PAPER

2-Step Frequency-Domain Iterative Channel Estimation for Training Sequence Inserted Single-Carrier Block Transmission

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SUMMARY In this paper, we propose a spectrally efficient frequency-domain channel estimation scheme suitable for training sequence inserted single-carrier (TS-SC) block transmission using frequency-domain equalization (FDE). The proposed scheme performs the channel estimation in two steps and allows the use of shorter TS (but, longer than the channel length) than the conventional channel estimation schemes. In the first step, the received TS having cyclic property is constructed for performing frequency-domain channel estimation and the improved channel estimate is obtained by using simple averaging of noisy channel estimates. In the second step, the maximum likelihood channel estimation is carried out iteratively by using both the TS and the estimated symbol sequence obtained in the first step. It is shown by computer simulation that the proposed 2-step frequency-domain iterative channel estimation scheme achieves a bit error rate (BER) performance close to perfect channel estimation even in a relatively fast fading environment.

key words: *single-carrier, frequency-domain equalization, channel estimation, training sequence*

1. Introduction

Broadband data services are demanded in next generation mobile communication systems. Since the wireless channel becomes severely frequency-selective as the transmission data rate increases, single-carrier (SC) systems suffer from inter-symbol interference (ISI) arising from severe frequency-selectivity of the channel [1]. Recently, to tackle ISI, simple one-tap frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion has been gaining popularity [2]–[4].

SC-FDE is a type of block transmission and discrete Fourier transform (DFT) is used at the receiver to transform the received signal block to the frequency-domain signal. In general, the cyclic prefix (CP) is inserted in front of each signal block to avoid the inter-block interference (IBI) and to make the received signal the circular convolution of the transmit signal block and the channel impulse response. Instead of CP insertion, known training sequence (TS) insertion [5]–[7] can be used. By using the same TS for all blocks, the TS in the previous block acts as the CP in the present block. Therefore, simple one-tap FDE can also be applied similar to the CP inserted SC (CP-SC) block transmissions; the difference is only the DFT size used at the receiver. Furthermore, as TS can be utilized for channel estimation, no pilot block is needed unlike CP-SC block transmissions.

Some studies on channel estimation which use the inserted TS for TS inserted SC (TS-SC) block transmissions are found in [6], [8]–[10]. By using a $2L$-symbol TS, where L denotes the channel length, the channel estimation can be done without suffering the interference from the data block. In addition, the use of a $2L$-symbol TS, which is constructed by repeating twice an L-symbol TS, enables the frequency-domain channel estimation by exploiting the circular property of the TS. However, the use of a $2L$-symbol TS reduces the transmission efficiency. The frequency-domain channel estimation scheme using an L-symbol TS, which was proposed in [10] performs the channel estimation in two steps. In the first step, the initial channel estimation is carried out while suppressing the interference from the data block to the TS by applying the rectangular windowing. After the interference suppression, DFT is applied to transform the zero-replaced received signal for performing the initial frequency-domain channel estimation. The DFT size equals to the sum of the data block length and the TS length (equals to that for FDE). In the second step, the channel estimation is carried out iteratively by using both the TS and the decision-feedback estimated symbol sequence.

Recently, we presented a new 2-step frequency-domain channel estimation scheme using an L-symbol TS [11]. Unlike [10], in the first step, the received TS having cyclic property is constructed by using the first L symbols and the last L symbols in the received signal block and then, DFT is applied to transform the constructed TS having cyclic property for the initial frequency-domain channel estimation. The DFT size required for the channel estimation is the same as the TS length.

The objective of this paper is to describe the principle of 2-step channel estimation scheme presented in [11] in detail. In the first step, the noise power needs to be estimated. In this paper, we propose a new noise power estimation which does not use the channel estimates while the noise power estimation of [10] requires the channel estimates. The performance of the proposed channel estimation scheme is assessed through computer simulations assuming a doubly (time- and frequency-) selective channel. It will be shown that the proposed 2-step channel estimation scheme is robust against the time selectivity (or the Doppler shift) of the channel. The performance comparison in terms of spectral efficiency is presented between the TS-SC transmission with the proposed 2-step channel estimation and the CP-SC transmission with the 2-step maximum likelihood channel

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estimation (MLCE) [12].

The rest of the paper is organized as follows. In Sect. 2, transmission system model of TS-SC block transmission with FDE is presented. Section 3 describes the proposed channel estimation scheme. In Sect. 4, the simulation results are presented, and finally Section 5 concludes the paper.

2. TS-SC Block Transmission with FDE

2.1 Transmission System Model

Figure 1 illustrates the transmitter/receiver structure considered in this paper, where the data block size and guard interval (GI) length are denoted by N_c and N_g, respectively. Symbol-spaced discrete time representation is used throughout the paper. At the transmitter, information bit sequence is transformed into a data-modulated symbol sequence, which is divided into a sequence of symbol blocks of N_c symbols. The n-th data symbol block can be expressed using the vector form as $\mathbf{d}^{(n)}=[d^{(n)}(0),\ldots,d^{(n)}(t),\ldots,d^{(n)}(N_c-1)]^T$, where $(.)^T$ expresses the transposition. Before the transmission, a TS of length N_g ($\geq L$) symbols is appended at the last part of each block instead of well-known CP. The n-th block $\mathbf{s}^{(n)}=[s^{(n)}(0),\ldots,s^{(n)}(t),\ldots,s^{(n)}(N_c+N_g-1)]^T$ to be transmitted is expressed as

$$\mathbf{s}^{(n)} = \begin{bmatrix} \mathbf{d}^{(n)} \\ \mathbf{u} \end{bmatrix}, \tag{1}$$

where $\mathbf{u}=[u(0),\ldots,u(n),\ldots,u(N_g-1)]^T$ denotes the TS vector which is identical for all blocks. Block structure of TS-SC block transmission is illustrated in Fig. 2. In TS-SC block transmission, in order to let a TS to play the role of CP, block size for DFT must be N_c+N_g symbols.

The transmitted signal block propagates through a frequency-selective fading channel and received at the receiver. The received signal block is transformed into the frequency-domain signal by N_c+N_g-point DFT. MMSE-FDE is carried out to obtain the equalized frequency-domain received signal. Finally, inverse DFT (IDFT) is applied to obtain the time-domain received signal block for data demodulation.

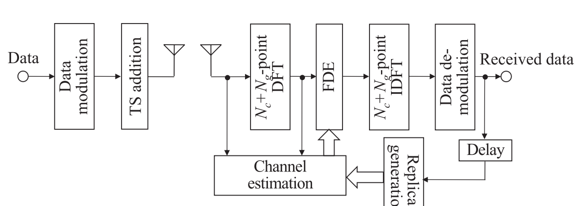


Fig. 1 Transmitter and receiver structure of TS-SC block transmission.

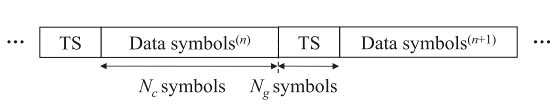


Fig. 2 Block structure of TS-SC block transmission.

2.2 Received Signal Representation

The propagation channel is assumed to be a frequency-selective block fading channel composed of L distinct symbol-spaced propagation paths. The channel impulse response $h^{(n)}(\tau)$ at the n-th block is given by

$$h^{(n)}(\tau) = \sum_{l=0}^{L-1} h_l^{(n)} \delta(\tau - \tau_l), \tag{2}$$

where $h_l^{(n)}$ and τ_l are respectively the complex-valued path gain with $E[\sum_{l=0}^{L-1} |h_l^{(n)}|] = 1$ and the time delay of the l-th path. The l-th path is assumed to has a time delay of l symbols, i.e. ($\tau_l = l$). The received signal vector during n-th block $\mathbf{y}^{(n)}=[y^{(n)}(0),\ldots,y^{(n)}(t),\ldots,y^{(n)}(N_c+N_g-1)]^T$ is expressed as

$$\mathbf{y}^{(n)} = \sqrt{2S}\,\mathbf{h}^{(n)}\mathbf{s}^{(n)} + \mathbf{z}^{(n)}, \tag{3}$$

where S is the average received signal power, $\mathbf{h}^{(n)}$ is the $(N_c+N_g)\times(N_c+N_g)$ channel impulse response matrix given as

$$\mathbf{h}^{(n)} = \begin{bmatrix}
h_0^{(n)} & & & & h_{L-1}^{(n)} & \cdots & h_1^{(n)} \\
h_1^{(n)} & h_0^{(n)} & & \mathbf{0} & & \ddots & \vdots \\
\vdots & h_1^{(n)} & \ddots & & & & h_{L-1}^{(n)} \\
h_{L-1}^{(n)} & \vdots & \ddots & \ddots & & & \\
 & h_{L-1}^{(n)} & & \ddots & \ddots & & \\
 & & \ddots & & \ddots & \ddots & \\
\mathbf{0} & & & h_{L-1}^{(n)} & \cdots & h_1^{(n)} & h_0^{(n)}
\end{bmatrix}, \tag{4}$$

and $\mathbf{z}^{(n)}=[z^{(n)}(0),\ldots,z^{(n)}(t),\ldots,z^{(n)}(N_c+N_g-1)]^T$ is the zero-mean additive white Gaussian noise (AWGN) vector. The t-th element, $z^{(n)}(t)$, of $\mathbf{z}$ is the AWGN having the variance $2\sigma^2$.

2.3 FDE

The received signal is transformed by N_c+N_g-point DFT into the frequency-domain signal $\mathbf{Y}^{(n)}=[Y^{(n)}(0),\ldots,Y^{(n)}(k),\ldots,Y^{(n)}(N_c+N_g-1)]^T$. $\mathbf{Y}^{(n)}$ is expressed as

$$\mathbf{Y}^{(n)} = \mathbf{F}_{N_c+N_g}\mathbf{y}^{(n)} = \mathbf{H}^{(n)}\mathbf{S}^{(n)} + \mathbf{Z}^{(n)}, \tag{5}$$

where $\mathbf{F}_K$ is the DFT matrix of size $K \times K$ given by

$$\mathbf{F}_K = \frac{1}{\sqrt{K}} \begin{bmatrix}
1 & 1 & \cdots & 1 \\
1 & e^{-j2\pi\frac{1\times1}{K}} & \cdots & e^{-j2\pi\frac{1\times(K-1)}{K}} \\
\vdots & \vdots & \ddots & \vdots \\
1 & e^{-j2\pi\frac{(K-1)\times1}{K}} & \cdots & e^{-j2\pi\frac{(K-1)\times(K-1)}{K}}
\end{bmatrix}. \tag{6}$$

$\mathbf{S}^{(n)}=[S^{(n)}(0),\ldots,S^{(n)}(k),\ldots,S^{(n)}(N_c+N_g-1)]^T = \mathbf{F}_{N_c+N_g}\mathbf{s}^{(n)}$ is the n-th frequency-domain transmit symbol block, $\mathbf{Z}^{(n)}=[Z^{(n)}(0),\ldots,Z^{(n)}(k),\ldots,Z^{(n)}(N_c+N_g-1)]^T = \mathbf{F}_{N_c+N_g}\mathbf{z}^{(n)}$

is the frequency-domain noise vector, and $\mathbf{H}^{(n)} = \sqrt{2S}\mathbf{F}_{N_c+N_g}\mathbf{h}^{(n)}\mathbf{F}^{H}_{N_c+N_g}$ is the frequency-domain channel matrix. $(.)^H$ is the Hermitian transpose operation. As $\mathbf{h}^{(n)}$ has circular property, the frequency-domain channel matrix $\mathbf{H}^{(n)}$ is diagonal. The k-th diagonal element of $\mathbf{H}^{(n)}$ is given by

$$H^{(n)}(k) = \sqrt{2S}\sum_{l=0}^{L-1} h_l^{(n)} \exp\left(-j2\pi k \frac{\tau_l}{N_c+N_g}\right). \quad (7)$$

FDE is carried out to obtain

$$\hat{\mathbf{Y}}^{(n)} = \mathbf{W}^{(n)}\mathbf{Y}^{(n)}, \quad (8)$$

where $\mathbf{W}^{(n)}=diag[W^{(n)}(0),\ldots,W^{(n)}(k),\ldots,W^{(n)}(N_c+N_g-1)]$ is the FDE weight matrix. In this paper, MMSE weight is used. The k-th diagonal element of $\mathbf{W}^{(n)}$ is given by [7]

$$W^{(n)}(k) = \frac{\{\tilde{H}^{(n)}(k)\}^*}{|\tilde{H}^{(n)}(k)|^2 + 2\tilde{\nu}}, \quad (9)$$

where $\tilde{H}^{(n)}(k)$ represents the channel gain estimate, $\tilde{\nu}$ is the noise power estimate, and $(.)^*$ denotes the complex conjugate operation.

$\hat{\mathbf{Y}}^{(n)}$ is transformed into a time-domain symbol block $\hat{\mathbf{s}}^{(n)}$ by (N_c+N_g)-point IDFT to obtain the decision variable vector $\hat{\mathbf{d}}^{(n)} = [\hat{s}^{(n)}(0), ..., \hat{s}^{(n)}(t), ..., \hat{s}^{(n)}(N_c-1)]^T$.

3. 2-Step Frequency-Domain Iterative Channel Estimation

Figure 3 shows the flow chart of the proposed 2-step frequency-domain channel estimation scheme, which consists of two steps. Channel estimation is carried out over N_B received symbol blocks, during which the channel gains are assumed to stay constant (i.e., $h_l^{(n)} = h_l$ and $H^{(n)}(k) = H(k)$, $n = 0 \sim N_B - 1$). In the first step, only received TSs are used. The received TS having cyclic property is constructed for performing frequency-domain channel estimation. The improved channel estimate is obtained by using simple averaging the instantaneous channel estimates over N_B blocks. Noise power is also estimated in the frequency-domain without using the channel estimates. MMSE-FDE and tentative symbol decision are carried out by using channel and noise power estimates obtained in the first step to generate the N_B transmitted symbol block replicas. Then, in the second step, the MLCE [12] is carried out iteratively by using both the TS and the estimated symbol sequence to improve channel estimation accuracy.

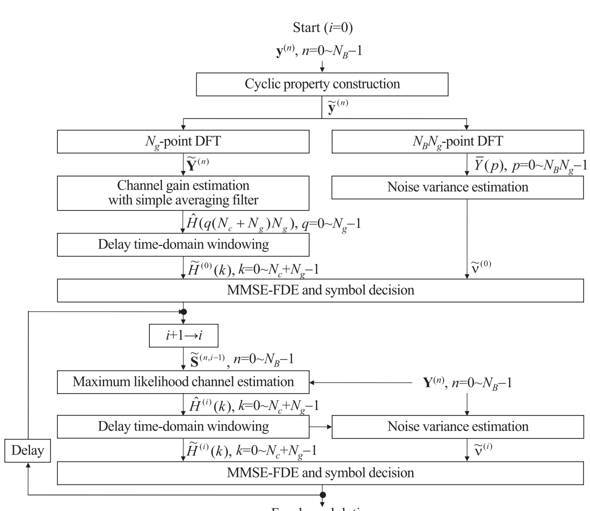


Fig. 3 2-step frequency-domain iterative channel estimation.

3.1 First Step

First, the received TS having the cyclic property is constructed, as shown in Fig. 4, by using the first $N_g - 1$ symbols and the last N_g ($N_g \geq L$) symbols of the received signal block of N_c+N_g symbols. The received TS having the cyclic property is obtained as

$$\tilde{y}^{(n)}(t) = \begin{cases} y^{(n)}(t) + y^{(n)}(t+N_c) & t = 0 \sim L-2 \\ y^{(n)}(t+N_c) & t = L-1 \sim N_g-1 \end{cases}. \quad (10)$$

Equation (10) can be rewritten by using the vector form as

$$\tilde{\mathbf{y}}^{(n)} = \sqrt{2S}\mathbf{h}_{N_g}\mathbf{u} + \mathbf{i}^{(n)} + \tilde{\mathbf{z}}^{(n)}, \quad (11)$$

where $\mathbf{h}_{N_g}$ is the $N_g \times N_g$ channel impulse response matrix which has the cyclic property similar to Eq. (4). Second and third terms respectively denote the interference from the data block and noise, which are respectively given by

$$\mathbf{i}^{(n)} = \sqrt{2S}\begin{bmatrix} h_0 & & & & h_{L-1}\cdots & h_1 \\ \vdots & \ddots & \mathbf{0}_{L-1,N_c-2L+2} & & \ddots & \vdots \\ h_{L-2}\cdots & h_0 & & & & h_{L-1} \\ & & \mathbf{0}_{N_g-L+1,N_c} & & & \end{bmatrix}\mathbf{d}^{(n)} \quad (12)$$

and

$$\tilde{\mathbf{z}}^{(n)} = \mathbf{z}^{(n)}_{[N_c;N_c+N_g-1]} + \begin{bmatrix} \mathbf{z}^{(n)}_{[0;L-1]} \\ \mathbf{0}_{N_g-L+1} \end{bmatrix}. \quad (13)$$

$\mathbf{0}_{K,K}$ and $\mathbf{0}_J$ represent a zero matrix of size $K \times K$ and zero vector of size $J\times 1$, respectively. $\mathbf{z}^{(n)}_{[t_1;t_2]}$ is the subvector of $\mathbf{z}^{(n)}$ defined as $\mathbf{z}^{(n)}_{[t_1;t_2]} = [z^{(n)}(t_1), ..., z^{(n)}(t), ..., z^{(n)}(t_2)]^T$.

$\tilde{\mathbf{y}}^{(n)}$ is transformed into the frequency-domain signal $\tilde{\mathbf{Y}}^{(n)} = [\tilde{Y}^{(n)}(0), ..., \tilde{Y}^{(n)}(q), ..., \tilde{Y}^{(n)}(N_g-1)]^T$ by N_g-point DFT as

$$\begin{aligned} \tilde{\mathbf{Y}}^{(n)} &= \mathbf{F}_{N_g}\tilde{\mathbf{y}}^{(n)} \\ &= \mathbf{H}_{N_g}\mathbf{U} + \mathbf{F}_{N_g}\mathbf{i}^{(n)} + \mathbf{F}_{N_g}\tilde{\mathbf{z}}^{(n)}, \end{aligned} \quad (14)$$

where $\mathbf{U}=[U(0),\ldots,U(q),\ldots,U(N_g-1)]^T = \mathbf{F}_{N_g}\mathbf{u}$ is the

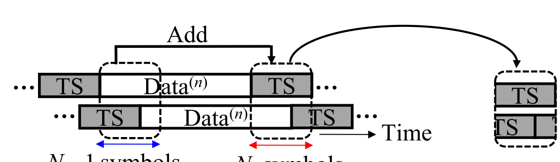


Fig. 4 Cyclic property construction for the received TS.

frequency-domain TS vector and $\mathbf{H}_{N_g} = \sqrt{2S}\mathbf{F}_{N_g}\mathbf{h}_{N_g}\mathbf{F}_{N_g}^H$ is the frequency-domain channel matrix. Due to the cyclic property of $\mathbf{h}_{N_g}$, the frequency-domain channel matrix $\mathbf{H}_{N_g}$ is also diagonal. The q-th ($q = 0 \sim N_g - 1$) diagonal element of $\mathbf{H}_{N_g}$ is given by

$$\begin{aligned} H_{N_g}(q) &= H\left(\frac{N_c+N_g}{N_g}q\right) \\ &= \sqrt{2S}\sum_{l=0}^{L-1} h_l \exp\left(-j2\pi\left(\frac{N_c+N_g}{N_g}q\right)\frac{\tau_l}{N_c+N_g}\right). \end{aligned} \tag{15}$$

From Eqs. (14) and (15), the channel gain estimates averaged over N_B blocks $\{\hat{H}(q(N_c+N_g)/N_g);\ q = 0 \sim N_g - 1\}$ are obtained as

$$\hat{H}\left(\frac{N_c+N_g}{N_g}q\right) = \frac{1}{N_B}\sum_{n=0}^{N_B-1}\frac{\tilde{Y}^{(n)}(q)}{U(q)}. \tag{16}$$

It can be understood that the channel gains at frequency $k = q(N_c+N_g)/N_g$, $q = 0 \sim N_g - 1$, can only be estimated by Eq. (16). Therefore, an interpolation technique is used to obtain $N_c + N_g$ channel gains for performing FDE. In this paper, we apply the delay time-domain windowing technique [13] for interpolation. First, N_g-point IDFT is applied to $\{\hat{H}(q(N_c+N_g)/N_g);\ q = 0 \sim N_g - 1\}$ to obtain the channel impulse response estimate $\{\tilde{h}(\tau);\ \tau = 0 \sim N_g - 1\}$. Then, N_c+N_g-point DFT is applied to $\{\tilde{h}(\tau);\ \tau = 0 \sim N_g - 1\}$ and filling zero's for $\tau = N_g \sim N_c+N_g-1$ to obtain $\{\tilde{H}^{(0)}(k);\ k = 0 \sim N_c + N_g - 1\}$ as

$$\begin{aligned} \tilde{H}^{(0)}(k) &= \sum_{\tau=0}^{N_g-1}\tilde{h}(\tau)\exp\left(-j2\pi k\frac{\tau}{N_c+N_g}\right) \\ &= \sum_{q=0}^{N_g-1} A\left(k - \frac{N_c+N_g}{N_g}q\right)\hat{H}\left(\frac{N_c+N_g}{N_g}q\right), \end{aligned} \tag{17}$$

where $A(x)$ is given by

$$\begin{aligned} A(x) &= \frac{1}{N_c+N_g}\exp\left(-j\pi(N_g-1)\frac{x}{N_c+N_g}\right) \\ &\quad\times\frac{\sin\left(\frac{\pi N_g x}{N_c+N_g}\right)}{\sin\left(\frac{\pi x}{N_c+N_g}\right)}. \end{aligned} \tag{18}$$

It can be understood from Eq. (18) that the interpolation using IDFT is equivalent to the high order polynomial interpolation with the coefficients $A(x)$.

MMSE-FDE requires the noise power estimate $\tilde{\nu}$. Noise power estimation is also carried out over N_B received symbol blocks as follows. First, the stacked received TS $\{\bar{y}(t);\ t = 0 \sim N_BN_g - 1\}$ of length of N_BN_g symbols is constructed as

$$\bar{y}(t) = \tilde{y}^{\left(\left\lfloor \frac{t}{N_g}\right\rfloor\right)}(t \bmod N_g), \tag{19}$$

where $\lfloor x \rfloor$ represents the largest integer smaller than or equal to x. An N_BN_g-point DFT is applied to transform the stacked received TS into N_BN_g frequency components $\{\bar{Y}(p);\ p = 0 \sim N_BN_g - 1\}$ as

$$\begin{aligned} \bar{Y}(p) &= \frac{1}{\sqrt{N_BN_g}}\sum_{t=0}^{N_BN_g-1}\bar{y}(t)\exp\left(-j2\pi p\frac{t}{N_BN_g}\right) \\ &= H(p)\bar{U}(p) + \bar{I}(p) + \bar{Z}(p), \end{aligned} \tag{20}$$

where $\bar{I}(p)$ and $\bar{Z}(p)$ represent the interference from the data block and noise components. $\bar{U}(p)$ is given by

$$\begin{aligned} \bar{U}(p) &= \frac{1}{\sqrt{N_BN_g}}\sum_{t=0}^{N_BN_g-1}u(t \bmod N_g)\exp\left(-j2\pi p\frac{t}{N_BN_g}\right) \\ &= \begin{cases} U(q) & \text{for } p = N_Bq, q = 0 \sim N_g - 1 \\ 0 & \text{otherwise} \end{cases}. \end{aligned} \tag{21}$$

As $\{\bar{Y}(p);\ p = 0 \sim N_BN_g - 1,\ p \neq N_Bq,\ q = 0 \sim N_g - 1\}$ contains the interference from the data block and noise components only, the noise power estimate $\tilde{\nu}^{(0)}$ is obtained as

$$\tilde{\nu}^{(0)} = \frac{1}{2}\frac{1}{N_BN_g - N_g}\left\{\sum_{p=0}^{N_BN_g-1}\left|\bar{Y}(p)\right|^2 - \sum_{q=0}^{N_g-1}\left|\bar{Y}(qN_B)\right|^2\right\}. \tag{22}$$

The above noise estimate contains the contribution from both the interference from the data block and noise, which contribute a channel estimation error.

3.2 Second Step

In the second step, the MLCE [12] is performed iteratively by using both the TS and the estimated symbol sequence. Below, the i-th ($i > 0$) iteration stage is explained. The channel gain estimates $\{\hat{H}^{(i)}(k);\ k = 0 \sim N_c + N_g - 1\}$ are obtained as

$$\hat{H}^{(i)}(k) = \frac{\sum_{n=0}^{N_B-1} Y^{(n)}(k)\{\tilde{S}^{(n,i-1)}(k)\}^*}{\sum_{n=0}^{N_B-1}|\tilde{S}^{(n,i-1)}(k)|^2}. \tag{23}$$

$\{\tilde{S}^{(n,i-1)}(k);\ n = 0 \sim N_B - 1,\ k = 0 \sim N_c + N_g - 1\}$ is the k-th frequency component of the symbol replica block for the n-th block in the previous iteration stage. Frequency-domain symbol replica block $\tilde{\mathbf{S}}^{(n,i-1)} = [\tilde{S}^{(n,i-1)}(0), ..., \tilde{S}^{(n,i-1)}(k), ..., \tilde{S}^{(n,i-1)}(N_c + N_g - 1)]^T$ is given as

$$\tilde{\mathbf{S}}^{(n,i-1)} = \mathbf{F}_{N_c+N_g}\begin{bmatrix} \tilde{\mathbf{d}}^{(n,i-1)} \\ \mathbf{u} \end{bmatrix}, \tag{24}$$

where $\tilde{\mathbf{d}}^{(n,i-1)} = [\tilde{d}^{(n,i-1)}(0), ..., \tilde{d}^{(n,i-1)}(t), ..., \tilde{d}^{(n,i-1)}(N_c - 1)]^T$ is the symbol replica block. In this paper, the symbol replica is generated using the log likelihood ratio (LLR) [14]–[16]. Using the MMSE-FDE output $\hat{d}(t)$ associated with $d(t)$, the

LLR for the x-th bit, $x = 0 \sim \log_2 X - 1$, in the t-th symbol, $t = 0 \sim N_c - 1$, is obtained where X is the modulation level. The soft symbol replica $\tilde{d}^{(i,n)}(t)$ is generated using the LLR as

$$\tilde{d}^{(i,n)}(t) = \begin{cases} \frac{1}{\sqrt{2}} \tanh\left(\frac{\lambda_0^{(i)}(t)}{2}\right) + j\frac{1}{\sqrt{2}} \tanh\left(\frac{\lambda_1^{(i)}(t)}{2}\right) \\ \text{for Quadrature phase shift keying} \\ \quad \text{(QPSK)} \\ \frac{1}{\sqrt{10}} \tanh\left(\frac{\lambda_0^{(i)}(t)}{2}\right)\left\{2 + \tanh\left(\frac{\lambda_1^{(i)}(t)}{2}\right)\right\} \\ +j\frac{1}{\sqrt{10}} \tanh\left(\frac{\lambda_2^{(i)}(t)}{2}\right)\left\{2 + \tanh\left(\frac{\lambda_3^{(i)}(t)}{2}\right)\right\} \\ \text{for 16 quadrature amplitude} \\ \quad \text{modulation (16QAM)} \end{cases}. \quad (25)$$

The LLR can be computed as

$$\begin{aligned} \lambda_x^{(i)}(t) &= \ln\left(\frac{p(b_{t,x} = 1)}{p(b_{t,x} = 0)}\right) \\ &\approx \frac{1}{2\rho^2}\left\{ \begin{array}{l} \left|\hat{d}(t) - \sqrt{\frac{2E_s}{T_s}} A d^{\min}_{b_{t,x}=0}\right|^2 \\ -\left|\hat{d}(t) - \sqrt{\frac{2E_s}{T_s}} A d^{\min}_{b_{t,x}=1}\right|^2 \end{array} \right\}, \end{aligned} \quad (26)$$

where $p(b_{t,x} = 1)$ and $p(b_{t,x} = 0)$ are *a posteriori* probabilities of the transmitted bit $b_{t,x} = 1$ and 0, respectively. $d^{\min}_{b_{t,x}=0}$ (or $d^{\min}_{b_{t,x}=1}$) is the most probable symbol that gives the minimum Euclidean distance from $\hat{d}(t)$ among all candidate symbols with $b_{t,x} = 0$ (or 1). A is the equivalent channel gain given by $(1/(N_c + N_g)) \sum_{k=0}^{N_c+N_g-1} W(k)\tilde{H}(k)$. $2\rho^2$ is the variance of the noise plus residual ISI and is given by [17]

$$\begin{aligned} 2\rho^2 = &\frac{1}{N_c + N_g} \sum_{k=0}^{N_c+N_g-1} |W(k)\tilde{H}(k)|^2 - A^2 \\ &+ \frac{\tilde{\nu}}{N_c + N_g} \sum_{k=0}^{N_c+N_g-1} |W(k)|^2. \end{aligned} \quad (27)$$

In the second step, the delay time-domain windowing technique is used to reduce the noise. $N_c + N_g$-point IDFT is applied to $\{\hat{H}^{(i)}(k);\ k = 0 \sim N_c + N_g - 1\}$ to obtain the channel impulse response estimate $\{\tilde{h}^{(i)}(\tau);\ \tau = 0 \sim N_c + N_g - 1\}$. Assuming that the channel impulse response length is less than the GI length (note that the noise is spread over an entire delay-time range), replacing $\tilde{h}^{(i)}(\tau)$ with zero's for $N_g \leq \tau \leq N_c + N_g - 1$ and applying $N_c + N_g$-point DFT to obtain the improved channel gain estimates $\{\tilde{H}^{(i)}(k);\ k = 0 \sim N_c + N_g - 1\}$.

After channel gain estimation, noise power estimate $\tilde{\nu}^{(i)}$ is obtained as [18]

$$\tilde{\nu}^{(i)} = \frac{1}{2}\frac{1}{N_B}\frac{1}{N_c + N_g}\sum_{n=0}^{N_B-1}\sum_{k=0}^{N_c+N_g-1} |Y^{(n)}(k) - \tilde{H}^{(i)}\tilde{S}^{(n,i-1)}(k)|^2. \quad (28)$$

$\tilde{\nu}^{(i)}$ includes the contribution from the channel estimation error.

In the second step, a series of channel estimation, MMSE-FDE and soft decision is repeated a sufficient number of times.

4. Computer Simulation Results

The BER performance with the proposed 2-step frequency-domain iterative channel estimation is evaluated by computer simulation. The simulation parameters are summarized in Table 1. QPSK or 16QAM is used for data modulation. $N_c = 64$, $N_g = 16$, and 16-path frequency-selective Rayleigh fading channel having uniform power delay profile are assumed. Although the block fading is assumed in Sect. 2 and 3, performance evaluations are done using a doubly selective fading channel model where the path gains time-vary symbol by symbol. The Chu sequence [19] is used for TS.

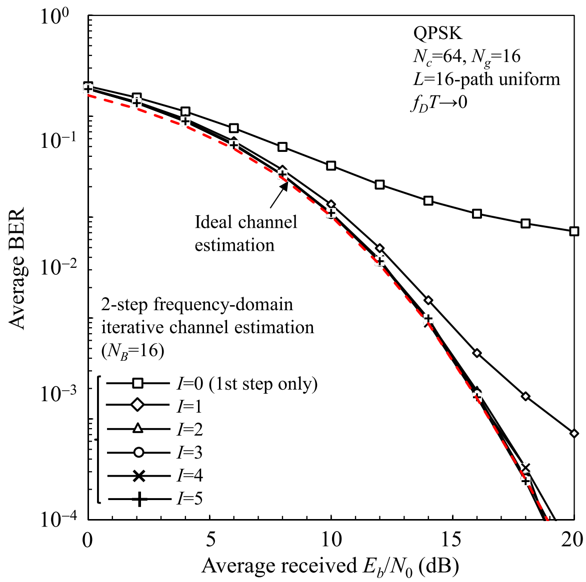
Figure 5 plots the BER performance of the proposed 2-step frequency-domain iterative channel estimation as a function of the average received $E_b/N_0(=(E_s/N_0)(1+N_g/N_c)/\log_2 X)$, where E_s is the symbol energy, N_0 is the one-sided noise power spectrum density, and X is the modulation level, with the number I of iterations as a parameter when $N_B = 16$. We have assumed the normalized Doppler frequency $f_D T_s \rightarrow 0$, where T_s is the symbol duration. It can be seen from Fig. 5 that by increasing I, the BER performance improves and approaches that with perfect channel estimation. In the first step of the channel estimation, the channel estimation accuracy is poor due to the interference from the data block. However, in the second step, as the channel estimation is repeated using both TS and the estimated data block, the channel estimation accuracy improves.

Figure 6 plots the BER performance of the proposed 2-step frequency-domain iterative channel estimation as a function of the average received E_b/N_0 with the averaging interval N_B in blocks as a parameter. The number of iteration is set to two ($I = 2$). It can be seen from Fig. 6 that the BER performance improves by increasing N_B. This is because as N_B increases, the noise and interference from the data block can be better suppressed.

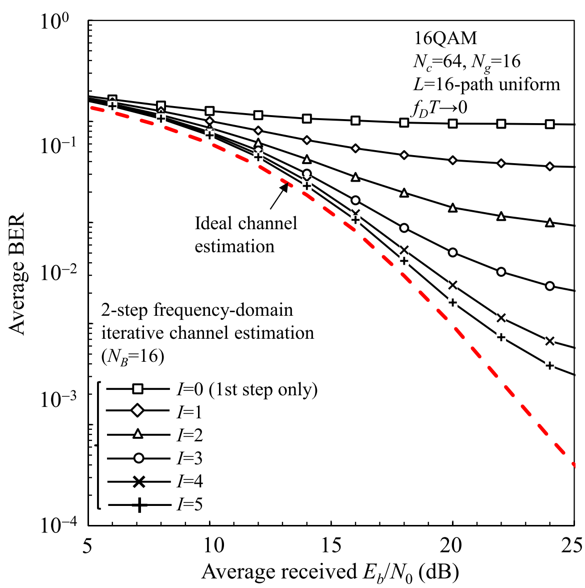
Figure 7 plots the required E_b/N_0 for achieving BER=10^{-3} as a function of I. By increasing N_B, smaller I can be used to reduce the E_b/N_0 gap from the perfect channel estimation. The combinations of N_B and I which achieve the

Table 1 Computer simulation condition.

Transmitter	Data modulation	QPSK, 16QAM
	Data symbol block length	N_c=64
	TS length	N_g=16
	TS	Chu sequence
Channel	Fading type	Frequency-selective Rayleigh
Receiver	Power delay profile	L=16 path uniform power delay profile
	Equalization	MMSE-FDE
	Channel estimation	2-step frequency-domain iterative channel estimation

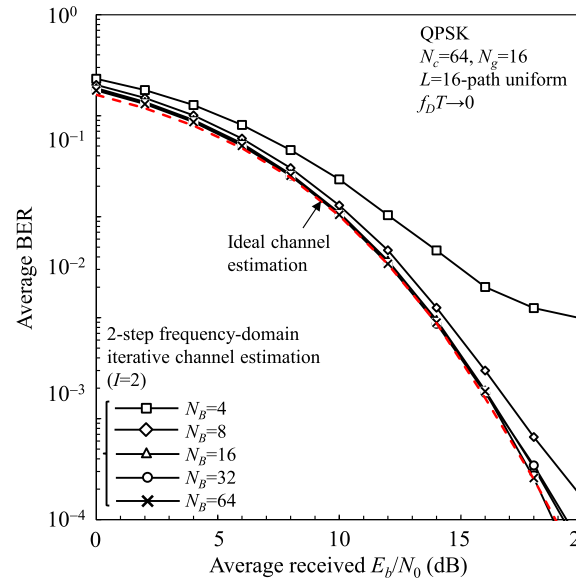


(a) QPSK

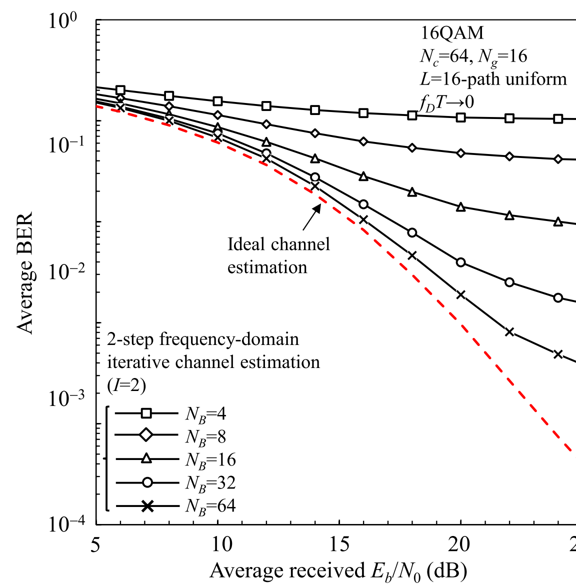


(b) 16QAM

Fig. 5 Impact of the number I of iterations.



(a) QPSK



(b) 16QAM

Fig. 6 Impact of the number N_B of blocks for averaging.

E_b/N_0 gap from the perfect channel estimation for achieving BER=10^{-3} within 0.5 dB are (N_B, I)=(64, 1), (32, 1), (16, 2), and (8, 3) for QPSK. For 16QAM, (N_B, I)=(64, 3) and (32, 4) can be used to achieve the E_b/N_0 gap from the perfect channel estimation for achieving BER=10^{-3} within 0.5 dB. The BER performance is very sensitive to the channel estimation accuracy as the Euclidean distance between the symbol candidates becomes smaller with higher data modulation scheme. Therefore, larger N_B and I are needed for higher data modulation.

So far, we have considered the case where the channel gains stay constant over the averaging interval N_B in blocks (i.e., $f_DT_s \to 0$). Below, we will consider the case where the channel gains vary symbol-by-symbol. Figure 8 shows the impact of normalized Doppler frequency f_DT_s on the average BER. Three cases of (N_B, I); (32, 1), (16, 2), and (8, 5) are plotted for QPSK. For 16QAM, two cases of (N_B, I); (64, 3) and (32, 4) are plotted. These are the combinations of N_B and I which achieve the E_b/N_0 gap from the perfect channel estimation for achieving BER=10^{-3} within 0.5 dB when $f_DT_s \to 0$. It can be seen from Fig. 8(a) that the BER of QPSK is almost constant when $f_DT_s \leq 5\times10^{-6}$, $f_DT_s \leq 2\times10^{-5}$, and $f_DT_s \leq 3\times10^{-5}$ for N_B = 32, 16, and 8, respectively. By increasing N_B, smaller number of iterations can be used due to better suppression of the noise and the interference from the data block, but the BER starts to increase because the tracking ability against fading tends to be lost.

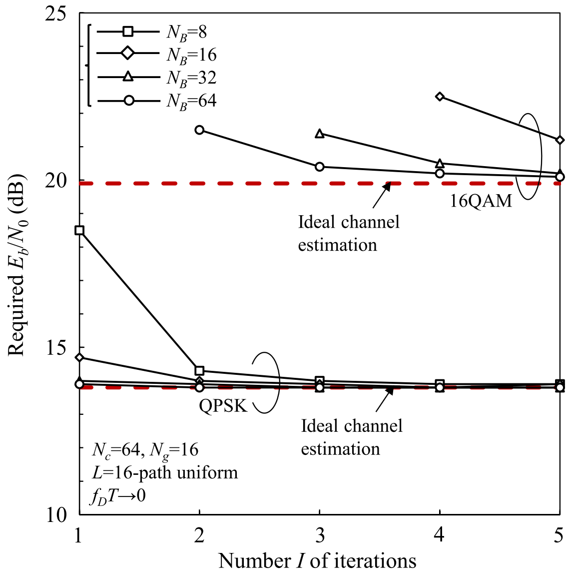
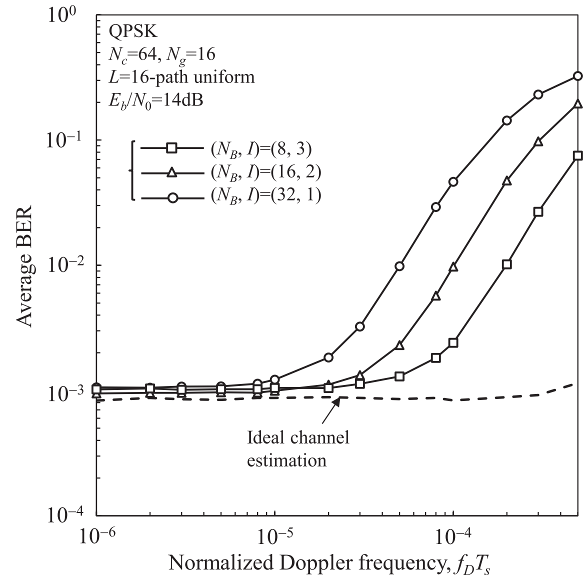
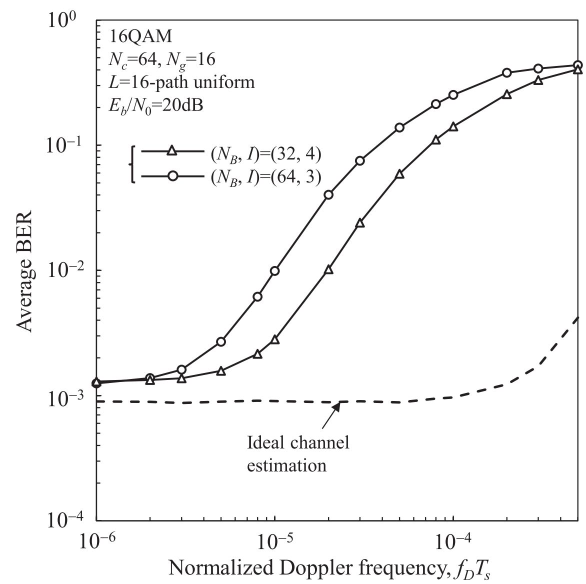


Fig. 7 Required E_b/N_0 for achieving BER=10^{-3}.



(a) QPSK



(b) 16QAM

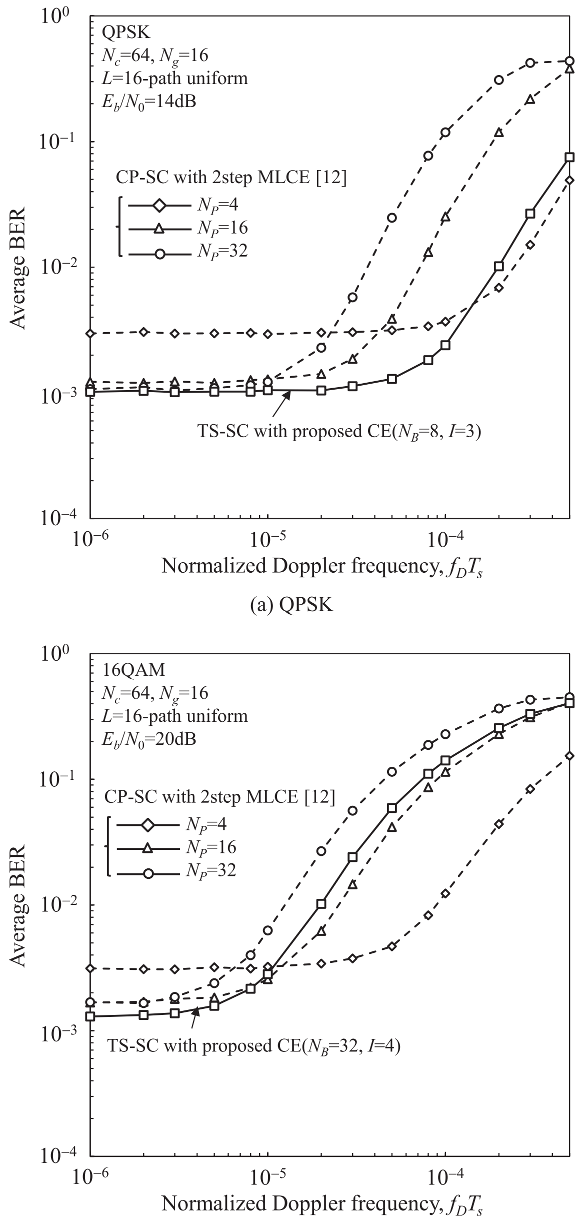
Fig. 8 Impact of normalized Doppler frequency.

Assuming 50 MHz signal bandwidth at the carrier frequency 2 GHz, the normalized Doppler frequency $f_DT_s = 3 \times 10^{-5}$ correspond to a travelling speed of 810 km/h. It can be said that for QPSK the proposed 2-step frequency-domain iterative channel estimation achieves a performance close to the perfect channel estimation even in a fast fading environment. It can be seen from Fig. 8(b) that the BER of 16QAM is almost constant when $f_DT_s \le 2\times10^{-6}$ and $f_DT_s \le 4\times10^{-6}$ for $N_B = 64$ and 32, respectively. Assuming 50 MHz signal bandwidth at the carrier frequency 2 GHz, the normalized Doppler frequency $f_DT_s = 4 \times 10^{-6}$ correspond to a travelling speed of 108 km/h. It can be said that the proposed 2-step frequency-domain iterative channel estimation robust against a relatively fast fading environment even in the case of 16QAM.

It is interesting to compare the TS-SC performance with CP-SC. Figure 9 illustrates the BER performances with TS-SC transmission and CP-SC transmission. For TS-SC transmission, the proposed 2-step channel estimation scheme with $N_B = 8$ (32) and $I = 3$ (4) is applied for QPSK (16QAM). For CP-SC transmission, 2-step MLCE [12] is applied. For channel estimation, one pilot block is transmitted every $N_p - 1$ blocks. Transmission efficiency degrades as N_p decreases. It can be seen form Fig. 9 that for QPSK, TS-SC with the proposed 2-step channel estimation provides almost the same tracking ability as the CP-SC with $N_p = 4$. Moreover, TS-SC provides 1.3 times higher transmission efficiency than the CP-SC with $N_p = 4$. For 16QAM, TS-SC with the proposed channel estimation provides about 1.1 times higher transmission efficiency while achieving the almost the same tracking ability.

5. Conclusion

In this paper, we proposed a 2-step frequency-domain channel estimation scheme suitable for TS-SC block transmission with MMSE-FDE. The proposed scheme performs the channel estimation in two steps and allows the use of shorter TS (but, longer than the channel length). In the first step, the received TS having cyclic property is constructed for performing frequency-domain channel estimation. The improved channel estimate is obtained by using simple averaging the instantaneous channel estimates over several blocks to suppress the interference from the data block and noise. In the second step, the channel estimation is repeated using both the TS and the estimated symbol sequence. We also proposed the noise power estimation scheme in the first step which does not require the channel estimates. It was demonstrated by computer simulation that by choosing properly the averaging interval N_B in blocks and number I of itera-



(a) QPSK

(b) 16QAM

Fig. 9 Comparison between TS-SC and CP-SC.

tions, the proposed 2-step frequency-domain iterative channel estimation achieves a BER performance close to the perfect channel estimation case even in a relatively fast fading environment. In addition, it was shown that the TS-SC transmission with the proposed channel estimation scheme provides higher transmission efficiency than the CP-SC transmission.

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