Spectrally Efficient Packet Recovery in Delay Constrained Rateless Coded Multihop Networks

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***Abstract*—Rateless codes have been found to be particularly attractive for decode and forward based relaying strategy in delay tolerant multihop networks. The latency performance of such networks is dependent on the worst links that results in the starvation of subsequent nodes with good channel conditions. The total delay suffered by such networks can be constrained by limiting the number of rateless coded transmissions, especially for applications with critical latency requirements. However, the performance of rateless codes deteriorates in such circumstances due to the lack of sufficient mutual information for successfully recovering the entire source packets. The fraction of source packets that can be recovered will depend on the encoded packets received across the transmission channel. This paper investigates the degradation in the performance of such rateless coded networks by deriving the average packet recovery rate.**

In order to improve the reliability in such delay constrained networks, a novel spectrally efficient transmission scheme for reliable multihop data transfer is proposed. The proposed scheme exploits the broadcast nature of wireless transmissions, which provides an inherent implicit feedback channel, to ensure the reliable delivery of information packets to the nodes in the network. Rather than allocating dedicated channels to feedback the packet recovery information, the implicit feedback channel determines such information which enhances the spectral efficiency. Further, the optimum number of packets recoverable within the specified delay constraint to reduce the reprocessing of lost packets across hops is analytically analysed in this paper.

***Index Terms*—Fountain/rateless codes, delay constrained multihop networks, implicit feedback assisted transmission.**

I. INTRODUCTION

FUTURE wireless networks are expected to provide high data rate with wide coverage and efficient spectrum utilisation. However, the overall transmission range is dependent on the battery powered nodes that constitute such networks. The energy expenditure and transmission delay for successful transmission in such networks increase with the distance between wireless nodes. Such limitations have prompted the use of multihop transmission as a potential technique for future networks to reliably deliver the data over large distances [1]–[3]. Multihop transmission breaks the path between source and destination nodes into several hops via relaying utilising intermediate nodes. The intermediate nodes are selected based on the routing protocol employed. By having much less path loss, multihop transmission reduces the interference levels, terminal radiation, and power consumption of nodes in the network [4], [5].

Multihop transmission emerging as a viable option for long distance communication traditionally relies on information transmitted by its immediate predecessor node in the previous hop, and the destination simply listens to the last node in the route. In this paper, such relaying is referred to as *conventional/simple* relaying, as is known from ad-hoc networking systems. By exploiting the nonlinearity of attenuation as a function of distance, conventional transmission benefit from a reduction in end-to-end pathloss. More recently, the concept of cooperation among intermediate nodes to create a virtual antenna array for improving the reliability and throughput of wireless networks has fostered the emergence of *cooperative* relaying. In such relaying, the destination decodes the information based on the spatially diverse signals received from several intermediate cooperating nodes. Such relaying where the resources are shared among multiple nodes has the potential to meet the growing demand of high data rate wireless networks with wider coverage and better spatial diversity to combat fading [2], [3]. Multihop transmission exploiting cooperative relaying is correspondingly referred to as *cooperative multihop transmission*. Different from conventional multihop transmission, the number of cooperating nodes should be optimized based on the permissible energy consumption of the network.

In order to ensure a reliable packet flow in multihop systems, the intermediate nodes process the overheard information based on the transmission technique employed. Of the forwarding strategies employed at the intermediate nodes, *decode, re-encode* and *forward* based transmission mitigate the effect of noise amplification and offers an improved performance. Fountain codes also known as rateless codes introduced in [6], [7] that offers a low complexity encoder/decoder is an attractive option for such systems. Further, the rateless property of recovering the source data from any subset of encoded packets with sufficient mutual information makes such coding technique ideal for network with multiple nodes. In such rateless coded networks, outage is never experienced as the intermediate nodes monitor the

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transmitting node virtually indefinitely. The application of rateless codes to such networks where the successful relays assist the source in transmissions have been studied in [8], [9]. In [10], a throughput optimal rateless coding scheme is proposed to relay Luby Transform (LT) codes across multiple nodes.

Most of the recent works on rateless coded networks are focussed on delay tolerant networks (DTNs) which are designed for reliable transmission but can tolerate long latency [11]–[13]. In DTNs, each of the intermediate nodes acts as independent rateless encoder/decoder, where it retrieves the original source packets and performs rateless encoding to transmit new encoded packets. The destination accumulates the mutual information and the process continues until sufficient mutual information is received to decode the source information. In spite of the capability to improve the system capacity and the wireless coverage in fading environments, rateless coded DTNs are inefficient and suffers high latency. Also the performance of DTNs depends on the intermediate link quality and the poor quality links introduce long/infinite delays that lead to starvation of subsequent hops with good channel conditions.

Mitigating latency becomes an important issue for future wireless networks owing to the explosive amount of delay-sensitive traffic such networks has to handle [14]–[16]. Further, the indefinite latency of rateless coded DTNs are detrimental for most real-time systems which also have a finite buffer. This has fostered the interest in employing rateless codes for delay constrained networks (DCNs). The existence of efficient and reliable rateless codes for DCNs under different channel conditions has been proved in [17]. Recently the authors have also analysed the impact of delay constraints on multihop rateless coded networks [18], [19]. In DCNs, the maximum transmission time is fixed to constrain the total delay. This in turn results in the maximum number of encoded packets transmitted per hop to be fixed. Such a scenario can be considered as a form of constrained incremental redundancy, and *partial* or *total* recovery of source packets is possible at the intermediate nodes depending on the channel conditions [20].

DCNs ensure a finite latency for multihop networks and mitigate the impact of poor quality links, but it does not guarantee the delivery of entire source data. In such networks, the receiver attempts to retrieve the maximum source packets from the encoded packets received over a finite amount of time. This paper analytically quantifies the delay constrained performance of rateless codes by deriving the packet recovery rate with finite number of rateless coded transmissions. Such an analysis facilitates the need for recovering the requisite number of source packets within the average delay permissible by the network and can be thought of as a quality of service (QoS) guarantee. Based on the application it might only be required to recover a large fraction of source packets instead of all the source packets. Selective retransmission upon failure (Hybrid-ARQ) schemes can be employed to recover the lost packets in such networks [21]. Rateless codes with feedback have been shown to offer better data recovery [22], [23]. In [22], a feedback is used to transmit systematic symbols for reviving the decoding process whenever the rateless decoder is stalled. For reliable and faster decoding, the intermediate packets retrieved at the receiver can be feedback to the rateless encoder for adjusting the degree distribution [23].

However, the use of an explicit feedback mechanism to signal the lost packets in rateless coded DCNs incurs significant bandwidth penalty. The spectral efficiency of such networks can be improved by minimizing traffic in the feedback channel. This motivates to exploit the implicit feedback concept in [24] to determine the lost packets in such multihop systems. The crux of the idea is to exploit the broadcast nature of wireless transmission which provides an additional dimension (an implicit feedback channel) to determine the lost packets per hop. The rateless encoder includes these packets along with the next block of information packets being processed by the nodes. This ensures the reliable delivery of source information to the nodes in the network and can greatly reduce the number of retransmission, which contributes to low end-to-end delays with high energy efficiency. Compared to other schemes where a dedicated feedback channel is allocated to signal this information, the proposed scheme is simple and does not require any extra bandwidth. Hence, it provides a low complexity and spectrally efficient alternative for reliable transmission in rateless coded multihop DCNs. The proposed scheme employing decomposed fountain codes can be beneficial to further lower the latency and guarantee the end-to-end reliability of the network [25], [26].

Even the use of an implicit feedback mechanism will incur the overhead of determining the lost packets at the transmitter. In order to reduce the reprocessing of lost packets in such networks, each node employs the hop distance information to ascertain the fraction of data it can reliably deliver. The optimum number of source packets that can be reliably delivered in rateless coded DCNs is analysed in this paper. This reduces the packet loss rate in such networks and improves its reliability.

The remainder of this paper is organised as follows. The system model employed for analysing the proposed scheme is discussed in Section II. Section III discusses the combinatorial analysis of rateless codes to determine the packets lost per hop in multihop rateless coded DCNs. Detailed discussion on the proposed scheme and its performance is given in Section IV. Numerical results demonstrating the performance improvement achievable by the proposed scheme is presented in Section V. Finally, concluding remarks and future extensions of the present work in Section VI wrap up this paper.

II. System Model

In this paper a multihop wireless network as depicted in Fig. 1 is considered, where a source $\mathcal{S}$, which generates a continuous stream of information bits, communicates with the destination $\mathcal{D}$ by hopping through the $w \times n$ intermediate relays $(\mathcal{R}_{ij})$. All nodes are equipped with a single omnidirectional antenna and are constrained by half duplex transmission.

A. Received Signal & Channel Model

The continuous stream of information bits ($\mathbf{u}$) generated by $\mathcal{S}$ are first grouped into blocks of size $p_k \times b$, where p_k is the number of packets per encoding set and b is the bit

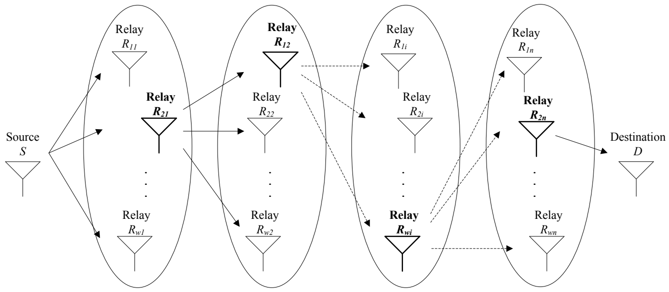


Fig. 1. System configuration with source node ($\mathcal{S}$), destination node ($\mathcal{D}$) and $m \times n$ relay nodes ($\mathcal{R}_{ij}$'s) with only one node forwarding per hop.

size of each data packet. At any instant t, the information block fed to the rateless encoder can be represented as $\mathbf{u}(t) = \begin{bmatrix} u_1 & u_2 & \cdots & u_{p_k} \end{bmatrix}^T$. The encoder continuously generates encoded packets c_i's of size $1 \times b$ until either sufficient mutual information is received for proper decoding or maximum predetermined number of coded packets (L) are transmitted. Each of the rateless coded packets is transmitted over orthogonal subchannels by employing orthogonal frequency division multiplexing (OFDM), where the subchannels experience relatively flat fading [27], [28]. Hence, the channel between any pair of nodes is assumed to be frequency flat, where the channel gain remains quasi-static for a fading block of one coded packet (i.e., up to b bits), and varies independently across packets. The corresponding received signal model over a quasi-static fading channel can be expressed as [24]

$$\mathbf{Y}_q = d_{pq}^{-\alpha/2} H_{pq} \mathbf{X}_p + \mathbf{N}_q \tag{1}$$

where $p \in \{\mathcal{S}, \mathcal{R}_{ij}\}$ and $q \in \{\mathcal{R}_{ij}, \mathcal{D}\}$, $\mathbf{X}_p \in \mathbb{C}^{1\times b}$ is the transmitted encoded packet, α captures the pathloss exponent, d_{pq} is the distance between transmitting node p and receiving node q (without loss of generality, shadowing is not considered in this paper), $H_{pq} \in \mathbb{C}^{1\times 1}$ is an element of the channel transfer matrix between nodes p and q, $\mathbf{N}_q \in \mathbb{C}^{1\times b}$ is the noise matrix which corresponds to additive white Gaussian noise (AWGN) with zero mean and variance N_0, and $\mathbf{Y}_q \in \mathbb{C}^{1\times b}$ is the received signal matrix. The entries of the channel matrix H are assumed to be independent and identically Nakagami-m distributed. Each node is assumed to transmit under equal power constraint, and hence $\mathbb{E}[|\mathbf{X}_p|^2] = \varepsilon \;\forall\; p$, where ε is the average energy per symbol during transmission. Equalization is performed on the received signal in (1) using conventional channel equalization methods such as zero forcing by assuming the availability of channel state information (CSI) only at the receiver. The transmitter is considered to be oblivious of a priori knowledge of the CSI and generates encoded packets bounded by either successful decoding or the permissible maximum delay.

For reliable transmission, a layered coding approach is considered, where error correction coding is applied to each rateless coded packet [29]. Assuming capacity achieving Gaussian codebooks[1] and in accordance with Shannon's theorem, the rateless coded packets transmitted through a channel can be decoded with vanishing error probability when the error correction code-rate R_c is less than the channel capacity C. The channel is considered to be in outage whenever this constraint is violated [30]. In an outage event, there is no guarantee that the transmitted encoded packet can be decoded without error and such packets are considered as erased. Hence, the quasi-static wireless channel can be treated to be in outage with probability

$$P_e^{pq} = \Pr[C < R_c] = \Pr\left\{ |H_{pq}|^2 < \frac{2^{R_c} - 1}{\gamma} \right\} \tag{2}$$

where $\gamma = \varepsilon/(d_{pq}^{\alpha} N_0)$. The channel outage probability P_e^{pq} across nodes p and q is simply the probability density function of $|H_{pq}|^2$; for Nakagami-m fading channels this is given as [31]

$$P_e^{pq} = 1 - \sum_{k=0}^{m-1} \frac{1}{k!} (m\sigma)^k \exp(-m\sigma) \tag{3}$$

where $\sigma = (2^{R_c} - 1)/\gamma$. When $m = 1$, the Nakagami channel reduces to a Rayleigh fading channel. The outage constraint enables the wireless channel to be treated as an equivalent erasure channel and the analysis for the rateless coded system can be performed using the same approach as in an erasure channel. For simplicity of exposition, binary phase shift keying (BPSK) modulation with coherent detection is used. The stream of received unerased rateless coded packets is then fed to the decoder.

B. Background on Rateless Coding

Rateless codes are modern flexible FEC codes which do not impose a fixed coding rate. An infinite number of encoded packets are generated through the rateless encoding process by linearly combining d randomly selected packets (chosen according to some degree distribution $\Omega(x) = \sum_{i=0}^{p_k} \Omega_i x^i$) from the message block $\mathbf{u}$ [32, ch. 50]. Each rateless code is completely defined by the degree distribution $\Omega(x)$, which is a discrete probability distribution over $[1, p_k]$. For simplicity of analysis a class of efficient rateless codes, random linear rateless codes with uniform degree distribution, is considered in this paper. Each rateless coded packet is further encoded with error-correction code and modulated before sequentially broadcasting over the channel, i.e. the transmitter serves as a perpetual fountain.

Initially, the source encodes the data packets and broadcasts it. The receivers tune into the on-going broadcast transmission at any arbitrary time, as rateless codes has the unique ability to recover the source message from any subset of encoded packets with sufficient mutual information. The received unerased encoded packets are then fed to the decoder. Each received encoded packet generates a column of the generator matrix G and the entire source packets can be recovered when the received generator matrix is full rank, i.e. $Rank(G) = p_k$. The decoder determines this by attempting to invert G using Gaussian elimination. The source ceases transmission upon reception of an acknowledgement (when any of the receivers has reliably decoded the data) or upon transmission of the permissible number of encoded packets constrained by the total delay. With delay constrained transmissions, the received encoded packets may not be sufficient to recover the entire

[1]This assumption is not very limiting, as many capacity achieving codes such as turbo and low-density parity-check (LDPC) codes have been reported [30].

source packets (depending on the channel conditions) and the maximum source packets are retrieved by performing partial decoding [33]. For reliable decoding, the index of source packets used to generate each encoded packet must be provided to the decoder. In order to ensure this, the required packet information is appended to the header of each encoded packet using an average of $\Omega'(1)\lceil \log_2 p_k \rceil$ bits [34]. The source packets that emerged from the decoding process are utilised for subsequent transmission and the corresponding receiver switches from reception to transmission. Nodes are designed to employ *independently generated rateless codes*, which is crucial for mutual information accumulation. This enables the receivers to combine the information flows from any subset of transmitters.

C. Routing & Network Configuration

The layered multihop network in Fig. 1 is assumed to be stable and has a predefined route from $\mathcal{S}$ to $\mathcal{D}$ via n hops. In the n-hop network topology, $(n-1)$ clusters are considered to randomly occupy the distance between the source and destination nodes, with w nodes per cluster. Cluster formulation is not considered in the present paper and it is assumed that the clusters are formulated by the previously known schemes in [35]. The relay nodes in the clusters are responsible for transmitting the information from the source to the destination node. The proper selection of a relay node from the w nodes in a cluster to forward the information is crucial in achieving a good end-to-end throughput. The best relay node is opportunistically selected by assuming that each node has some knowledge about its position as well as the position of the destination node [36], [37]. This distance information is utilized to prioritize the nodes in a cluster and the key idea is to forward the information along the shortest path. However, the number of rateless coded packets received at the nodes is random and is based on independent wireless links from the source node. During instances of successful decoding (recover the entire source packets), an acknowledgment (ACK) is transmitted by the highest priority node in the set of successful receivers. This node acts as the relay node and suppresses all the other lower priority nodes attempts to forward the data packet. During instances of partial decoding, the nodes in the cluster coordinate among themselves to determine the set of most successful receivers and the corresponding highest priority node forwards the information. The transmission thus proceeds from hop-to-hop and continues until the message reaches $\mathcal{D}$.

In this paper two multihop transmission technique, conventional and cooperative transmission for hop-by-hop data transfer is considered. The transmission distance between source and destination nodes remain the same for both transmission techniques but it differs with respect to signal processing at the nodes. In conventional multihop transmission, the receiving nodes process the information transmitted by the node in the previous hop alone. This scheme only benefits from reduced pathloss. However, as the transmissions are broadcast by nature, the rateless coded packets are received by all the nodes within the transmission range of the transmitter. Hence, the receiving nodes can also receive (weak) signals from other transmitters in the previous hops and cooperatively decode from all the received information. Such cooperative transmission not only benefits from power savings by pathloss reduction, but also from the spatial diversity of the received signals. In this paper, only two diverse signals received from the previous two transmitting nodes are processed by the nodes for cooperative multihop transmission. The received encoded packets are buffered by the nodes until either the mutual information is sufficient to decode the entire source packets or the permissible transmission time has elapsed. Successful reception of entire transmitted packets is signalled by an acknowledgment (ACK) signal. The hard delay deadline causes some packets to be dropped/lost across the hop and only the successfully recovered packets are transmitted in the subsequent hops. This process is repeated until the message reaches $\mathcal{D}$.

III. Combinatorial Analysis of Delay Constrained Rateless Codes

Most of the real-time applications demand high bandwidth together with an acceptable QoS which cannot be guaranteed by the best-effort delivery networks such as DTNs. In this context, rateless coded DCNs can be employed to have a finite latency with a specified QoS. By constraining the number of transmissions, DCNs cannot guarantee complete data recovery as the received mutual information (depending on the channel conditions) may not be sufficient to recover the entire source packets. Rateless coded DCNs are thereby characterized by the reliability of information delivered to the destination, which in turn is determined by the packets lost across the number of hops. The performance of DCNs based on the average packet loss with finite number of transmissions (dependent on the permissible delay) for conventional and cooperative multihop transmission are analysed in this section. Rateless coding is performed on the p_k source packets by employing a uniform degree distribution, $\Omega(x)$. The present analysis is independent of the employed degree distribution and therefore the same technique can be extended to other (more practical) degree distributions, such as Robust Soliton distribution [6].

When transmissions are performed over a wireless channel with outage probability P_e^{pq} across nodes p and q, the number of encoded packets received unerased is random and is related to the number of transmissions, T. Consider exactly k packets are to be received in T transmissions, which implies the unerased reception of the final transmission, which occurs with a probability $(1-P_e^{pq})$. Then, the probability of receiving the k^{th} unerased encoded packet on the T^{th} transmission can be computed as

$$P_k^T\left(P_e^{pq}\right) = \left(1-P_e^{pq}\right)\binom{T-1}{k-1}\left(P_e^{pq}\right)^{T-k}\left(1-P_e^{pq}\right)^{(k-1)} \quad (4)$$

For a rateless coded system, the probability of successfully recovering p_k source packets is the probability of the corresponding $p_k \times i$ received generator matrix G to be of rank p_k, where i is the number of received unerased encoded packets. From the cumulative mass function (CMF) for successful decoding [38], the corresponding probability mass function (PMF) for an equiprobable binary generator matrix to be of rank p_k after receiving i unerased packets can be expressed as

$$P_{rank}^{p_k}(i) = \begin{cases} 0 & \text{if} \quad i < p_k \\ \prod_{j=0}^{p_k-1} \left(1 - 2^{j-i}\right) & \text{if} \quad i = p_k \\ \frac{2^{-i}(2^{p_k}-1)}{1-2^{p_k-i}} \prod_{j=0}^{p_k-1} \left(1 - 2^{j-i+1}\right) & \text{if} \quad i > p_k \end{cases} \quad (5)$$

For DCNs, the channel drop-off rate or the packet loss rate is defined as the ratio of packets lost per hop to the transmitted number of packets, which can be expressed as

$$\delta = \frac{p_k^j - \acute{p}_k^j}{p_k^j} \quad (6)$$

where p_k^j and $\acute{p}_k^j \in \{0 \text{ to } p_k^j\}$ represents the number of source packets and the instantaneous number of packets recovered in the j^{th} hop, respectively. The average number of packets that can be recovered ($\acute{p}_k^j$) is a function of the rank of the received generator matrix and can be computed by evaluating its expectation over the entire range, i.e.

$$\mathbb{E}[\acute{p}_k^j] = \sum_{p_s} p_s \Pr\left(\acute{p}_k^j = p_s\right) \quad (7)$$

where $\Pr(\acute{p}_k^j = p_s)$ is the PMF. For analytical simplicity, the number of packets decoded correctly (p_s) is assumed to be equal to the rank of the received generator matrix. In reality, the number of first degree encoded packets (coded packets that directly corresponds to source packets) obtained by the rateless decoding process will be less than or equal to the rank due to the linear combination of transmitted packets.

Analytical evaluation of DCNs based on the number of packets recovered for conventional and cooperative multihop systems are done by computing the corresponding probability mass functions as given below.

A. $\hat{p}_k^j$ for conventional multihop transmission

In order to compute the PMF of the number of packets recovered for conventional DCNs, there are two cases to be considered. For the first case, the receiver is able to decode correctly the entire source packets, i.e. the received generator matrix is full rank within the maximum permissible number of transmissions (L). Then, the corresponding PMF of decoding p_k^j packets transmitted along a channel with outage probability P_e^{pq} across the transmitting node p and receiving node q by the total probability theorem [31] can be computed as

$$\Pr\left[Rank(G) = p_k^j\right] = \sum_{l=p_k^j}^{L} \sum_{i=p_k^j}^{l} P_{rank}^{p_k^j}(i) P_i^l\left(P_e^{pq}\right) \quad (8)$$

The first summation is for the number of transmissions required to have a full rank matrix at the destination where the minimum limit is p_k^j (as at least p_k^j transmissions are required to decode p_k^j source packets from as many encoded packets). The second summation represents the reception of p_k^j or more unerased packets from corresponding source transmissions. Equation (8) corresponds to the probability of receiving a full rank generator matrix from the unerased encoded packets received from equivalent source transmissions.

For the second case, the receiver is unable to recover the entire transmitted packets within the specified duration. The receiver thereby decodes only a subset of p_k^j source packets within the maximum number of transmissions (L). By the total probability theorem, the PMF of decoding only $p_s(< p_k^j)$ packets within L source transmissions can be given as

$$\Pr[Rank(G) = p_s] = \sum_{i=p_s}^{L} P_{lr}\left(p_s, i, p_k^j\right) \times \binom{L}{i} \times (1 - P_e^{pq})^i \left(P_e^{pq}\right)^{(L-i)} \quad (9)$$

where $p_s \in \{0 \text{ to } (p_k^j - 1)\}$ and $P_{lr}(p_s, i, p_k^j)$ is the PMF of receiving a low rank submatrix (p_s) from a higher dimension matrix of order $p_k^j \times i$. The detailed steps to evaluate $P_{lr}(p_s, i, p_k^j)$ are shown in the Appendix. Equation (9) signifies the reception of a generator matrix of rank p_s from the i unerased encoded packets received from L source transmissions.

B. $\hat{p}_k^j$ for cooperative multihop transmission

For simplicity of analysis, only two cooperating nodes are considered and the first transmitting node is taken as $\mathcal{S}$ which communicates with the receiver $\mathcal{D}$ by cooperating with the second node $\mathcal{R}$. Based on the packet decodability, two cases exist for delay constrained cooperative transmission similar to conventional transmission. For the first case, $\mathcal{D}$ is able to decode the entire p_k^j source packets. Within this case, the entire encoded packets to decode the source packets can be received from $\mathcal{S}$ (via direct transmission, provided $\mathcal{R}$ is unable to decode with corresponding source transmissions), and hence no relaying gain is achieved. Then by the total probability theorem, the probability of receiving the entire encoded packets by direct transmission from L or less number of transmissions is similar to (8) and can be computed as

$$\Pr_{Direct}\left(L, P_e^{SR}, P_e^{SD}\right) = \sum_{l=p_k^j}^{L} \left(\sum_{i=p_k^j}^{l} P_{rank}^{p_k^j}(i) P_i^l\left(P_e^{SD}\right) \right) \times \left(1 - \Pr_{RCoop}\left(l-1, P_e^{SR}\right)\right) \quad (10)$$

where $\Pr_{RCoop}(l-1, P_e^{SR})$ is the probability of $\mathcal{R}$ entering into cooperation mode with $(l-1)$ or less number of transmissions along the SR link. The corresponding probability can be evaluated as

$$\Pr_{RCoop}\left(l-1, P_e^{SR}\right) = \sum_{T=p_k^j}^{l-1} \Pr_{p_k^j}\left(T, P_e^{SR}\right) \quad (11)$$

where $\Pr_{p_k^j}(T, P_e^{SR})$ is the probability that $\mathcal{R}$ has received p_k^j linearly independent encoded packets from T transmissions across the SR link (T basically captures the instant when the received generator matrix at $\mathcal{R}$ has reached full rank); which can be computed as

$$\Pr_{p_k^j}\left(T, P_e^{SR}\right) = \sum_{i=p_k^j}^{T} P_{rank}^{p_k^j}(i) P_i^T\left(P_e^{SR}\right) \quad (12)$$

Once the relay is able to decode, $\mathcal{R}$ cooperates with $\mathcal{S}$ in transmitting the rateless coded packets. Given that the transmissions at each node are constrained to a maximum of L, the maximum number of packets received at $\mathcal{D}$ is $2L$ (L packets from both $\mathcal{S}$ and $\mathcal{R}$). By the total probability

$$\Pr_{Coop}\left(L,P_e^{SR},P_e^{SD},P_e^{RD}\right)=\sum_{l=p_k^j}^{2L}\sum_{T_s=\left(max\left((l-L),p_k^j\right)\right)}^{L}\sum_{i=max(0,(l-L))}^{min((l-1),T_s)}\sum_{T_r=l-i}^{L}P_{rank}^{p_k^j}(l)\binom{T_s}{i}\left(1-P_e^{SD}\right)^i\left(P_e^{SD}\right)^{(T_s-i)}\times P_{l-i}^{T_r}\left(P_e^{RD}\right)\times\Pr_{p_k^j}\left(T_s,P_e^{SR}\right) \tag{13}$$

$$\Pr_{Direct}\left(p_s,L,P_e^{SR},P_e^{SD},P_e^{RD}\right)=\sum_{l=p_s}^{L}P_{lr}\left(p_s,l,p_k^j\right)\binom{L}{l}\left(1-P_e^{SD}\right)^l\left(P_e^{SD}\right)^{(L-l)}\times\left[\left(1-\Pr_{RDec>p_s}\left(L,P_e^{SR}\right)\left(1-\left(P_e^{RD}\right)^L\right)\right)\right] \tag{16}$$

$$\Pr_{Coop}\left(p_s,L,P_e^{SR},P_e^{SD},P_e^{RD}\right)=\sum_{l=p_s}^{2L}P_{lr}\left(p_s,l,p_k^j\right)\sum_{i=max(0,l-L)}^{min(l-1,L)}\binom{L}{i}\left(1-P_e^{SD}\right)^i\left(P_e^{SD}\right)^{(L-i)}\times$$
$$\left(\binom{L}{l-i}\left(1-P_e^{RD}\right)^{l-i}\left(P_e^{RD}\right)^{(L-l+i)}\times\Pr_{RDec>(p_s+1)}\left(L,P_e^{SR}\right)+\Pr_{RDec}\left(p_s,L,P_e^{SR}\right)\times\sum_{v=(l-i)}^{L}P_v^L\left(P_e^{RD}\right)\right) \tag{18}$$

theorem, the probability of receiving the encoded packets through cooperation can be computed as in (13), where the first summation represents the reception of l encoded packets. The second and third summation represents the reception of i unerased encoded packets from T_s source transmissions, where the limits are obtained by considering all probable combinations. The final summation represents the number of relay transmissions required for receiving the remaining $(l-i)$ unerased encoded packets. Equation (13) can be interpreted as the probability for the cooperative reception of p_k^j linearly independent packets from l encoded packets obtained from T_s and T_r, source and relay transmissions, respectively.

By using (10) and (13), the PMF of the number of packets required for receiving a full rank matrix at $\mathcal{D}$ with cooperation in DCNs can be given as

$$\Pr\left[Rank(G)=p_k^j\right]=\Pr_{Direct}\left(L,P_e^{SR},P_e^{SD}\right)+\Pr_{Coop}\left(L,P_e^{SR},P_e^{SD},P_e^{RD}\right) \tag{14}$$

For the second case, $\mathcal{D}$ is able to correctly decode a subset p_s $(< p_k^j)$ of source packets in the j^{th} hop. The encoded packets for decoding p_s packets can be received from $\mathcal{S}$, $\mathcal{R}$, or a combination of both. The entire encoded packets for decoding p_s packets can be received from $\mathcal{S}$ when either the SR channel is unreliable ($P_e^{SR} \gg P_e^{SD}$, so $\mathcal{R}$ is unable to decode more than p_s packets from L source transmissions) or the entire transmissions from relay are erased due to an unreliable RD link. The probability of $\mathcal{R}$ decoding more than p_s packets across the SR link in L transmissions by $\mathcal{S}$ can be evaluated as

$$\Pr_{RDec>p_s}\left(L,P_e^{SR}\right)=\sum_{i=p_s}^{p_k^j-1}\sum_{j=i}^{L}P_{lr}\left(i,j,p_k^j\right)\times\binom{L}{j}\left(1-P_e^{SR}\right)^j\left(P_e^{SR}\right)^{(L-j)}+\Pr_{RCoop}\left(L,P_e^{SR}\right) \tag{15}$$

where the first term represent the recovery of p_s or more packets (but less than the entire source packets) from L source transmissions, and the second term represents successful decoding of all the source packets at $\mathcal{R}$ (receiving a full rank generator matrix at $\mathcal{R}$). Then using (15), the probability of decoding p_s packets at $\mathcal{D}$ by direct transmission can be evaluated as in (16).

The cooperative scenario of receiving encoded packets at $\mathcal{D}$ from both $\mathcal{S}$ and $\mathcal{R}$ has to be addressed in two contexts. First one is the occurrence of the event that $\mathcal{R}$ has recovered more than (p_s+1) packets from L source transmissions which can be computed similar to (15). The second event is the decoding of exactly p_s source packets at $\mathcal{R}$ from p_s or more unerased encoded packets received from L source transmissions, which can be evaluated as

$$\Pr_{RDec}\left(p_s,L,P_e^{SR}\right)=\sum_{i=p_s}^{L}P_{lr}\left(p_s,i,p_k^j\right)\times\binom{L}{i}\left(1-P_e^{SR}\right)^i\left(P_e^{SR}\right)^{L-i} \tag{17}$$

Then, the corresponding probability for the cooperative scenario can be computed using (15) and (17), and is given in (18). In equation (18), the first summation represents the reception of l unerased encoded packets from L transmissions by $\mathcal{S}$ and $\mathcal{R}$. The second summation signifies the reception of i packets out of the l unerased encoded packets at $\mathcal{D}$ from L source transmissions. The remaining $(l-i)$ unerased encoded packets are received from $\mathcal{R}$. Relay $\mathcal{R}$ cooperates with transmission only when it has successfully decoded p_s or more source packets. The $(l-i)$ packets are received from L transmission by $\mathcal{R}$ only when it has decoded more than p_s source packets. Equation (18) can be interpreted as the probability of receiving p_s linearly independent encoded packets at $\mathcal{D}$. The PMF of the number of packets required for correctly decoding p_s source packets at $\mathcal{D}$ with the number of transmissions by $\mathcal{S}$ and $\mathcal{R}$ constrained to L can be given from (16) and (18) as

$$\Pr\left[Rank(G)=p_s\right]=\Pr_{Direct}\left(p_s,L,P_e^{SR},P_e^{SD},P_e^{RD}\right)+\Pr_{Coop}\left(p_s,L,P_e^{SR},P_e^{SD},P_e^{RD}\right) \tag{19}$$

Therefore, the PMF of recovering p_s source packets when the channel usage per node is constrained to a maximum of L can be evaluated using (8) and (9), (14) and (19) for conventional and cooperative relaying cases as

$$\Pr\left(\acute{p}_k^j = p_s\right) = \begin{cases} \Pr\left[Rank(G) = p_k^j\right] & \text{if } p_s = p_k^j \\ \Pr\left[Rank(G) = p_s\right] & \text{else} \end{cases} \quad (20)$$

By evaluating (7) using (20), $\hat{p}_k^j$ the average number of

IV. Implicit Feedback Assisted Multihop Transmission Scheme

By constraining the number of transmissions DCNs ensures a finite latency at the cost of system reliability. As observed in the previous analysis, some packets are lost across the hops in rateless coded DCNs based on the channel conditions and there should be some kind of recovery mechanism to improve the reliability of such networks. A naïve method is to use a dedicated feedback channel to communicate the lost packets across the hops and selectively retransmit those packets. However, the use of dedicated feedback channels to signal the lost packets will lower the throughput of the entire system. The spectral efficiency of multihop networks can be enhanced by minimizing traffic in the feedback links. This motivates the extension of the spectrally efficient transmission scheme proposed in [24] into DCNs. The crux of the idea is to minimize the required feedback by exploiting the broadcast nature of wireless transmissions. Earlier in [39] it was revealed how such a feedback channel can optimize the throughput in cooperative networks. To the best of authors' knowledge, the approach of using such an implicit feedback channel in rateless coded DCNs is first of its kind.

As explained in Section II, an ACK is transmitted during instances of total packet recovery. For all other instances, the transmission proceeds for the maximum permissible amount to constrain the total delay. Non-reception of an acknowledgment within the permissible time will apprise the transmitter of the packet loss. During such instances, the transmitter utilises the proposed implicit feedback assisted multihop transmission scheme to enhance the reliability of the network. The focus of this paper is on maintaining the reliability of multihop networks by constraining to the average delay of the network. The average delay constraint can be thought of as a QoS guarantee and can be achieved by processing only a subset (or none) of the lost packets based on data recovery at the nodes to maintain the required performance.

The transmission strategy of the proposed scheme for a conventional multihop network comprising of nodes N_T and N_{T+1}; and cooperative multihop network comprising of nodes N_T, N_{T+1}, and N_{T+2} illustrated in Fig. 2 is explained in Algorithm 1. For the proposed scheme, transmission is initiated by $\mathcal{S}$ and all nodes within its transmissions range receive the information. For the cooperative scenario, the nodes process the information received from previous two transmitting nodes to enhance the reliability. Conversely, the nodes process the information from only a single transmitting node for conventional transmission. The encoded packets are transmitted for the permissible channel usage or till any node in the receiving set successfully decodes the entire source packets. Total data recovery is signalled by transmitting an ACK signal and that node takes over the transmission. The present network considers non-reception of ACKs as negative acknowledgments and the best node in the next hop takes

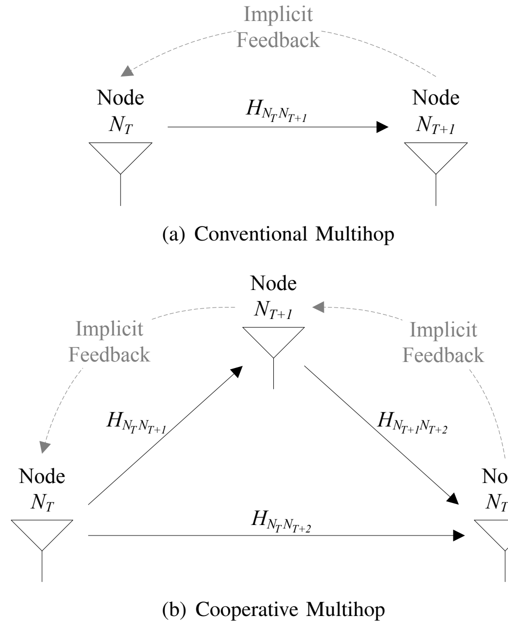


Fig. 2. Network configuration of the proposed scheme with the best relay node as the transmitter at each hop.

over the transmission during such instances. In such cases (non-reception of an ACK), the eavesdropping flag (E_d) is set to 1 and the initial source node eavesdrop the subsequent transmission. As the transmissions are broadcast by nature, the transmitting node in the previous hop listens to the present transmission and determines the lost packets by comparing its source packets with those received. The successfully recovered source packets can be determined by verifying the header information of the rateless coded packets as it contains the indices of packets that were XORed to generate the current packet. After determining the lost packets, these packets are included in the next block of information processed by this node which improves the hop-by-hop reliability of the network. However, the unreliable nature of the wireless medium will result in cases when none of the information packets are recovered by any of the nodes in the next hop. During such instances, the nodes remain silent and the eavesdropping node determines that none of the nodes in the receiving set has recovered any information. The transmission proceeds by lowering the code-rate of the outer error-correction code to ensure a reliable data flow. This will effectively lower the throughput of the network. However, adapting the code-rate of the error-correction code with respect to channel conditions will enhance the spectral efficiency and maintain hop-by-hop data flow in multihop networks [24]. This process proceeds from hop-to-hop till the message is delivered to $\mathcal{D}$.

From the above description it can be inferred that nodes need to eavesdrop only during the non-reception of an ACK signal. Those packets that are not recovered during the transmission process are included with the next block of information. In short, the system reliability is guaranteed hop-by-hop by integrating the implicit feedback channel to rateless coded DCNs. However, the overhead for such a scheme is the bits that need to be appended to the header of each rateless coded packet to identify the encoded source packets. This will

Algorithm 1: Transmission algorithm of the proposed implicit feedback assisted multihop transmission scheme

Initialization: Transmit Node: N_T, Receiver Nodes: $N_R \in \{N_{T+1}, N_{T+2}\}$ and $E_d = 0$;
while *delay* < *permitted* **do**
  Node N_T transmits encoded packets;
  if $E_d = 1$ **then**
    Node N_{T-1} eavesdrop to determine the lost packets and appends those to the next set;
  end
  if *ACK from node* N_R **then**
    Proceed to initialization step and set $N_T = N_R$, $N_R \in \{N_{T+1}, N_{T+2}\}$ and $E_d = 0$;
  end
end
Proceed to initialization step and set $N_T = N_{T+1}$, $N_R \in \{N_{T+2}, N_{T+3}\}$ and $E_d = 1$;
This process is repeated till $N_R \in \mathcal{D}$

require an average of $\Theta(\log_2 p_k)$ bits to be appended to each rateless coded packet. Even though the overhead introduced by these additional bits is considerably smaller compared to the information size ($p_k \ll b$), the lost packet reprocessing can be mitigated by selecting only appropriate number of source packets for rateless coding. This selection process is based on the channel conditions and the permissible maximum channel usage. Optimum number of source packets will lower the processing complexity at the nodes but requires the knowledge of average channel conditions. One way to ensure this is by utilising the hop distance information.

Optimum Source Packets Per Hop

The number of packets that need to be reprocessed by the transmitting node in DCNs is dependent on the link quality and the permissible latency. With rateless coding, the receiver can successfully decode the information from slightly more encoded packets than the p_k^j source packets [6], [7]. Thereby, the number of source packets p_k^j selected for hop j is very critical. A large p_k^j will result in more source packets being reprocessed, whereas a small p_k^j increases the queuing delay at the transmitting node which also degrades the system performance. Therefore, the number of source packets processed should be optimized based on the link quality and the permissible latency of the network. The average number of source packets $\overline{P}_k^j$, that can be decoded per hop is random which should be optimized based on the outage probability across the links and the maximum permissible channel usage. By considering all possible cases, the average number of packets decodable for hop j ($\overline{P}_k^j$) can be computed as

$$\overline{P}_k^j = \mathbb{E}\left[P_k^j\right] = \sum_{p_k^j} p_k^j \Pr\left(P_k^j = p_k^j\right) \tag{21}$$

where P_k^j represents the number of source packets that can be decoded successfully at hop j and $\Pr(P_k^j = p_k^j)$ is the corresponding PMF. The average number of packets that need to be encoded to minimize packet drop-off rate per hop for conventional and cooperative transmission are evaluated in the following section.

A. $\overline{P}_k^j$ for conventional multihop transmission

For conventional multihop transmission, the nodes process the encoded packets received from only one transmitting node. Then, the probability of receiving l unerased encoded packets from L transmissions can be given as

$$\Pr\left(l, P_e^{pq}\right) = \binom{L}{l} \left(1 - P_e^{pq}\right)^l \left(P_e^{pq}\right)^{(L-l)} \tag{22}$$

Given that l encoded packets are received at $\mathcal{D}$, the probability of decoding p_k^j source packets in hop j of a rateless coded DCN is the probability that the received $p_k^j \times l$ generator matrix is full ranked. Consequently, the PMF of decoding p_k^j source packets in a conventional multihop network constrained by maximum of L transmissions can be expressed as

$$\Pr_{Direct}\left(P_k^j = p_k^j\right) = \sum_{l=p_k^j}^{L} \Pr\left(l, P_e^{pq}\right) \times P_{rank}^{p_k^j}(l) \tag{23}$$

B. $\overline{P}_k^j$ for cooperative multihop transmission

For the cooperative scenario, the source packets can be received from either of the transmitting nodes $\mathcal{S}$ or $\mathcal{R}$, or a combination of both. The number of packets that can be recovered is dependent on the channel conditions indicated by the probabilities P_e^{SR}, P_e^{SD} and P_e^{RD}. For the case when $\mathcal{D}$ receives a larger fraction of encoded packets from $\mathcal{S}$ i.e., $P_e^{SD} \ll P_e^{SR}$ owing to a reliable SD link, the PMF is similar to the one obtained in (23). The maximum number of packets decodable at $\mathcal{D}$ with cooperation from $\mathcal{R}$ will be a function of the packet decodability at $\mathcal{R}$, which can be evaluated using (21) and (23) with $P_e^{pq} = P_e^{SR}$. The number of packets decodable at $\mathcal{D}$ for such a cooperative scenario is dependent on the RD link. The PMF of decoding p_k^j source packets where the encoded packets can be received from $\mathcal{S}$ and $\mathcal{R}$ in a delay constrained scenario can be computed as in (24). In equation (24), the first summation represents the reception of a full rank matrix from L transmissions by $\mathcal{S}$ and $\mathcal{R}$. The second summation represents the reception of i unerased packets from L source transmissions. The above equation can be interpreted as the probability of receiving l linearly independent encoded packets from L transmissions by $\mathcal{S}$ and $\mathcal{R}$. From these l packets received at $\mathcal{D}$, i packets are from L source transmissions and the remaining $(l - i)$ packets are received from L relay transmissions, provided $\mathcal{R}$ has decoded p_k^j or more packets from corresponding source transmissions. By using (23) and (24), the PMF of decoding p_k^j source packets in a cooperative multihop network constrained by maximum of L channel usage per node can be expressed as

$$\Pr\left(P_k^j = p_k^j\right) = \Pr\left[P_e^{SD} < P_e^{SR}\right] \times \Pr_{Direct}\left(P_k^j = p_k^j\right) + \left(1 - \Pr\left[P_e^{SD} < P_e^{SR}\right]\right) \times \Pr_{Coop}\left(P_k^j = p_k^j\right) \tag{25}$$

The expected number of packets $\overline{P}_k^j$ decodable for DCNs can be computed by substituting (23) and (25) into (21) for conventional and cooperative multihop transmission, respectively.

$$\Pr_{Coop}\left(P_k^j = p_k^j\right) = \sum_{l=p_k^j}^{2L} P_{rank}^{p_k^j}(l) \sum_{i=\max(0,l-L)}^{\min(l,L)} \binom{L}{i} \left(1-P_e^{SD}\right)^i \left(P_e^{SD}\right)^{(L-i)} \binom{L}{l-i} \left(1-P_e^{RD}\right)^{(l-i)} \left(P_e^{RD}\right)^{(L-l+i)} \left(\sum_{v=p_k^j}^{L} \sum_{z=p_k^j}^{v} P_{rank}^{p_k^j}(z) \binom{L}{z} \left(1-P_e^{SR}\right)^z \left(P_e^{SR}\right)^{(L-z)}\right) \quad (24)$$

V. Numerical Results

In this section, the numerical and empirical results of the proposed scheme are analysed. The nodes comprising the multihop network are connected with each other through wireless links with n hops and w nodes per cluster. Two nodes per cluster ($w = 2$) is considered for simulations with the best relay forwarding the information. The intermediate nodes are considered to be fixed and collinear across the n hops. The intermediate nodes thus divide the direct path between source and destination nodes into equal-length segments. Such a configuration serves to validate the analytical results and illustrate the benefits that can be realized with an optimal placement of the nodes. For a fair comparison with direct transmission, the overall distance of all hops is normalized to the distance between the source and destination nodes. The pathloss between nodes are computed as discussed in Section II and a typical pathloss exponent (α) value of 3.5 is chosen to model an urban environment.

Monte Carlo simulation exercise based on the realistic communication environment discussed in Section II is performed. Noiseless ACK is assumed for simplicity. Without loss of generality, the transmitter and the receiver are supposed to use a deterministic random generator for rateless coding. Thus, the receiver can easily synchronise with the transmitter. Further, the packet length (b) is set to 256 bits with the number of source packets (p_k) taken as 12. The number of rateless coded transmissions (L) is limited to 20. For analytical simplicity, BPSK with coherent detection for transmission and reception is utilised. For simulations, channel coding is not utilised and hence the rate (R_c) is taken as 1. The packets received through the channel are considered as erased based on its outage characteristics as specified in Section II. The unerased encoded packets are retained in the receiver buffer till either sufficient mutual information is received to recover the entire source packets (received generator matrix is full rank) or maximum delay has elapsed. The performance of the n-hop network is quantified with respect to the average received SNR across the direct link (SD link).

A. Throughput of the multihop network

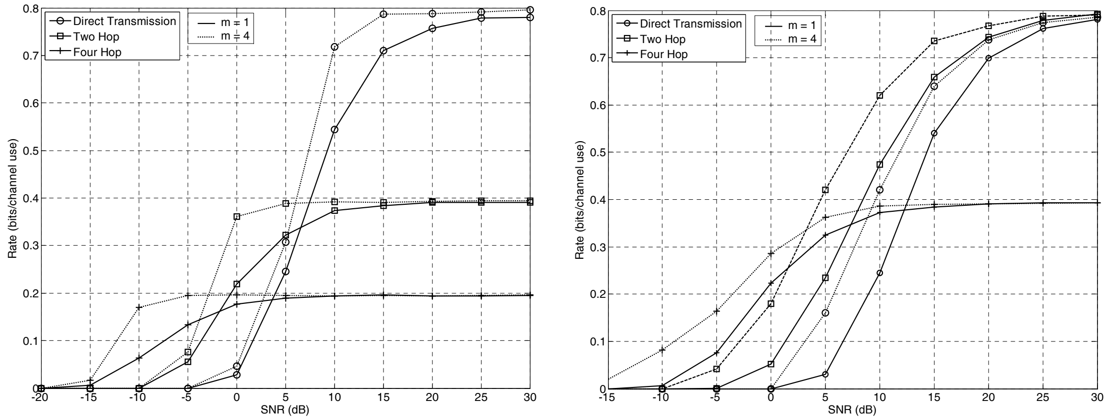
An important measure of the performance of the system under consideration is the throughput which is defined as the ratio of decoded bits to the total number of bits transmitted. The number of bits that are transmitted is dependent on the channel characteristics and the total delay permissible by the network. Figure 3 illustrates the throughput performance of the analyzed rateless coded multihop system for conventional and cooperative transmission. It can be seen that direct transmission is feasible only at high SNRs especially when the source and destination are separated by a large distance. The performance can be improved further by multihopping. It is also observed that the optimum number of hops is dependent on the operating SNR and channel characteristics. Higher number of hops are required to achieve the same performance in a Rayleigh fading channel ($m = 1$) as compared to Nakagami-4 fading channels, which has less severe fading characteristics. Further, as SNR improves lesser number of hops is optimal due to the accumulative effect of the number of bits required for decoding at each hop. Cooperative transmission benefits from the transmit diversity and coding gain, which improves the throughput of such networks. It is observed that two-hop cooperative transmission offers a higher throughput than direct transmission (diversity benefit).

B. Packet reception rates for delay constrained transmission

DCNs limit the number of rateless coded transmissions or channel usage (L) to a specific value in order to constrain the overall delay to that permissible by the network. Although this takes care of the issues related to infinite delay when the link between nodes is unreliable, it does not guarantee the successful delivery of the entire set of p_k source packets. As explained in Section III, there exists a trade-off between overall delay and reliability. Figure 4 illustrates the variation of percentage of packets received with SNR for conventional and cooperative multihop DCNs. The numerical results (dashed-dotted lines) obtained in Section III for conventional (direct transmission) and cooperative multihop (two hop) DCNs are compared with simulated performance (straight lines) in a Rayleigh fading channel ($m = 1$). The numerical analysis derived in this paper serves as the upper bound, as the number of packets decodable will be less than or equal to the rank of the received generator matrix due to the linear combination of packets transmitted. It is observed that packet reception improves with the reliability of links (as SNR increases). Figure 4 reveals that less severe fading environment ($m = 4$) results in better reliability. Further, it can be observed that more number of nodes forwarding the information results in an improvement in link reliability, which alleviates the packet reception rate, due to the collinear placement of nodes. A significant variation of packet reception is observed with cooperation as the probability of unreliable links decreases with increasing number of links available for transmission (diversity benefit). Cooperation among nodes thereby results in better reliability.

C. Average end-to-end delay performance of the proposed scheme

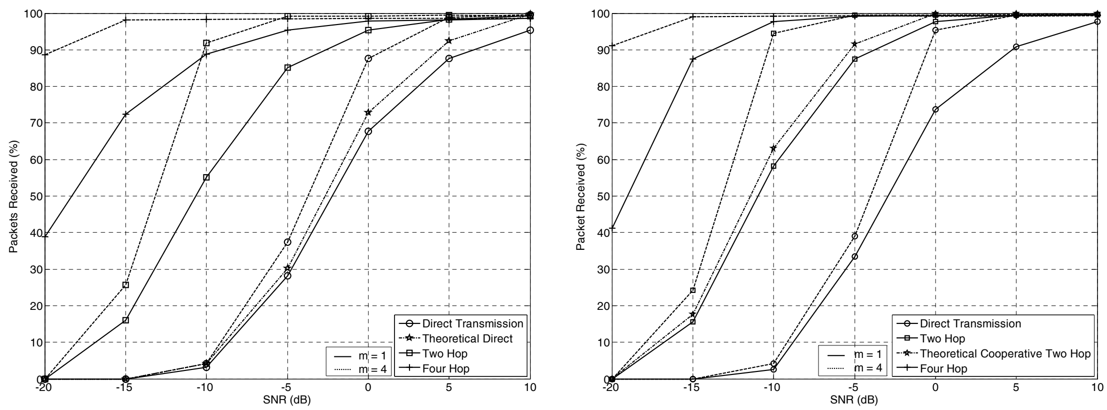
The end-to-end delay performance can be evaluated by computing the number of encoded packets transmitted for



(a) Conventional Multihop

(b) Cooperative Multihop

Fig. 3. Throughput performance of the proposed multihop system with varying number of hops compared with received SNR across SD link in a Rayleigh ($m = 1$) and Nakagami-4 fading channel.



(a) Conventional Multihop

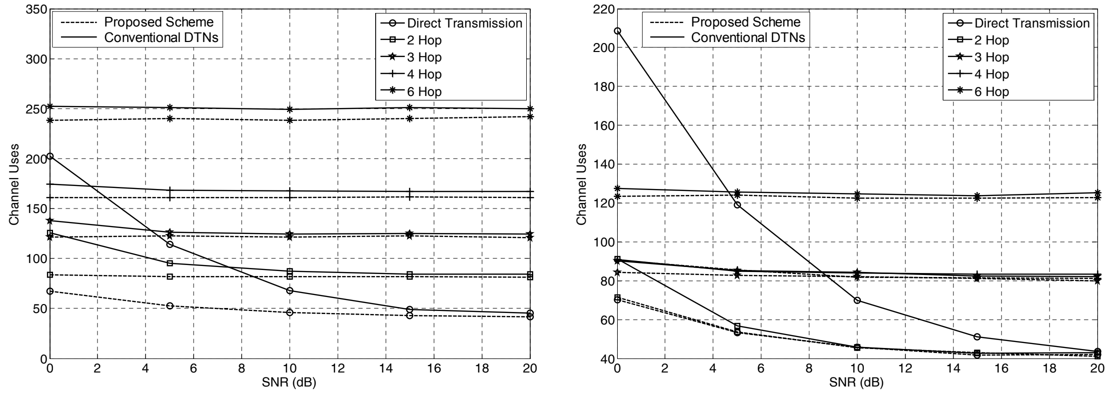
(b) Cooperative Multihop

Fig. 4. Reliability of information retrieval in DCNs expressed as percentage of packets retrieved for corresponding SNRs in a Rayleigh ($m = 1$) and Nakagami-4 fading channel.

successful decoding, which is measured by the channel usage. One encoded packet is transmitted per channel use. The results on average end-to-end delay performance with varying SNRs and varying number of hops are plotted in Figures 5 and 6, respectively.

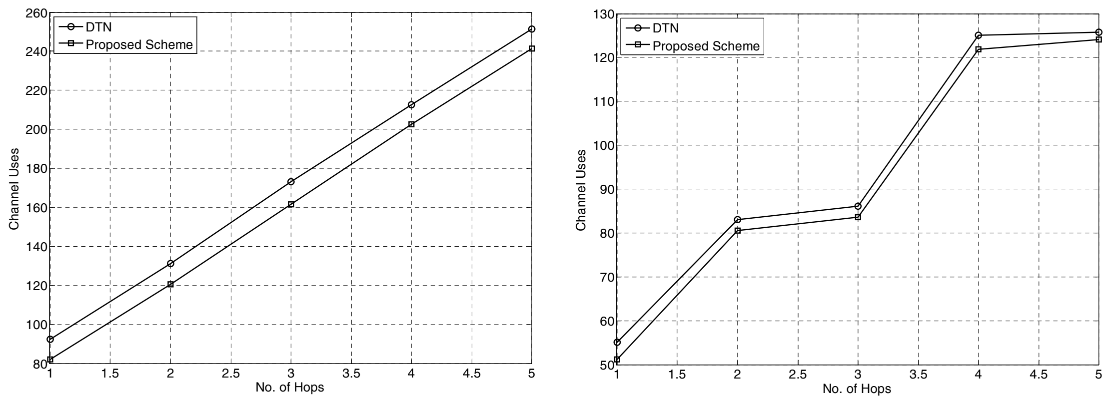
Figure 5 illustrates the variation of channel usage for the proposed implicit feedback assisted rateless coded DCNs (broken lines) and DTNs (straight lines) for corresponding SNRs in a Rayleigh fading channel ($m = 1$). It can be observed that the average channel usage for the proposed scheme is significantly lower than that of DTNs, especially at low SNRs. This can be attributed to the fact that the proposed scheme considers hop-by-hop optimization by constraining the number of transmissions per hop and the packets lost across the hops are transmitted in the subsequent transmissions. In DTNs the number of transmissions required is very large, when the SNR is low or when the links are least reliable. It can be observed that at these low SNRs multihopping is the best method of communication. As the link reliability increases (at high SNRs), the number of transmissions required decreases and the optimal number of hops for transmission also decreases. At very high SNRs, most of the packets are received unerased which results in the reception of sufficient mutual information from direct transmissions and it becomes optimal. This further result in both DTNs and the proposed scheme to utilise similar number of transmissions at high SNRs. Multihopping becomes less attractive at high SNRs as it has a accumulative effect on the number of slots required. At high SNRs i^{th} hop and $(i+1)^{\text{th}}$ hop, where i is even, will yield almost similar performance, as the receiving node cooperates with only previous two transmitting nodes. This can be further verified by evaluating the performance with respect to varying number of hops, which is illustrated in Fig. 6.

The average end-to-end delay performance of the proposed scheme and DTNs for varying number of hops in a Rayleigh fading channel ($m = 1$) at a fixed SNR of 5 dB is illustrated in Fig. 6. The channel usage for conventional multihop transmission shows a linear variation as it processes the encoded packets received from only a single transmitting node. This results in the channel usages to have a cumulative effect with the number of hops. However for the cooperative scenario, the encoded packets received from previous two transmitting nodes are utilised for decoding at the receiver. The diversity benefit offered by such cooperation among nodes yields a better performance. At high SNRs when the direct link is very reliable, the cooperating nodes are not utilised and thereby i^{th}



(a) Conventional Multihop (b) Cooperative Multihop

Fig. 5. Average end-to-end delay performance of the proposed scheme and DTNs for varying SNRs in a Rayleigh ($m = 1$) fading channel.



(a) Conventional Multihop (b) Cooperative Multihop

Fig. 6. Average end-to-end delay performance of the proposed scheme and DTNs for varying hops at an SNR of 5 dB in a Rayleigh ($m = 1$) fading channel.

hop and $(i+1)^{\text{th}}$ hop, where i is even, will yield almost similar performance.

D. Performance with optimum number of source packets

In DCNs, the number of source packets decodable is dependent on the permissible maximum number of rateless coded packets that can be transmitted (L) within the permissible delay and the link quality between the nodes. Packet loss in such networks will result in the reprocessing of these packets along with the next block of information to enhance the reliability. This will increase the processing overhead at the nodes. The number of packets that need to be reprocessed can be minimized by optimising the number of source packets based on the channel conditions and permissible number of encoded transmissions (L), as discussed in Section IV. Each node utilizes the average channel outage probability and maximum channel usage (L) parameters to determine the average number of packets decodable per hop by evaluating (21) using (23) and (25). The corresponding theoretical average number of packets decodable for a particular SNR when the maximum permissible coded transmissions are limited to 15 is illustrated in Table I. It is observed that lesser number of packets is transmitted by conventional multihop transmission at low SNRs owing to the low capacity links. The diversity benefit offered by cooperation among nodes permits the transmission of more packets in cooperative multihop transmission.

TABLE I
THEORETICAL OPTIMAL NUMBER OF SOURCE PACKETS FOR VARYING SNRS FOR CONVENTIONAL DIRECT AND COOPERATIVE TWO-HOP TRANSMISSION IN A RAYLEIGH ($m = 1$) FADING CHANNEL

SNR (dB)	Conventional	Cooperative
0	2	9
5	6	9
10	8	9
15	9	10
20	9	10

Figure 7 illustrates the average percentage of packets that need to be reprocessed with the optimal number of source packets computed in Table I. It can be observed that the number of packets that need to be reprocessed is reduced by adaptively selecting the number of source packets. As observed in Figure 7, packet reprocessing can be eliminated by selecting about 75% of the theoretical optimal number of source packets. This is due to the fact that the number of packets decoded at the receiver will be less than or equal to the rank of the received generator matrix. Hence, the optimal packets derived in Section IV serves as the upper bound, which justifies the selection of a lower number of source packets to minimize packet reprocessing at the nodes.

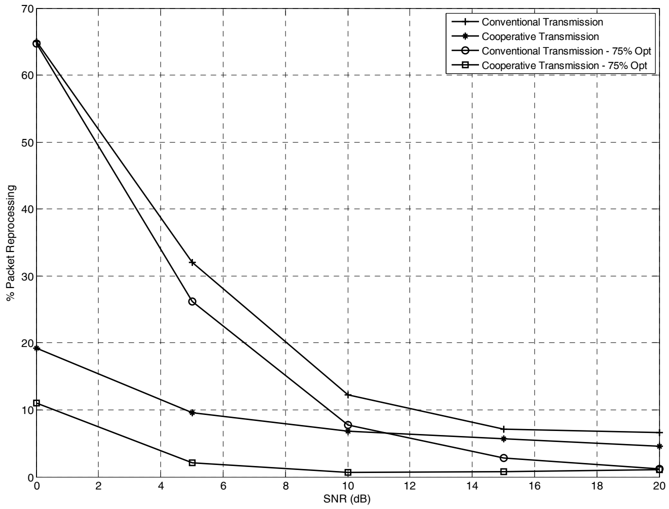


Fig. 7. Average percentage of packets that need to be reprocessed with number of source packets equal to the theoretical optimal value computed in Table I in a Rayleigh ($m = 1$) fading channel.

VI. CONCLUSION

In this paper, the impact of delay constraints on multihop networks employing rateless codes is analysed. The number of packets dropped across hops is dependent on the delay and the link quality between nodes. Increasing the number of hops between a fixed source and destination node necessarily improves the reliability of multihop network. But it brings down the throughput of the system as the number of encoded packets required for decoding has an accumulative effect with increasing number of hops. The number of transmissions per hop and the number of hops should be selected appropriately to minimize the total delay and packet loss.

A multihop reliable data transmission scheme is proposed in this context to minimise the packet loss in such delay constrained networks. The proposed scheme exploits the broadcast nature of the wireless medium to detect the packets lost across the hop which are then reprocessed with the next set of data packets. Further, the number of data packets processed at the transmitter is optimised for the delay and link quality to reduce reprocessing at the nodes in the previous hop. Numerical results obtained in this paper are verified through extensive simulation studies which show that the spectrally efficient proposed scheme can achieve high reliability with low end-to-end delays.

APPENDIX

PMF OF RECEIVING A LOW RANK SUBMATRIX FROM A HIGHER DIMENSION MATRIX

Let $GF(2)$ be a Galois Field of size 2. Each new fountain coded packet is associated with an encoding vector over $GF(2)$ of dimension p_k, where each packet is obtained by the linear combination of p_k source packets as explained in Section II. The p_k source packets can be recovered if p_k linearly independent packets are received i.e., the $Rank(G)$ is p_k.

As an all zero encoding vector does not contain any information, it is assumed that an all zero column is not generated by the encoder. Then, it is certain to receive a rank 1 matrix from a non-zero encoding set with one column. With the next encoding set there are 2 columns that are dependent among them, then the probability of having two linearly independent columns can be obtained as $\left(1-\frac{1}{2^{p_k}}\right)$ [40]. By extending this property for R_p received vectors, the probability of receiving l independent columns with $l=p_k=R_p$ is

$$P_{lr}(l,R_p,p_k) = \prod_{i=0}^{l-1}\left(1-\frac{2^i-1}{2^{p_k}}\right) \tag{26}$$

Then as R_p increases, there are $2^{\left(R_p-l\right)}$ columns that are dependent among them. Then, the probability of having l linearly independent columns from R_p received vectors is

$$P_{lr}(l,R_p,p_k)=C\times 2^{\left(R_p-l\right)-p_k}\times\prod_{i=0}^{l-1}\left(1-\left(2^i-1\right)2^{-p_k}\right) \tag{27}$$

where C is a constant. By evaluation, the constant C can be approximated as a geometric series with a common ratio of 2 and initial term to be $2^{\left(R_p-l\right)}$. The number of terms between initial and final terms can be computed as

$$k = (R_p - l)\times(l-1)+1 \tag{28}$$

Thus, the constant C can be computed as

$$C \approx 2^{\left(R_p-l\right)}\times\left(2^k-1\right) \tag{29}$$

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