

Sparse least mean fourth algorithm for adaptive channel estimation in low signal-to-noise ratio region

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SUMMARY

Both least mean square (LMS) and least mean fourth (LMF) are popular adaptive algorithms with application to adaptive channel estimation. Because the wireless channel vector is often sparse, sparse LMS-based approaches have been proposed with different sparse penalties, for example, zero-attracting LMS and L_p -norm LMS. However, these proposed methods lead to suboptimal solutions in low signal-to-noise ratio (SNR) region, and the suboptimal solutions are caused by LMS-based algorithms that are sensitive to the scaling of input signal and strong noise. Comparatively, LMF can achieve better solution in low SNR region. However, LMF cannot exploit the sparse information because the algorithm depends only on its adaptive updating error but neglects the inherent sparse channel structure. In this paper, we propose several sparse LMF algorithms with different sparse penalties to achieve better solution in low SNR region and take the advantage of channel sparsity at the same time. The contribution of this paper is briefly summarized as follows: (1) construct the cost functions of the LMF algorithm with different sparse penalties; (2) derive their lower bounds; and (3) provide experiment results to show the performance advantage of the propose method in low SNR region. Copyright © 2013 John Wiley & Sons, Ltd.

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KEY WORDS: least mean fourth (LMF); zero-attracting LMF (ZA-LMF); reweighted ZA-LMF (RZA-LMF); L_p -norm LMF (LP-LMF); L_0 -norm LMF (L0-LMF); adaptive channel estimation

1. INTRODUCTION

Broadband transmission has become an indispensable technique for high-speed wireless communications [1, 2]. Accurate channel estimation is required at the receiver for coherence detection. Because of broadband high-speed data transmission, channel impulse response is often sparse, which is supported by a few dominant taps [3–5]. A typical example of sparse channel is shown in Figure 1. Many sparse channel estimation methods [6–13] have been proposed in the last decade. Unlike these proposed linear methods, we investigate adaptive channel estimation methods in this paper. Because least mean square (LMS)-based adaptive channel estimation or system identification has attracted many attention because of its simplicity of implementation, to exploit the sparse channel structure information, various sparse LMS algorithms have been proposed [14–17]. In [14], zero-attracting LMS (ZA-LMS) has been proposed by Chen *et al.*, using L_1 -norm sparse penalty on mean square error (MSE). However, ZA-LMS only exploits limited sparse information. Motivated by the reweight L_1 -norm sparse signal recovery algorithm [18], Chen *et al.* proposed an improved algorithm, which is termed as reweighted zero-attracting LMS

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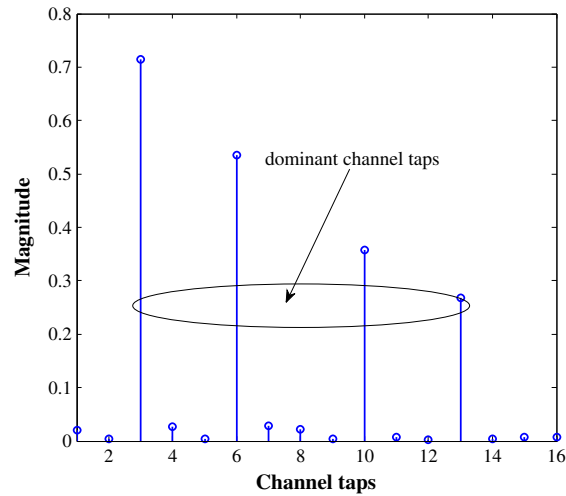


Figure 1. An example of an FIR-based sparse channel.

(RZA-LMS) algorithm [14], to further exploit the sparsity. Beside that L_1 -norm sparse penalty on LMS, Taheri and Vorobyov proposed L_p -norm LMS algorithm [15] to improve the sparse channel estimation performance. In addition, Gu *et al.* proposed L_0 -norm LMS (L0-LMS) algorithm in [16] and provided performance analysis in [19]. Later, Gui *et al.* applied the L0-LMS algorithm on adaptive sparse channel estimation [17] to further improve estimation performance. Even though these proposed methods can take the channel sparsity effectively, estimation performance is unstable in low signal-to-noise ratio (SNR) region. The common drawback of these algorithms is that LMS is sensitive to the scaling of input signal and noise, especially in low SNR region.

To deal with two drawbacks of the LMS-based sparse algorithms, adaptive algorithms using higher order moments of the estimation error have been proposed and proved to perform better than LMS in terms of the MSE. A typical algorithm is the least mean fourth (LMF) [20] algorithm using fourth-order moments of estimation error signal. The LMF-based adaptive channel estimation is shown in Figure 2. Comparing with LMS algorithm, LMF can mitigate random signal scaling and the effect of strong noise. In addition, LMF algorithm outperforms the LMS algorithm in achieving a better tradeoff between convergence speed and estimation performance. The reason for this is that the stability of LMF depends on both input signal power and noise power, whereas the stability of LMS-based algorithms depend only on the input signal power. In the low SNR region, LMF algorithm can achieve better estimator using small gradient descend step-size [18]. Therefore, the obvious advantage of LMF is that it can achieve better performance, and the disadvantage of LMF algorithm is the slow convergence speed in high SNR region.

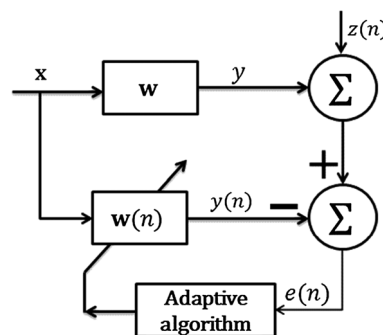


Figure 2. Adaptive channel estimation using adaptive algorithm.

In this paper, on the basis of the above observation, we propose several sparse LMF algorithms using different sparse penalties to exploit channel sparsity and achieve better performance than LMS-based sparse algorithms. The proposed algorithms are termed as zero-attracting LMF (ZA-LMF), reweighted zero-attracting LMF (RZA-LMF), L_p -LMF (LP-LMF), and L_0 -LMF (L0-LMF). Different from sparse LMS algorithms in [14–17], we construct novel LMF-based cost functions for each algorithm and then derive their update equations. The performances of the proposed algorithms are evaluated by the Monte Carlo simulation.

This paper is organized as follows. Section 2 reviews the standard LMF algorithm. In Section 3, we construct LMF-based cost functions and propose several adaptive sparse algorithms with sparse penalties. In Section 4, Monte Carlo simulation results for the MSE performance are presented to confirm the effectiveness of sparse LMF algorithms. Concluding remarks and future works are presented in Section 5.

2. OVERVIEW OF STANDARD LMF ALGORITHM

Assuming that training signal $\mathbf{x}(n)$ is input to a system with unknown sparse FIR-based channel vector $[w_0, w_1, \dots, w_{N-1}]^T$, its observed signal $y(n)$ is given by

$$y(n) = \mathbf{w}^T \mathbf{x}(n) + z(n) \quad (1)$$

where $\mathbf{x}(n) = [x(n), x(n-1), \dots, x(n-N+1)]^T$ denotes the vector of training signal $x(n)$ and $z(n)$ is the observation noise, which is assumed to be independent from $\mathbf{x}(n)$. The object of LMF algorithm is to adaptive estimate the unknown channel vector using the training signal $\mathbf{x}(n)$ and the output $y(n)$. Let $\mathbf{w}(n)$ be the estimated coefficient vector of the adaptive filter at iteration n . In the standard LMF, the cost function $L(n)$ is defined as

$$L(n) = \frac{1}{4} e^4(n) \quad (2)$$

where

$$e(n) = y(n) - \mathbf{w}^T(n) \mathbf{x}(n) \quad (3)$$

is the instantaneous update estimation error at n -th step. The channel vector is then updated by

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu \frac{\partial L(n)}{\partial \mathbf{w}(n)} = \mathbf{w}(n) + \mu e^3(n) \mathbf{x}(n) \quad (4)$$

where μ is the step-size parameter that tradeoffs stability and rate of convergence of the LMF algorithm.

3. SPARSE LMF ALGORITHMS

From Equation (3), we can find that the standard LMF algorithm cannot exploit sparse structure in an unknown sparse channel vector. To take the advantage of the sparsity, we introduce different sparse penalties on the gradient update equation in Equation (3). For a better understanding, an update equation of the sparse LMF can be concluded as

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \text{adaptive update error} + \text{sparse penalty} \quad (5)$$

Here, the sparse penalties act as L_p -norm ($0 \leq p \leq 1$) sparse constraints. In the following, adaptive sparse channel estimation approaches will be realized by four algorithms: ZA-LMF, RZA-LMF, LP-LMF, and L0-LMF.

3.1. ZA-LMF algorithm

Zero-attracting LMF is an adaptive sparse algorithm using L_1 -norm penalty. At first, the cost function of ZA-LMF is constructed by

$$L_{ZA}(n) = \frac{1}{4}e^4(n) + \lambda_{ZA}\|\mathbf{w}(n)\|_1 \quad (6)$$

where λ_{ZA} is a regularization parameter to balance the fourth-order identification error and sparse penalty of filter vector \mathbf{w} . The corresponding update equation of ZA-LMF is given by

$$\begin{aligned} \mathbf{w}(n+1) &= \mathbf{w}(n) - \mu \frac{\partial L_{ZA}(n)}{\partial \mathbf{w}(n)} \\ &= \mathbf{w}(n) + \mu e^3(n)\mathbf{x}(n) - \rho_{ZA}\text{sgn}(\mathbf{w}(n)) \end{aligned} \quad (7)$$

where $\rho_{ZA} = \mu\lambda_{ZA}$ and $\text{sgn}(\cdot)$ is a componentwise function is defined as

$$\text{sgn}(\mathbf{w}) = \begin{cases} w_i/|w_i|, & w_i \neq 0 \\ 0, & w_i = 0 \end{cases} \quad (8)$$

In Equation (7), the second term attracts the filter coefficients to zero, which speed up convergence when the majority of the channel coefficients in \mathbf{w} are zero.

3.2. RZA-LMF algorithm

Zero-attracting LMF algorithm cannot distinguish between zero taps and non-zero taps because all the taps are forced to zero uniformly; therefore, its performance will degrade in less sparse channel vectors. Motivated by the reweighted L_1 -norm minimization recovery algorithm [18], Chen *et al.* has proposed a heuristic approach to reinforce the zero attractor, which was termed as the reweighted zero-attracting LMS (RZA-LMS) [14]. On the basis of this idea, we construct a RZA-LMF cost function as

$$L_{RZA}(n) = \frac{1}{4}e^4(n) + \lambda_{RZA} \sum_{i=1}^N \log(1 + \varepsilon_{RZA}|w_i(n)|) \quad (8)$$

where λ_{RZA} is the regularization parameter that controls the fourth-order error and channel sparseness and ε_{RZA} . The corresponding update equation is

$$\begin{aligned} \mathbf{w}(n+1) &= \mathbf{w}(n) - \mu \frac{\partial L_{RZA}(n)}{\partial \mathbf{w}(n)} \\ &= \mathbf{w}(n) + \mu e^3(n)\mathbf{x}(n) - \mu\lambda_{RZA}\varepsilon_{RZA} \sum_{i=0}^{N-1} \frac{\text{sgn}(|w_i(n)|)}{1 + \varepsilon_{RZA}|w_i(n)|} \\ &= \mathbf{w}(n) + \mu e^3(n)\mathbf{x}(n) - \rho_{RZA} \frac{\text{sgn}(|\mathbf{w}(n)|)}{1 + \varepsilon_{RZA}|\mathbf{w}(n)|} \end{aligned} \quad (9)$$

where $\rho_{RZA} = \mu\lambda_{RZA}\varepsilon_{RZA}$. Note that the second term in Equation (9) attracts the filter coefficients $w_i(n)$, $i = 1, 2, \dots, N$, whose magnitudes are less than $1/\varepsilon_{RZA}$ to zeros.

3.3. LP-LMF algorithm

When the L_p -norm sparse penalty is introduced to LMF-based adaptive sparse channel estimation, its cost function is given by

$$L_{LP}(n) = \frac{1}{4}e^4(n) + \lambda_{LP}\|\mathbf{w}(n)\|_p \tag{10}$$

where $\lambda_{LP} > 0$ is a regularization parameter that trades off the estimation error and sparsity of channel vector. The corresponding update equation of LP-LMF is derived as

$$\begin{aligned} \mathbf{w}(n+1) &= \mathbf{w}(n) - \mu \frac{\partial L_{LP}(n)}{\partial \mathbf{w}(n)} \\ &= \mathbf{w}(n) + \mu e^3(n)\mathbf{x}(n) - \rho_{LP} \frac{\|\mathbf{w}(n)\|_p^{1-p} \text{sgn}(\mathbf{w}(n))}{\varepsilon_{LP} + \|\mathbf{w}(n)\|_p^{1-p}} \end{aligned} \tag{11}$$

where $\rho_{LP} = \mu\lambda_{LP}$ and $\varepsilon_{LP} > 0$.

3.4. L0-LMF algorithm

L_0 -norm penalty is also considered to be used in LMF cost function to produce sparse solution because this penalty term forces the small nonzero filter coefficients of $\mathbf{w}(n)$ to approach zero. The cost function of L0-LMF is given by

$$L_{L0}(n) = \frac{1}{4}e^4(n) + \lambda_{L0}\|\mathbf{w}(n)\|_0 \tag{12}$$

where $\lambda_{L0} > 0$ is a regularization parameter. Because solving the L_0 -norm minimization is a non-polynomial hard problem, we replace it with an approximate continuous function

$$\|\mathbf{w}\|_0 \approx \sum_{i=0}^{N-1} (1 - e^{-\beta|w_i|}) \tag{13}$$

According to the approximate function, L0-LMF cost function can be rewritten as

$$L_{L0}(n) = \frac{1}{4}e^4(n) + \lambda_{L0} \sum_{i=0}^{N-1} (1 - e^{-\beta|w_i|}) \tag{14}$$

Therefore, the update equation of L0-LMF-based adaptive sparse channel estimation is given by

$$\begin{aligned} \mathbf{w}(n+1) &= \mathbf{w}(n) - \mu \frac{\partial L_{L0}(n)}{\partial \mathbf{w}(n)} \\ &= \mathbf{w}(n) + \mu e^3(n)\mathbf{x}(n) - \rho_{L0}\beta \text{sgn}(\mathbf{w}(n))e^{-\beta|w_i|} \end{aligned} \tag{15}$$

where $\rho_{L0} = \mu\lambda_{L0}$. It is worth mentioning that the exponential function in Equation (15) will cause high computational complexity. To reduce the computational complexity, the first-order Taylor series expansion of exponential function is used, given by

$$e^{-\beta|w|} = \begin{cases} 1 - \beta|w|, & \text{when } |w| \leq 1/\beta \\ 0, & \text{others} \end{cases} \tag{16}$$

The modified update equation of L0-LMF can be written as

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu e^3(n)\mathbf{x}(n) - \rho_{L0}J(\mathbf{w}(n)) \tag{17}$$

where $J(w)$ is defined as

$$J(w) = \begin{cases} 2\beta^2 - 2\beta \operatorname{sgn}(w), & \text{when } |w| \leq 1/\beta \\ 0, & \text{others} \end{cases} \tag{18}$$

4. COMPUTER SIMULATIONS

In this section, we will compare the MSE performance of the proposed channel estimators using 1000 independent Monte Carlo runs for averaging. The FIR-based channel length is set as $N=16$, and its number of nonzero coefficients is set as $k=1, 2, 4$, and 6, respectively, and the nonzero coefficients follow Gaussian distribution while their positions are randomly allocated within the length of w and is subjected to $E[w_2^2] = 1$. The received SNR is defined as $\text{SNR} = 10\log(E_0/\sigma_n^2)$, where $E_0=1$ is the transmitted signal power. In the simulation, SNR is set as 5 db. All of the step sizes of gradient descend and regularization parameters are listed in Table 1. The sparse channel estimation performance is evaluated by average MSE standard, which is defined as

$$\text{MSE}\{\mathbf{w}(n+1)\} = E[\|\mathbf{w} - \mathbf{w}(n)\|_2^2] \tag{19}$$

At first, the MSE performance of LMF is evaluated with gradient descend step-size μ as shown in Figure 3. It can be observed that as the step-size μ decreases, MSE performance of LMF is improved while the convergence speed is decreased. To balance the convergence speed and estimation performance, we

Table I. Simulation parameters of LMS(F)-based algorithms.

	$\mu_{\text{LMS(F)}}$	$\lambda_{\text{ZA(F)}}$	$\lambda_{\text{RZA(F)}}$	$\lambda_{\text{LP(F)}}$	$\lambda_{\text{L0(F)}}$
LMS-based algorithms	$5e-2 = 0.05$	$5e-3 = 0.005$	$5e-3 = 0.005$	$5e-3 = 0.005$	$5e-2 = 0.005$
LMF-based algorithms	$5e-3 = 0.005$	$5e-3 = 0.005$	$5e-3 = 0.005$	$5e-4 = 0.0005$	$5e-3 = 0.005$

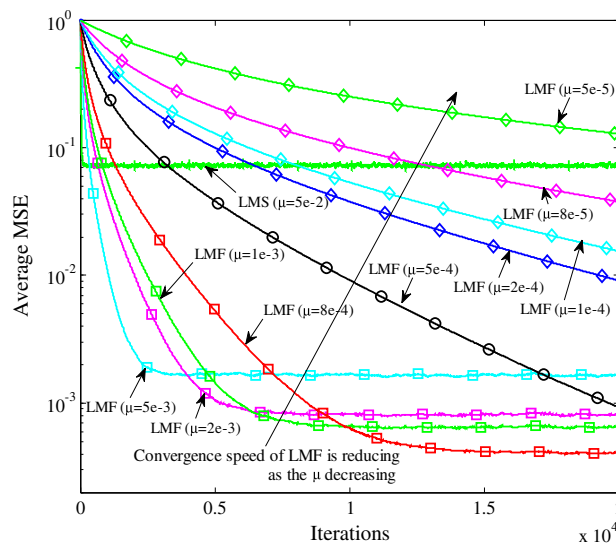


Figure 3. MSE performance versus gradient descend step-size (SNR = 5 dB).

choose the step-size of LMF as $\mu = 5e-3 = 0.005$, which is shown in Figure 3. Because sparse LMF algorithms are based on the standard LMF algorithm, step-size is also set as $\mu = 5e-3 = 0.005$ for all the algorithms used in the computer simulations.

Secondly, we compare all the LMF-based sparse channel estimation methods with different number of nonzero coefficients; the simulation results are shown in Figures 4–7. In Figure 4, the number of nonzero channel coefficient is $K = 1$; sparse LMF algorithms achieve much better MSE performance than sparse LMS algorithms, especially the MSE performance of LP(0)-LMF is obviously better than the other methods because the two algorithms can exploit more sparse structure information in a sparse FIR system. Of course, LP(0)-LMS algorithm can also exploit better MSE performance than (R)ZA-LMS algorithm. However, standard LMS algorithm cannot obtain optimal solution in low SNR regime. As the number of nonzero channel coefficient increases, the MSE performance curves of LP(0)-LMF are close to the performance curve of the

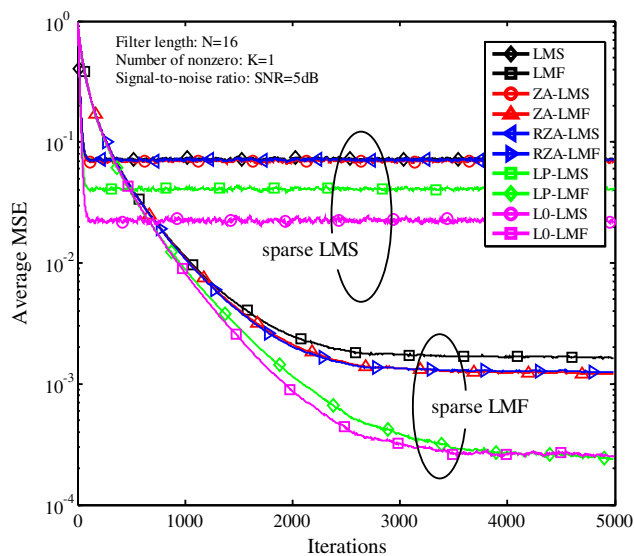


Figure 4. Performance comparison ($K = 1$).

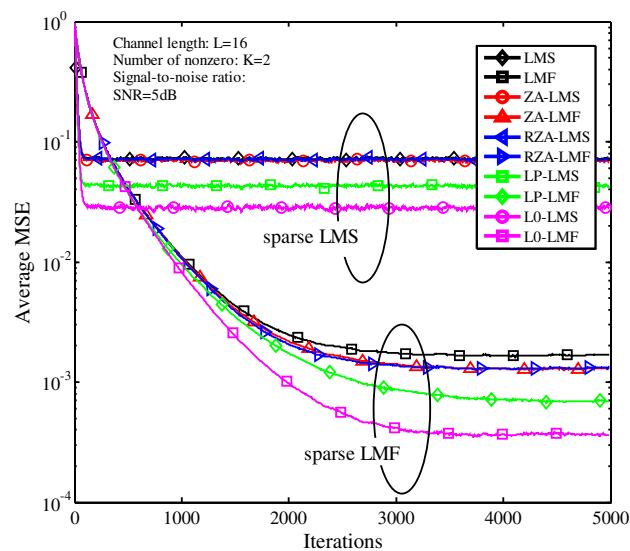
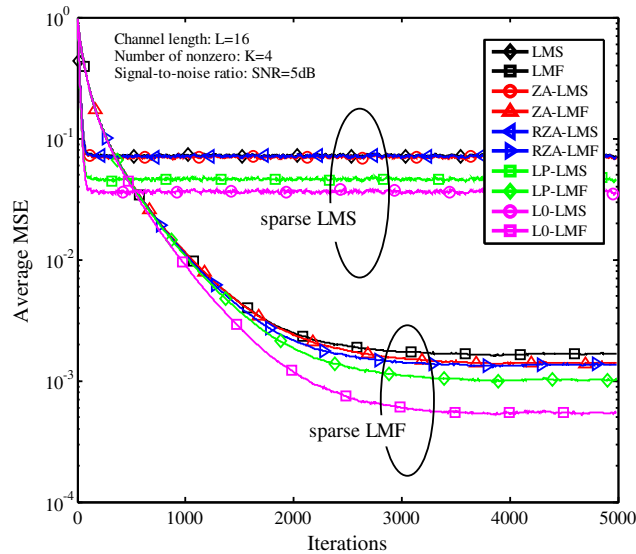
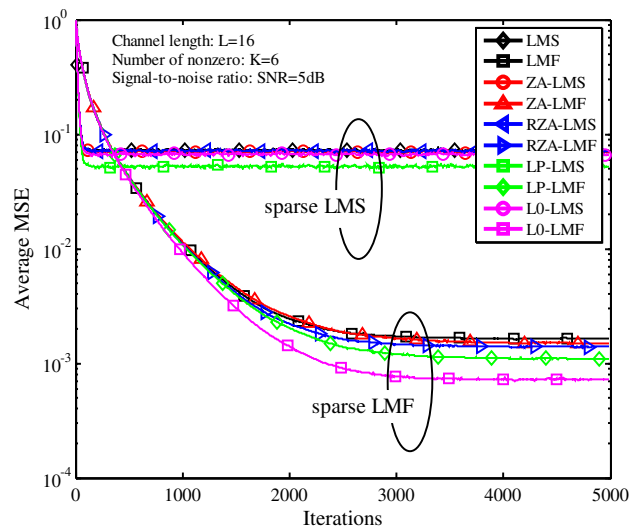


Figure 5. Performance comparison ($K = 2$).

Figure 6. Performance comparison ($K = 4$).Figure 7. Performance comparison ($K = 6$).

standard LMF. According to these simulation results, the effectiveness of our proposed adaptive sparse channel estimation methods has been verified.

5. CONCLUSION

Sparse LMF algorithms have been proposed for adaptive sparse channel estimation. Comparing with sparse LMS methods, our proposed LMF algorithms can achieve better performance in low SNR region. We constructed different cost functions with different sparse penalties on standard LMF algorithm, and the effectiveness of the proposed sparse LMF algorithms was confirmed by Monte Carlo simulation. Sparse LMF performs better in low SNR, while sparse LMS performs better in high SNR region. Therefore, adaptive switching between the two depending on the SNR may be useful in practical systems, and its switching criterion is an interest for our future work.

APPENDIX

Because both performance analysis and convergence speed are very important for LMF algorithm, the interested readers can refer to the detailed derivations in [21, 22]. In addition, the global stabilization of linear LMF algorithm is derived in [23]. In this paper, the proposed algorithms are based on the standard LMF algorithm. Similarly, the performance and convergence analysis are given briefly as follows.

APPENDIX A. CONVERGENCE ANALYSIS.

Let $\mathbf{v}(n)$ denote the sparse channel estimation error vector, which is defined as

$$\mathbf{v}(n) = \mathbf{w}(n) - \mathbf{w} \tag{A1}$$

where \mathbf{w} is the actual sparse channel vector. Then, $e(n)$ in Equation (3) can be rewritten as

$$e(n) = z(n) - \mathbf{v}^T(n)\mathbf{x}(n) \tag{A2}$$

If we subtract \mathbf{w} from both sides in Equation (4), then the updating error can be written as

$$\begin{aligned} \mathbf{v}(n+1) &= \mathbf{v}(n) + \mu\mathbf{x}(n)e^3(n) \\ &= \mathbf{v}(n) + \mu\mathbf{x}(n)[z(n) - \mathbf{v}^T(n)\mathbf{x}(n)]^3 \\ &= \mathbf{v}(n) + \mu\mathbf{x}(n)z^3(n) - 3\mu\mathbf{v}(n)\mathbf{x}(n)\mathbf{x}^T(n)z^2(n) \\ &\quad + 3\mu z(n)\mathbf{x}(n)[\mathbf{v}^T(n)\mathbf{x}(n)]^2 - \mu\mathbf{x}(n)[\mathbf{v}^T(n)\mathbf{x}(n)]^3 \\ &\approx \mathbf{v}(n) - 3\mu\mathbf{v}(n)\mathbf{x}(n)\mathbf{x}^T(n)z^2(n) \end{aligned} \tag{A3}$$

Note that the updating error in Equation (A3) neglects the effects of the high-order terms of $\mathbf{v}(n)$ on the basis of the assumption that $\lim_{n \rightarrow \infty} \mathbf{v}(n) = 0$ [21], that is, $\lim_{n \rightarrow \infty} [\mathbf{v}^T(n)\mathbf{x}(n)]^3 = 0$. Assume $z(n)$ is Gaussian distributed with zero-mean and variance $E[z^2(n)] = \sigma_z^2$. In addition, $z(n)$ is assumed to have an even probability density function, that is, $f_z(n) = f_z(-n)$. Then, it is easy to found that all the odd moments of $z(n)$ are equal to zero, that is, $E[z(n)] = E[z^3(n)] = 0$. Taking the expectation of both sides in Equation (A3), we can obtain

$$E[\mathbf{v}(n+1)] = (\mathbf{I} - 3\mu\sigma_z^2\mathbf{R})E[\mathbf{v}(n)] \tag{A4}$$

where $E[\cdot]$ denotes the statistical expectation operation. The necessary condition for the convergence of LMF algorithm in Equation (4) is

$$-1 < 1 - 3\mu\sigma_z^2\lambda_{\max} < 1 \tag{A5}$$

Therefore, the step size μ should satisfy

$$0 < \mu < \frac{2}{2\sigma_z^2\lambda_{\max}} \tag{A6}$$

where λ_{\max} is the largest eigenvalue of the correlation matrix $\mathbf{R} = E[\mathbf{x}(n)\mathbf{x}^T(n)]$, which is the autocorrelation of the input signal $\mathbf{x}(n)$. From the necessary condition in Equation (A6), it is worth noting that the stability of LMF depends on both input signal power of adaptive filter and the noise power. Assume that scaling of training signal is invariant, the step-size will be increased when the SNR increases. However, LMF algorithm using large gradient descend step-size easily incurs unstable performance. In other words, the step-size will be reduced to ensure stable performance when the SNR decreases, especially in low SNR region.

APPENDIX B. PERFORMANCE ANALYSIS.

By using $\mathbf{v}(n)$ in Equation (A1), under the independent assumption, the $(n + 1)$ -th MSE performance is of $\mathbf{w}(n + 1)$ evaluated by

$$\begin{aligned} \mathbf{C}(n + 1) &= \lim_{n \rightarrow \infty} \mathbf{E}[\mathbf{v}(n + 1)\mathbf{v}^T(n + 1)] \\ &= \lim_{n \rightarrow \infty} \mathbf{E}[\mathbf{v}(n)\mathbf{v}^T(n)] + \mu \mathbf{E}[e^3(n)(\mathbf{v}(n)\mathbf{v}^T(n) + \mathbf{x}(n)\mathbf{v}^T(n))] + \mu^2 \mathbf{E}[e^6(n)\mathbf{x}(n)\mathbf{v}^T(n)] \\ &= \mathbf{C}(n) - 3\mu[\sigma_z^2 + \text{Tr}(\mathbf{RC}(n))][\mathbf{RC}(n) + \mathbf{C}(n)\mathbf{R}] + \mu^2[E(z^6(n)\mathbf{R}) \\ &\quad + 15 E(z^4(n))][\text{Tr}(\mathbf{RC}(n) + 2\mathbf{RC}(n)\mathbf{R})] \end{aligned} \tag{B1}$$

where $\mathbf{C}(n) = \mathbf{E}[\mathbf{v}(n)\mathbf{v}^T(n)]$ denotes recursive correlation matrix of $\mathbf{v}(n)$. This MSE equation shows that algorithm convergence speed is decided by initial conditions, noise power, and gradient descend step-size.

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