IEICE TRANS. COMMUN., VOL.E97–B, NO.2 FEBRUARY 2014

PAPER

Training Sequence Inserted OFDM Transmission with MMSE-FDE

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SUMMARY Orthogonal frequency division multiplexing (OFDM) has been attracting much attention because of its robustness against frequency selective fading. Instead of well-known cyclic prefix (CP) insertion, known training sequence (TS) insertion can be used for OFDM block transmission (called TS-OFDM). In this paper, we propose a new receiver design, which can obtain the frequency diversity gain through the use of frequency-domain equalization (FDE) for TS-OFDM. A conditional bit error rate (BER) analysis of the proposed FDE is presented. The average BER performance of the TS-OFDM signal transmission in a frequency-selective Rayleigh fading channel is evaluated by the Monte-Carlo numerical computation method using the derived conditional BER and is confirmed by computer simulation. Numerical and computer simulation results show the proposed TS-OFDM with FDE improves BER and throughput performance of TS-OFDM compared to the conventional TS-OFDM receiver due to the frequency diversity gain. It is also shown that the proposed TS-OFDM with FDE is more robust against imperfect channel estimation than the conventional TS-OFDM receiver.
key words: *OFDM, frequency-domain equalization, training sequence*

1. 1. Introduction

Broadband data services are demanded in next generation mobile communication systems. Since the mobile wireless channel is composed of many propagation paths with different time delays, the broadband channel becomes severely frequency-selective [1]. To mitigate the problems resulting from the severe frequency-selectivity of the channel, orthogonal frequency division multiplexing (OFDM) [2], [3] has been attracting much attention. OFDM is block transmission using a number of orthogonal subcarriers. Before the transmission, the cyclic prefix (CP) is often inserted into guard interval (GI) placed in front of each OFDM signal to make the received OFDM signal a circular convolution of the transmitted OFDM signal and the channel impulse response. Each data symbol in a block is transmitted in parallel using a different orthogonal subcarrier and hence, simple one-tap frequency-domain equalization (FDE) (i.e., zero-forcing (ZF)-FDE) can be used.

Instead of CP insertion, known training sequence (TS) insertion [4]–[6] can be used. The TS can be utilized for channel estimation and therefore, no pilot block is needed or no pilot subcarrier is needed unlike conventional CP inserted OFDM transmission schemes, which can improve the transmission efficiency [5]. For the reception of TS-OFDM signals, the overlap and add (OLA) processing was proposed [4], [7]. The OLA copies the GI part of the next OFDM signal and adds it to the beginning of the present OFDM signal to make the received OFDM signal to be a circular convolution of the transmitted OFDM signal and the channel impulse response. After OLA processing, the same signal processing as CP-OFDM (i.e, discrete Fourier transform (DFT) and simple one-tap ZF-FDE) can be applied. However, the conventional TS-OFDM with OLA processing cannot obtain the frequency diversity gain similar to the CP-OFDM.

In this paper, we propose a novel receiver design, which can obtain the frequency diversity gain through the use of FDE for TS-OFDM. In the proposed scheme, FDE is performed over an OFDM symbol block plus TS. The TS inserted to the end of previous OFDM symbol block acts as a CP of the present OFDM symbol block plus TS. The conditional bit error rate (BER) analysis of the proposed FDE is presented. The average BER performance in a frequency-selective Rayleigh fading channel is evaluated by Monte-Carlo numerical computation method using the derived conditional BER and is confirmed by computer simulation of the TS-OFDM signal transmission.

Packet access will be the core technology of the next generation mobile data communication systems. High-speed packet transmissions can be realized by the use of hybrid automatic repeat request (HARQ). HARQ using incremental redundancy (IR) strategy is known to achieve the high throughput performance [8]. In this paper, the throughput performance of TS-OFDM with HARQ is also presented. Since the TS-OFDM with the proposed FDE can obtain the frequency diversity gain, the throughput of HARQ with IR strategy can be improved compared to the TS-OFDM with the OLA processing.

The rest of the paper is organized as follows. In Sect. 2, transmission system model of the TS-OFDM with FDE is presented. Section 3 provides the proposed FDE for TS-OFDM and derives the conditional BER analysis. In Sect. 4, the performance evaluation is shown. In addition to the performance evaluation in the perfect channel estimation case, we investigate the impact of imperfect the channel estimation on the proposed FDE. Finally Sect. 5 concludes the paper.

Manuscript received April 16, 2013.
Manuscript revised September 1, 2013.
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DOI: 10.1587/transcom.E97.B.476

2. TS-OFDM Transmission with FDE

2.1 Transmission System Model

Transmission system model of TS-OFDM with FDE is illustrated in Fig. 1. We consider an OFDM transmission with N_c subcarriers. At the transmitter, information bit sequence is transformed into a data-modulated symbol sequence. Then, the data-modulated symbol sequence is divided into a sequence of symbol blocks of N_c symbols each. The data symbol block is expressed using the vector form as $\mathbf{D}=[D(0),\ldots,D(i),\ldots,D(N_c-1)]^T$, where $(.)^T$ expresses the transposition. The data symbol block $\mathbf{D}$ is transformed into the time-domain OFDM symbol block $\mathbf{d}=[d(0),\ldots,d(t),\ldots,d(N_c-1)]^T$ by using an N_c-point inverse DFT (IDFT), which is given by

$$\mathbf{d} = \mathbf{F}_{N_c}^H \mathbf{D}, \tag{1}$$

where $(.)^H$ is the Hermitian transpose operation and $\mathbf{F}_K$ is the DFT matrix of size $K \times K$ given by

$$\mathbf{F}_K = \frac{1}{\sqrt{K}} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{-j2\pi\frac{1\times1}{K}} & \cdots & e^{-j2\pi\frac{1\times(K-1)}{K}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j2\pi\frac{(K-1)\times1}{K}} & \cdots & e^{-j2\pi\frac{(K-1)\times(K-1)}{K}} \end{bmatrix}. \tag{2}$$

Before the transmission, the TS of length N_g ($\geq L$) samples is appended at the end of each OFDM symbol block. The propagation channel is assumed to be a frequency-selective block fading channel composed of sample-spaced L distinct propagation paths. The block $\mathbf{s}=[s(0),\ldots,s(t),\ldots,s(N_c+N_g-1)]^T$ to be transmitted is expressed as

$$\mathbf{s} = \begin{bmatrix} \mathbf{d} \\ \mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{N_c}^H \mathbf{D} \\ \mathbf{u} \end{bmatrix}. \tag{3}$$

where $\mathbf{u}=[u(0),\ldots,u(t),\ldots,u(N_g-1)]^T$ denotes the TS vector which is identical for all blocks. The TS-OFDM block structure is illustrated in Fig. 2. The difference from CP-OFDM transmission is that CP is replaced by TS.

The signal block is transmitted over a frequency-selective fading channel. The conventional FDE with OLA copies the GI part of the next OFDM signal and adds it to the beginning of the present OFDM signal. After OLA processing, the received OFDM signal is decomposed into N_c frequency components by N_c-point DFT and then, simple one-tap ZF-FDE is performed as in the case of CP-OFDM.

On the other hand, in the proposed FDE, the received signal is transformed by N_c+N_g-point DFT into the frequency-domain signal and then, minimum mean square error (MMSE) based FDE is carried out. After MMSE-FDE, N_c+N_g-point IDFT is applied to obtain the time-domain received TS-OFDM signal block. This time-domain received TS-OFDM signal block can be divided into two parts; a first N_c-sample signal block which corresponds to the OFDM data symbol block and last N_g-sample block which corresponds to the TS. Therefore, finally, OFDM demodulation is performed by applying N_c-point DFT to first N_c-sample signal block. Through a series of N_c+N_g-point DFT, MMSE-FDE, N_c+N_g-point IDFT, and OFDM demodulation, proposed TS-OFDM transmission can get the frequency diversity gain.

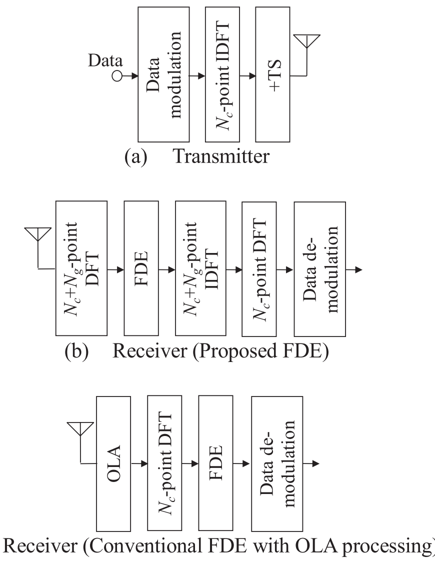

Fig. 1 System model of TS-OFDM with FDE.

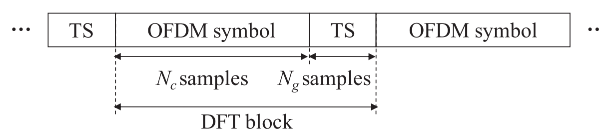

Fig. 2 Transmit block structure.

2.2 Received Signal Representation

The OFDM symbol block is transmitted over a frequency-selective fading channel. The channel impulse response $h(\tau)$ can be expressed as

$$h(\tau) = \sum_{l=0}^{L-1} h_l \delta(\tau - \tau_l), \tag{4}$$

where h_l and τ_l are respectively the complex-valued path gain with $E[\sum_{l=0}^{L-1}|h_l|^2] = 1$ and the time delay of the l-th path. We assume the l-th path has a time delay of l samples, i.e. ($\tau_l = l$). The received signal block $\mathbf{y}=[y(0),\ldots,y(t),\ldots,y(N_c+N_g-1)]^T$ can be expressed as

$$\mathbf{y} = \sqrt{\frac{2E_s}{T_s}}\mathbf{hs} + \mathbf{n}, \tag{5}$$

where $E_s$and T_s are respectively the symbol energy and duration, $\mathbf{h}$ is the $(N_c+N_g)\times(N_c+N_g)$ channel impulse response matrix given as

$$\mathbf{h}=\begin{bmatrix} h_0 & & & & h_{L-1} & \cdots & h_1 \\ h_1 & h_0 & & \mathbf{0} & & \ddots & \vdots \\ \vdots & h_1 & \ddots & & & & h_{L-1} \\ h_{L-1} & \vdots & \ddots & \ddots & & & \\ & h_{L-1} & & \ddots & \ddots & & \\ & & \ddots & & \ddots & \ddots & \\ \mathbf{0} & & & h_{L-1} & \cdots & h_1 & h_0 \end{bmatrix}, \tag{6}$$

and $\mathbf{n}=[n(0),\ldots,n(t),\ldots,n(N_c+N_g-1)]^T$ is the noise vector. The t-th element, $n(t)$, of $\mathbf{n}$ is the zero-mean additive white Gaussian noise (AWGN) having the variance $2N_0/T_s$ with N_0 being the one-sided noise power spectrum density.

3. FDE for TS-OFDM

In this section, first, we show the conventional FDE with OLA processing [4], [7] in Sect. 3.1. The proposed FDE for TS-OFDM is shown in Sect. 3.2. In Sect. 3.3, a theoretical conditional BER analysis is presented for the given channel condition.

3.1 Conventional FDE with OLA Processing

For the conventional FDE with OLA processing, the TS component needs to be removed from the received signal before data symbol detection. Figure 3 illustrates the OLA processing. First, the received OFDM signal having the cyclic property is constructed as

$$\bar{y}(t)=\begin{cases} y(t)+y(t+N_c) & t=0\sim N_g-1 \\ y(t) & t=N_g\sim N_c-1 \end{cases}. \tag{7}$$

Since the receiver knows the TS, the received distorted TS $\hat{u}(t)$, t=0$\sim$ N_g-1, can be generated as

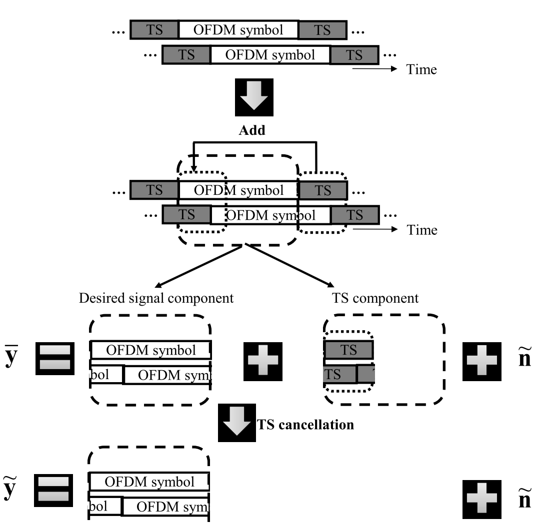

Fig. 3 OLA processing.

$$\hat{u}(t)=\sum_{l=0}^{L-1} h_l u(t-\tau_l), \tag{8}$$

which also has the cyclic property (note that the ideal channel estimation is assumed). Then, the TS is subtracted from the received OFDM signal to obtain

$$\tilde{y}(t)=\begin{cases} \bar{y}(t)-\hat{u}(t) & t=0\sim N_g-1 \\ \bar{y}(t) & t=N_g\sim N_c-1 \end{cases}, \tag{9}$$

which also has the cyclic property. Equation (9) can be rewritten by using the vector form as

$$\tilde{\mathbf{y}}=\sqrt{\frac{2E_s}{T_s}}\mathbf{h}_{N_c}\mathbf{F}_{N_c}^H\mathbf{D}+\tilde{\mathbf{n}}, \tag{10}$$

where $\mathbf{h}_{N_c}$ is the $N_c\times N_c$ channel impulse response matrix which has the circular property similar to (6). The second term denotes the noise, which is given by

$$\tilde{\mathbf{n}}=\mathbf{n}_{[0;N_c-1]}+\begin{bmatrix} \mathbf{n}_{[N_c;N_c+N_g-1]} \\ \mathbf{0}_{N_c-N_g} \end{bmatrix}, \tag{11}$$

where $\mathbf{0}_J$ represents a zero vector of size $J\times 1$ and $\mathbf{n}_{[t_1;t_2]}$ is the subvector of $\mathbf{n}$ defined as $\mathbf{n}_{[t_1;t_2]}=[n(t_1),...,n(t),...,n(t_2)]^T$. From (10), it can be understood that since the received OFDM signal is a circular convolution of the transmitted OFDM signal and the channel impulse response, the same signal processing as the CP-OFDM can be applied.

By applying N_c-point DFT to $\tilde{\mathbf{y}}$, $\tilde{\mathbf{y}}$ is transformed into the frequency-domain signal $\tilde{\mathbf{Y}}=[\tilde{Y}(0),...,\tilde{Y}(i),...,\tilde{Y}(N_c-1)]^T$ given by

$$\begin{aligned}\tilde{\mathbf{Y}}&=\mathbf{F}_{N_c}\tilde{\mathbf{y}}\\ &=\sqrt{\frac{2E_s}{T_s}}\mathbf{H}_{N_c}\mathbf{D}+\tilde{\mathbf{N}},\end{aligned} \tag{12}$$

where $\tilde{\mathbf{N}}=[\tilde{N}(0),...,\tilde{N}(i),...,\tilde{N}(N_c-1)]^T=\mathbf{F}_{N_c}\tilde{\mathbf{n}}$ is the frequency-domain noise vector and $\mathbf{H}_{N_c}=\mathbf{F}_{N_c}\mathbf{h}_{N_c}\mathbf{F}_{N_c}^H$ is the channel gain matrix. Due to the circulant property of $\mathbf{h}_{N_c}$, the channel gain matrix $\mathbf{H}_{N_c}$ is diagonal. The i-th (i=0$\sim$ N_c-1) diagonal element of $\mathbf{H}_{N_c}$ is given by

$$H_{N_c}(i)=\sum_{l=0}^{L-1} h_l\exp\left(-j2\pi i\frac{\tau_l}{N_c}\right). \tag{13}$$

FDE is carried out to obtain

$$\hat{\mathbf{Y}}=\mathbf{W}_{OLA}\tilde{\mathbf{Y}}, \tag{14}$$

where $\mathbf{W}_{OLA}=diag[W_{OLA}(0),\ldots,W_{OLA}(k),\ldots,W_{OLA}(N_c+N_g-1)]$ is the FDE weight matrix. The k-th diagonal element of $\mathbf{W}_{OLA}$ is given by

$$W_{OLA}(k)=\frac{H^*(k)}{|H(k)|^2}. \tag{15}$$

After OLA processing, the same signal processing as the CP-OFDM can be applied. The conventional TS-OFDM

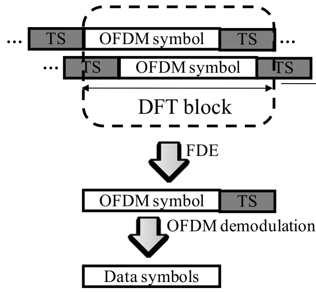

Fig. 4 The principle of the proposed FDE.

with OLA processing cannot obtain the frequency diversity gain similar to the CP-OFDM. Also, the transmission performance may be worse than CP-OFDM due to the increased noise by OLA processing.

3.2 Proposed FDE

For TS-OFDM, the same N_g-sample TS is inserted to the end of each N_c-sample OFDM symbol block. Therefore, it can be viewed as a CP of an $N_c + N_g$-sample block as shown in Fig. 4. The frequency-domain signal block obtained by $(N_c + N_g)$-point DFT $\mathbf{Y}=[Y(0),\ldots,Y(k),\ldots,Y(N_c+N_g-1)]^T$ is expressed as

$$\begin{aligned}\mathbf{Y} &= \mathbf{F}_{N_c+N_g}\mathbf{y}\\ &= \sqrt{\tfrac{2E_s}{T_s}}\mathbf{HS}+\mathbf{N}\end{aligned}, \tag{16}$$

where $\mathbf{S}=[S(0),\ldots,S(k),\ldots,S(N_c+N_g-1)]^T = \mathbf{F}_{N_c+N_g}\mathbf{s}$ represents the frequency-domain representation of transmit block, which contains N_c-sample OFDM symbol block and N_g-sample TS, $\mathbf{N}=[N(0),\ldots,N(k),\ldots,N(N_c+N_g-1)]^T = \mathbf{F}_{N_c+N_g}\mathbf{n}$ is the frequency-domain noise vector, and $\mathbf{H} = \mathbf{F}_{N_c+N_g}\mathbf{h}\mathbf{F}^H_{N_c+N_g}$ is the channel gain matrix. Due to the circulant property of $\mathbf{h}$, the channel gain matrix $\mathbf{H}$ is diagonal. The k-th diagonal element of $\mathbf{H}$ is given by

$$H(k) = \sum_{l=0}^{L-1} h_l \exp\left(-j2\pi k\frac{\tau_l}{N_c+N_g}\right). \tag{17}$$

FDE is carried out to obtain

$$\hat{\mathbf{Y}} = \mathbf{WY}, \tag{18}$$

where $\mathbf{W}=diag[W(0),\ldots,W(k),\ldots,W(N_c+N_g-1)]$ is the FDE weight matrix. The k-th diagonal element of $\mathbf{W}$ is given by

$$W(k) = \frac{H^*(k)}{|H(k)|^2+(E_s/N_0)^{-1}}. \tag{19}$$

$\mathbf{W}$ of (19) is the weight which minimizes the mean square error (MSE) between the transmit block $\mathbf{S}$ and output of the FDE $\hat{\mathbf{Y}}$.

$\hat{\mathbf{Y}}$ is transformed into a time-domain TS-OFDM signal block $\hat{\mathbf{s}} = [\hat{s}(0), ..., \hat{s}(t), ..., \hat{s}(N_c+N_g-1)]^T$ by (N_c+N_g)-point IDFT as

$$\hat{\mathbf{s}} = \mathbf{F}^H_{N_c+N_g}\hat{\mathbf{Y}}. \tag{20}$$

The time-domain TS-OFDM signal block $\hat{\mathbf{s}}$ can be divided into two parts; first N_c-sample signal block which corresponds to the OFDM data symbol block and last N_g-sample block which corresponds to the TS. Therefore, to obtain the decision variable vector $\hat{\mathbf{D}} = [\hat{D}(0), ..., \hat{D}(i), ..., \hat{D}(N_c-1)]^T$, N_c-point DFT is applied to transform N_c-sample signal block $\{\hat{s}(0), ..., \hat{s}(t), ..., \hat{s}(N_c-1)\}$ into N_c frequency components as

$$\hat{\mathbf{D}} = \mathbf{F}_{N_c}[\hat{s}(0), ..., \hat{s}(t), ..., \hat{s}(N_c-1)]^T. \tag{21}$$

In principle, the above described FDE process for TS-OFDM is similar to the FDE process for single-carrier (SC) block transmission [9], [10] except for the last DFT operation. Since the received TS-OFDM signal using N_c orthogonal subcarriers is decomposed by N_c+N_g-point DFT into more than N_c frequency components, the frequency diversity gain can be obtained unlike the CP-OFDM and the conventional TS-OFDM with OLA processing. It should be noted that the inter-symbol interference (ISI) is also produced. It should also be noted that since additional IDFT and DFT are required, the computational complexity of the proposed FDE increases compared to the conventional TS-OFDM with OLA processing. The computational complexities of the proposed FDE and the OLA processing are compared. The complexity is defined here as the number of complex multiply operations. The overall computational complexity of the proposed FDE is the sum of the complexities required for N_c+N_g-point DFT, weight multiplication, TS cancellation, N_c+N_g-point IDFT, and N_c-point DFT. On the other hand, the overall computational complexity of the OLA processing is the sum of the complexity required for TS cancellation, N_c-point DFT, and weight multiplication. In general, the number N_c of subcarriers in OFDM transmission is set to a power of 2 and therefore, the well-known efficient DFT algorithm (i.e., fast Fourier transform (FFT)) can be applied. The proposed FDE requires N_c+N_g-point DFT and IDFT and the number N_c+N_g may not necessary be a power of 2. However, the prime factor FFT algorithm [11] can be applied in this case. When N_c=64 and N_g=16, the overall computational complexity of the proposed FDE per block becomes 2,135 while that of the OLA processing is 768 and hence, the computational cost of the proposed FDE is about 2.8 times higher than the OLA processing.

3.3 BER Analysis

From (16)–(21), the i-th subcarrier component $\hat{D}(i)$, i=0~N_c-1, can be written as

$$\begin{aligned}\hat{D}(i) = &\frac{1}{\sqrt{N_c}}\frac{1}{\sqrt{N_c+N_g}}\sqrt{\frac{2E_s}{T_s}}\sum_{k=0}^{N_c+N_g-1}\hat{H}(k)S(k)\\ &\times\exp\left[j\pi(N_c-1)\left(\frac{k}{N_c+N_g}-\frac{i}{N_c}\right)\right]\Phi(k,i)\\ &+\frac{1}{\sqrt{N_c}}\frac{1}{\sqrt{N_c+N_g}}\sum_{k=0}^{N_c+N_g-1}\hat{N}(k)\end{aligned}$$

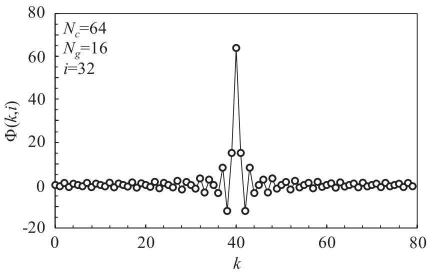

Fig. 5 $\Phi(k, i = 32)$ for $N_c = 64$ and $N_g = 16$.

$$\times \exp\left[j\pi(N_c - 1)\left(\frac{k}{N_c + N_g} - \frac{i}{N_c}\right)\right]\Phi(k, i), \tag{22}$$

where $\hat{H}(k) = W(k)H(k)$ and $\hat{N}(k) = W(k)N(k)$ and

$$\Phi(k, i) = \begin{cases} N_c \text{ if } k = (1 + N_g/N_c)i \\ \dfrac{\sin \pi N_c\left(\dfrac{k}{N_c + N_g} - \dfrac{i}{N_c}\right)}{\sin \pi\left(\dfrac{k}{N_c + N_g} - \dfrac{i}{N_c}\right)} \text{ otherwise} \end{cases}. \tag{23}$$

As an example, $\Phi(k, i{=}32)$ for N_c=64, N_g=16 is plotted in Fig. 5.

Since the first term is decompose into the desired signal, residual ISI, and TS components, (22) can be rewritten as

$$\hat{D}(i) = \sqrt{\frac{2E_s}{T_s}}\left(\frac{1}{N_c}\frac{1}{N_c + N_g}\sum_{k=0}^{N_c+N_g-1}\hat{H}(k)\Phi^2(k, i)\right)D(i)$$
$$+\mu_{TS}(i) + \mu_{ISI}(i) + \mu_{noise}(i), \tag{24}$$

where second, third, and fourth components are respectively denote the TS, residual ISI, and noise components. If the ideal knowledge of channel state information is assumed, TS component can be cancelled perfectly (i.e., $\mu_{TS}(i)$=0) and this can be performed simply in the frequency-domain as

$$\hat{\mathbf{Y}} = \mathbf{WY} - \sqrt{\frac{2E_s}{T_s}}\mathbf{WHF}_{N_c+N_g}\begin{bmatrix}\mathbf{0}\\ \mathbf{u}\end{bmatrix}. \tag{25}$$

Residual ISI and noise components are given by

$$\begin{cases} \mu_{ISI}(i) = \sqrt{\dfrac{2E_s}{T_s}}\displaystyle\sum_{\substack{i'=0\\ \neq i}}^{N_c-1} d(i')\exp\left[j\pi(N_c - 1)\frac{i - i'}{N_c}\right] \\ \quad\times\left(\dfrac{1}{N_c}\dfrac{1}{N_c + N_g}\displaystyle\sum_{k=0}^{N_c+N_g-1}\hat{H}(k)\Phi(k, i)\Phi(k, i')\right) \\ \mu_{noise}(i) = \dfrac{1}{\sqrt{N_c}}\dfrac{1}{\sqrt{N_c + N_g}}\displaystyle\sum_{k=0}^{N_c+N_g-1}\hat{N}(k) \\ \quad\times\exp\left[j\pi(N_c - 1)\left(\dfrac{k}{N_c + N_g} - \dfrac{i}{N_c}\right)\right]\Phi(k, i) \end{cases}. \tag{26}$$

We assume the Quadrature phase shift keying (QPSK) data modulation. It can be understood from (24) that $\hat{D}(i)$ is a complex-valued random variable with mean $\sqrt{2E_s/T_s}(1/N_c)(1/(N_c{+}N_g))\sum_{k=0}^{N_c+N_g-1}\hat{H}(k)\Phi^2(k, i)D(i)$. Approximating $\mu_{ISI}(i)$ and $\mu_{noise}(i)$ as a zero mean complex-valued Gaussian variable, $\mu(i) = \mu_{ISI}(i) + \mu_{noise}(i)$ can be treated as a new zero mean complex-valued Gaussian variable. The variance $2\sigma_\mu^2(i)$ of $\mu(i)$ is given by $2\sigma_{\mu_{ISI}}^2(i) + 2\sigma_{\mu_{noise}}^2(i)$. From (26), $\sigma_{\mu_{ISI}}^2(i)$ and $\sigma_{\mu_{noise}}^2(i)$ can be derived as

$$\begin{cases} \sigma_{\mu_{ISI}}^2(i) = \dfrac{E_s}{T_s}\displaystyle\sum_{\substack{i'=0\\ \neq i}}^{N_c-1}\left|\begin{matrix}\dfrac{1}{N_c}\dfrac{1}{N_c + N_g} \\ \times\displaystyle\sum_{k=0}^{N_c+N_g-1}\hat{H}(k)\Phi(k, i)\Phi(k, i')\end{matrix}\right|^2 \\ \sigma_{\mu_{noise}}^2(i) = \dfrac{1}{N_c}\dfrac{1}{N_c + N_g}\dfrac{N_0}{T_s}\displaystyle\sum_{k=0}^{N_c+N_g-1}|W(k)\Phi(k, i)|^2 \end{cases}. \tag{27}$$

The conditional BER for the given E_s/N_0 and $\mathbf{H}$ can be given by [1]

$$p_b\left(i, \frac{E_s}{N_0}, \mathbf{H}\right) = \frac{1}{2}erfc\left[\sqrt{\frac{1}{4}\gamma\left(i, \frac{E_s}{N_0}, \mathbf{H}\right)}\right], \tag{28}$$

where $erfc(x)$ is the complementary error function and $\gamma(i, E_s/N_0, \mathbf{H})$ is the conditional signal-to-interference plus noise power ratio (SINR), which is given by

$$\gamma\left(i, \frac{E_s}{N_0}, \mathbf{H}\right) = \frac{\dfrac{2E_s}{T_s}\left|\dfrac{1}{N_c}\dfrac{1}{N_c + N_g}\displaystyle\sum_{k=0}^{N_c+N_g-1}\hat{H}(k)\Phi^2(k, i)\right|^2}{\sigma_{\mu_{ISI}}^2(i)}$$
$$= \frac{\dfrac{2E_s}{N_0}\left|\dfrac{1}{N_c}\dfrac{1}{N_c + N_g}\displaystyle\sum_{k=0}^{N_c+N_g-1}\hat{H}(k)\Phi^2(k, i)\right|^2}{\left[\begin{matrix}\dfrac{E_s}{N_0}\displaystyle\sum_{\substack{i'=0\\ \neq i}}^{N_c-1}\left|\dfrac{1}{N_c}\dfrac{1}{N_c + N_g}\displaystyle\sum_{k=0}^{N_c+N_g-1}\hat{H}(k)\Phi(k, i)\Phi(k, i')\right|^2 \\ +\dfrac{1}{N_c}\dfrac{1}{N_c + N_g}\displaystyle\sum_{k=0}^{N_c+N_g-1}|W(k)|^2\Phi^2(k, i)\end{matrix}\right]}. \tag{29}$$

The theoretical average BER is numerically computed by averaging (28) over all possible $\mathbf{H}$ and is confirmed by computer simulation in the next section.

4. Performance Evaluation

The numerical and simulation conditions are summarized in Table 1. QPSK data modulation, N_c=64, N_g=16, and 8-path frequency-selective block Rayleigh fading channel having uniform power delay profile are assumed.

Table 1 Numerical and simulation condition.

Transmitter	Data modulation	QPSK
	Data symbol block length	N_c=64
	TS length	N_g=16
	TS	Chu sequence [12]
Channel	Fading type	Frequency-selective block Rayleigh
	Power delay profile	L=8 path uniform power delay profile
Receiver	Equalization	OLA, Proposed FDE
	Channel estimation	Ideal / 2-step frequency-domain iterative channel estimation [13]

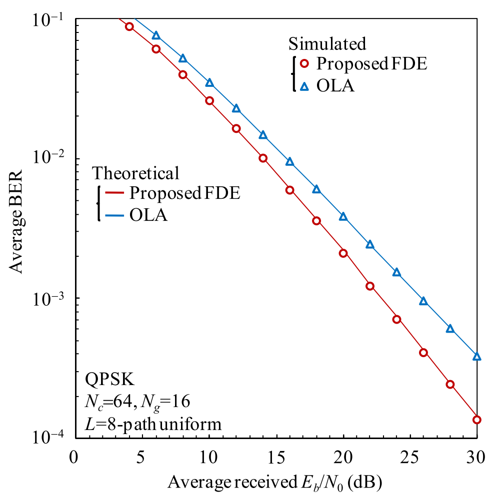

Fig. 6 Average BER performance.

4.1 Average BER Performance

Figure 6 plots the theoretical and simulated average BER performances of TS-OFDM with the proposed FDE as a function of the average received $E_b/N_0(=(E_s/N_0)(1+N_g/N_c)/2)$. Ideal channel estimation is assumed. For comparison, the BER performance when using OLA processing is also plotted. It can be seen from Fig. 6 that a fairly good agreement between the theoretical and simulated results is seen. The proposed FDE can improve the BER performance of TS-OFDM than the conventional OLA processing. This is because the frequency diversity gain can be obtained.

4.2 HARQ Throughput Performance

Type II HARQ is considered. In type II HARQ-OFDM, the uncoded packet is sent at the initial transmission and consequently, no coding gain can be obtained. Therefore, it is desirable to exploit the channel frequency selectivity in the first transmission. This is possible by our proposed FDE. In this section, the type II HARQ throughput performance of TS-OFDM is evaluated by computer simulation. We employ

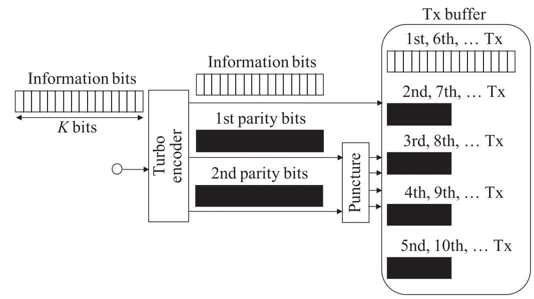

Fig. 7 HARQ type II S-P4.

a rate 1/3 turbo encoder using two (13, 15) recursive systematic convolutional (RSC) component encoders HARQ type II S-P4 [14] as illustrated in Fig. 7. Log-MAP decoding with 6 iterations is assumed. The packet size is set to K=512. The turbo encoder outputs the systematic bit sequence and two parity bit sequences. These sequences are punctured into five sequences (including systematic bit sequence) by the puncturing matrices given by

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \tag{30}$$

where the 1st, 2nd, and 3rd rows denote the puncturing pattern for the systematic bit sequence, 1st parity bit sequence, and 2nd parity bit sequence, respectively. For the first transmission, only the systematic bit sequence is transmitted. At the receiver, data decision and error detection are performed. If any error is detected in the received packet, second transmission is requested from the receiver by sending a negative acknowledgement (NACK) signal. When the NACK signal is received at the transmitter, the second packet (consisting of the punctured parity bit sequence) is transmitted. At the receiver, turbo decoding is carried out by using the first and second received packets. If any error is detected after turbo decoding, the NACK signal is transmitted again. One of the punctured parity bit sequences is transmitted each time the NACK signal is received at the transmitter until the 5th packet transmission. After the 5th packet transmission, the same packet is retransmitted.

The log likelihood ratio (LLR) is used as the soft-input in the turbo decoder. From (23)–(26), the LLR of the x-th (x=0~ $X-1$) bit associated with the i-th symbol (X is the number of bits per symbol and $i = N_c - 1$) in a block is given as

$$\lambda_x(i) = \ln\left(\frac{p(b_{i,x}=1)}{p(b_{i,x}=0)}\right)$$
$$= \frac{\left|\hat{D}(i) - \sqrt{\frac{2E_s}{T_s}}\left(\frac{1}{N_c}\frac{1}{N_c+N_g}\sum_{k=0}^{N_c+N_g-1}\hat{H}(k)\Phi^2(k,i)\right)d^{\min}_{b_{i,x}=0}\right|}{2\sigma_{\mu_{ISI}}(i) + 2\sigma_{\mu_{noise}}(i)}$$

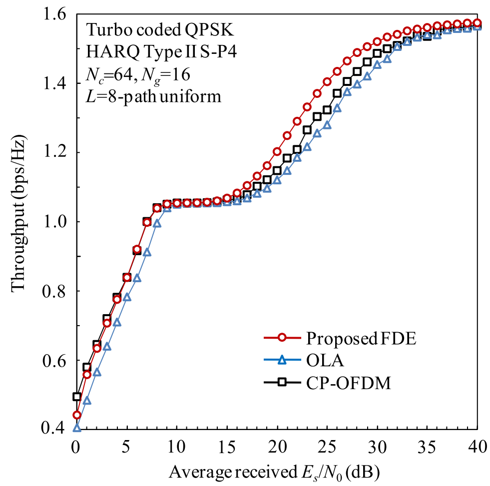

Fig. 8 Throughput performance.

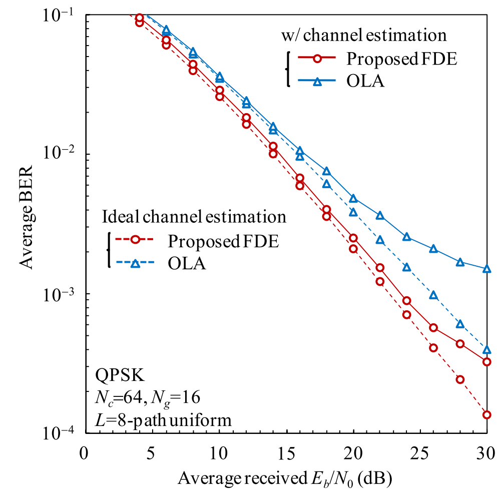

Fig. 9 Impact of the channel estimation.

$$-\frac{\left|\hat{D}(i)-\sqrt{\frac{2E_s}{T_s}}\left(\frac{1}{N_c}\frac{1}{N_c+N_g}\sum_{k=0}^{N_c+N_g-1}\hat{H}(k)\Phi^2(k,i)\right)d^{\min}_{b_{i,x}=1}\right|}{2\sigma_{\mu_{ISI}}(i)+2\sigma_{\mu_{noise}}(i)}, \tag{31}$$

where $p(b_{i,x}{=}0)$ and $p(b_{i,x}{=}1)$ are the *a posteriori* probabilities of the transmitted bit $b_{i,x}$ being $b_{i,x}{=}0$ and $b_{i,x}{=}1$, respectively, and $d^{\min}_{b_{i,x}=0}$ (or $d^{\min}_{b_{i,x}=1}$) is the symbol having the shortest Euclidean distance from $\hat{D}(i)$ and whose x-th bit in 0 (or 1).

The throughput performance of TS-OFDM with HARQ using the proposed FDE is plotted in Fig. 8 as a function of average received symbol energy-to-noise power spectrum density ratio E_s/N_0. For comparison, the throughput performances of TS-OFDM using the OLA processing and CP-OFDM are also plotted. It can be seen form Fig. 8 that the proposed FDE can achieve better throughput performance than the conventional OLA processing and improve the throughput compared to the CP-OFDM in high E_s/N_0 region due to the frequency diversity gain. TS-OFDM with the proposed FDE provides at most the 15% improvement in throughput compared to TS-OFDM with OLA processing and the 6% improvement in throughput compared to CP-OFDM, respectively.

4.3 Impact of Channel Estimation

The average BER performance of TS-OFDM using the proposed FDE with the actual channel estimation scheme is investigated. The 2-step frequency-domain channel estimation scheme which we have proposed in [13] is used for channel estimation. The number of iterations and the number of blocks to be used for averaging are set to 0 and 64, respectively. We have assumed the normalized Doppler frequency $f_DT_s \rightarrow 0$. For comparison, the BER performance when using OLA processing is also plotted. It can be seen from Fig. 9 that the proposed FDE provides lower BER even when the channel estimation error is present and is more robust against the channel estimation error than the conventional FDE with OLA processing. The E_b/N_0 loss from the ideal channel estimation case for BER=$2{\times}10^{-3}$ is about 0.8 dB for the proposed FDE, but about 3.2 dB for the conventional FDE with OLA processing.

5. Conclusion

In this paper, we proposed a new receiver design for TS-OFDM with FDE which can obtain the frequency diversity gain. We presented the theoretical BER analysis and confirmed it by computer simulation. It was shown that the proposed FDE improves BER and throughput performances of TS-OFDM compared to the conventional FDE with OLA processing. It was also shown that the TS-OFDM with the proposed FDE is more robust against the imperfect channel estimation than the conventional FDE with OLA processing.

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