Γ can be approximated as a geometric series with a common ratio of 2 and initial term to be $2^{(R_p - l)}$. The number of terms between initial and final terms can be computed as

$$k = (R_p - l) \times (l - 1) + 1. \quad (27)$$

Thus, constant Γ can be computed as

$$\Gamma \approx 2^{(R_p - l)} \times (2^k - 1). \quad (28)$$

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