Analog SC-FDE using SSB technique

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Abstract: In order to improve performance while keeping high spectrum efficiency of analog signal transmission, we recently proposed an analog single-carrier transmission with frequency-domain equalization (analog SC-FDE). In this paper, in order to improve the spectrum efficiency, analog SC-FDE using single sideband (SSB) technique is proposed. A theoretical analysis of normalized mean square error (NMSE) performance is carried out and confirmed by computer simulation. We show that analog SC-FDE using SSB technique achieves better NMSE performance than both conventional SSB transmission and analog SC-FDE while doubling spectrum efficiency of analog SC-FDE.

Keywords: analog signal transmission, SSB, FDE

Classification: Wireless Communication Technologies

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1   Introduction

Recently, we proposed a novel analog single-carrier transmission with frequency-domain equalization (analog SC-FDE) [1]. In analog SC-FDE, the frequency components obtained by discrete Fourier transform (DFT) are mapped over a broader bandwidth which experiences frequency-selective fading. At a receiver, one-tap FDE [2] is adopted to take advantage of frequency-selective fading channel (i.e., frequency diversity). Multi-access using analog SC-FDE is possible based on the principle of frequency-division multiple access (FDMA) [3]. We showed in [1] that analog SC-FDE can lower the normalized mean square error (NMSE) compared to the conventional analog signal transmission.

Since the frequency components in upper sideband (USB) and lower sideband (LSB) of the analog signal have a complex conjugate relation, the whole spectrum can be reproduced from one of these sidebands. In this paper, in order to improve the spectrum efficiency, we exploit a characteristic of analog signal and propose an analog SC-FDE using SSB technique. The proposed scheme applies DFT, spectrum shaping filter, spectrum mapping, inverse DFT (IDFT), and cyclic prefix (CP) insertion before transmission. At the receiver, the original analog signal spectrum is reproduced by spectrum copying after one-tap FDE. A theoretical analysis of NMSE performance is presented and confirmed by computer simulation.

There are two different points between analog SC-FDE and analog SC-FDE using SSB technique. First, the spectrum shaping filter keeps both LSB and USB in analog SC-FDE, however, it removes the USB frequency components in analog SC-FDE using SSB technique. Secondly, the spectrum copying is used in analog SC-FDE using SSB technique for reproducing the removed USB components by exploiting the complex conjugate relation between LSB and USB.
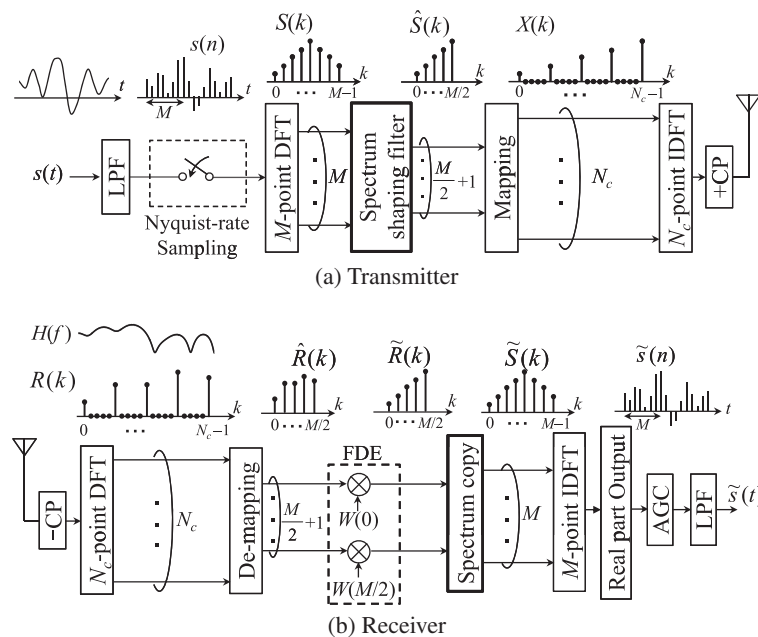

Fig. 1.   Transmission system model of analog SC-FDE.

2 Analog SC-FDE using SSB technique

The system model of analog SC-FDE using SSB technique is shown in Fig. 1.

2.1 Transmit signal representation

After limiting bandwidth by low-pass filter (LPF), the analog signal $s(t)$ is sampled at the Nyquist rate of $1/T$. Then, the sample sequence is grouped into a sequence of signal blocks of M samples each. Each signal block $\{s(n); n = 0 \sim M - 1\}$ is directly transformed by M-point DFT into frequency-domain signal block $\{S(k); k = 0 \sim M - 1\}$ as

$$
\begin{aligned}
S(k) &= \frac{1}{\sqrt{M}} \sum_{n=0}^{M-1} s(n) \exp\left\{-j2\pi k \frac{n}{M}\right\} \\
&= \left[\frac{1}{\sqrt{M}} \sum_{n=0}^{M-1} s(n) \exp\left\{-j2\pi(M-k)\frac{n}{M}\right\}\right]^* = S^*(M-k).
\end{aligned} \tag{1}
$$

In the first step of analog SC-FDE using SSB technique, the USB frequency components are removed as illustrated in Fig. 2. The k-th frequency component of SSB signal $\hat{S}(k)$, $k = 0 \sim M/2$, is expressed as

$$
\hat{S}(k) = \sqrt{Q} \times \frac{1}{\sqrt{M}} \sum_{n=0}^{M-1} s(n) \exp\left(-j2\pi k \frac{n}{M}\right), \tag{2}
$$

where $\sqrt{Q} = \sqrt{M/(M/2 + 1)}$ is normalization factor to keep the transmit power same as in analog SC-FDE. The frequency components obtained by DFT operation are orthogonal to each other and thereby giving no interference with each other.

In the second step, the frequency components of LSB are mapped over a broader bandwidth having $N_c(\gg M)$ subcarriers to take advantage of channel frequency-selectivity. In this paper, distributed mapping [3] is used, where the frequency components of LSB $\{\hat{S}(k); k = 0 \sim M/2\}$ are mapped uniformly over the bandwidth having N_c orthogonal subcarriers $\{X(k'); k' = 0 \sim N_c - 1\}$ as

$$
X(k') = \begin{cases} \hat{S}(k), & k' = k \times \lfloor N_c/(M/2 + 1) \rfloor \\ 0, & \text{otherwise} \end{cases}, \tag{3}
$$

where $\lfloor x \rfloor$ is the largest integer less than or equal to x.

The resultant frequency-domain signal having N_c subcarriers is transformed by N_c-point IDFT back into time-domain signal $\{x(n); n = 0 \sim N_c - 1\}$ at a rate of $1/T_s = (N_c/M) \times 1/T$. $x(n)$ is expressed as

$$
x(n) = \frac{1}{\sqrt{N_c}} \sum_{k=0}^{N_c-1} X(k) \exp\left(j2\pi n \frac{k}{N_c}\right). \tag{4}
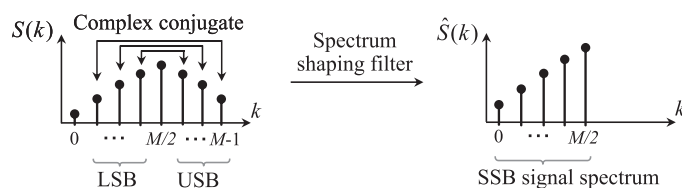$$

Fig. 2. Spectrum of SSB signal generated by spectrum shaping filter.

Finally, the last N_g samples of the time-domain signal block are copied as a CP and inserted into the beginning of each transmit signal block. The discrete-time signal block $\{\tilde{x}(n); n = -N_g \sim N_c - 1\}$ after inserting CP can be expressed as

$$\tilde{x}(n) = \sqrt{2P}x(n \bmod N_c). \tag{5}$$

where P is the average transmit power.

2.2 Received signal representation

A quasi-static L-path channel is assumed, where the channel impulse response $h(\tau)$ can be expressed as

$$h(\tau) = \sum_{l=0}^{L-1} h_l \delta(\tau - \tau_l). \tag{6}$$

In Eq. (6), h_l and τ_l are the complex-valued path gain with $E[\sum_{l=0}^{L-1} |h_l|^2] = 1$ ($E[.]$ denotes the expectation operation) and the sample-spaced time delay of the l-th path (i.e., $\tau_l = l$), respectively. The assumption of quasi-static channel means that the path gains stay constant during the signal transmission period of one block.

Assuming that the maximum time delay of channel is shorter than CP length, the discrete-time received signal block $\{r(n); n = -N_g \sim N_c - 1\}$ is expressed as

$$r(n) = \sum_{l=0}^{L-1} h_l \tilde{x}(n - \tau_l) + z(n), \tag{7}$$

where $z(n)$ is a complex Gaussian noise with zero-mean and variance $2N_0/T_s$ with N_0 being the single-sided power spectrum density of additive white Gaussian noise (AWGN). After removing CP, each received signal block is decomposed by N_c-point DFT into N_c orthogonal frequency components $\{R(k); k = 0 \sim N_c - 1\}$. $R(k)$ is expressed as

$$R(k) = \sqrt{2P}H(k)X(k) + \Pi(k), \tag{8}$$

where $H(k)$ and $\Pi(k)$ are respectively the channel gain and the noise given by

$$\begin{cases} H(k) = \sum_{l=0}^{L-1} h_l \exp\left(-j2\pi k \dfrac{\tau_l}{N_c}\right) \\ \Pi(k) = \dfrac{1}{\sqrt{N_c}} \sum_{n=0}^{N_c-1} z(n) \exp\left(-j2\pi k \dfrac{n}{N_c}\right) \end{cases}. \tag{9}$$

The de-mapping is the reverse operation of the spectrum mapping done at the transmitter in order to pick out the desired SSB frequency components $\{\hat{R}(k); k = 0 \sim M/2\}$ and the corresponding channel gain $\{\hat{H}(k); k = 0 \sim M/2\}$. The one-tap FDE is performed to restore the LSB frequency components $\{\tilde{R}(k); k = 0 \sim M/2\}$ as

$$\tilde{R}(k) = W(k)\hat{R}(k) = \sqrt{2P}W(k)\hat{H}(k)\hat{S}(k) + W(k)\hat{\Pi}(k) = \sqrt{2P}\tilde{H}(k)\hat{S}(k) + \tilde{\Pi}(k), \tag{10}$$

where $\hat{\Pi}(k)$ is the noise after de-mapping and $W(k)$ is the MMSE weight [2] described as

$$W(k) = \hat{H}^*(k)/\{|\hat{H}(k)|^2 + (Q\Gamma)^{-1}\}. \tag{11}$$

In Eq. (11), $\Gamma = PT_s/N_0$ and $[.]^*$ denotes complex conjugate operation.

Next, by exploiting the complex conjugate relation between LSB and USB, the spectrum copying is carried out to reproduce the whole spectrum of original analog signal $\{\tilde{S}(k); k = 0 \sim M - 1\}$. This is the reverse operation of the spectrum shaping done at the transmitter. $\tilde{S}(k)$ is expressed as

$$\tilde{S}(k) = \begin{cases} \tilde{R}(k), & k = 0 \sim M/2 \\ \tilde{R}^*(M - k), & k = M/2 + 1 \sim M - 1 \end{cases}. \quad (12)$$

The resultant frequency-domain signal $\{\tilde{S}(k); k = 0 \sim M - 1\}$ is transformed by M-point IDFT back into the time-domain signal. Taking out its real part and applying fast automatic gain control (AGC) [4] give the demodulated signal $\{\tilde{s}(n); n = 0 \sim M - 1\}$ as

$$\tilde{s}(n) = \frac{1}{\epsilon} \times \text{Re}\left\{ \frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} \tilde{S}(k) \exp\left(j2\pi n \frac{k}{M} \right) \right\} = s(n) + \text{Re}\{\mu_{\text{ISI}}(n) + \mu_{\text{noise}}(n)\}, \quad (13)$$

where ϵ, $\mu_{\text{ISI}}(n)$, and $\mu_{\text{noise}}(n)$ are respectively the AGC normalization factor, residual inter-sample interference, and noise. They are given by (their derivations are omitted for the brevity)

$$\begin{cases} \epsilon = \frac{\sqrt{2PQ}}{M} \left\{ \sum_{k=0}^{M/2} \tilde{H}(k) + \sum_{k=1}^{M/2-1} \tilde{H}^*(k) \right\} \\ \mu_{\text{ISI}}(n) = \frac{1}{\epsilon} \frac{\sqrt{2PQ}}{M} \times \\ \times \left\{ \sum_{k=0}^{M/2} \tilde{H}(k) \left[\sum_{\substack{n'=0 \\ n' \neq n}}^{M-1} s(n') \exp\left(j2\pi k \frac{n - n'}{M} \right) \right] + \sum_{k=1}^{M/2-1} \tilde{H}^*(k) \left[\sum_{\substack{n'=0 \\ n' \neq n}}^{M-1} s(n') \exp\left(-j2\pi k \frac{n - n'}{M} \right) \right] \right\} \\ \mu_{\text{noise}}(n) = \frac{1}{\epsilon} \frac{1}{\sqrt{M}} \left\{ \sum_{k=0}^{M/2} \tilde{\Pi}(k) \exp\left(j2\pi n \frac{k}{M} \right) + \sum_{k=1}^{M/2-1} \tilde{\Pi}^*(k) \exp\left(-j2\pi n \frac{k}{M} \right) \right\} \end{cases}$$
$$(14)$$

Finally, the analog signal $\tilde{s}(t)$ is reconstructed by LPF from sequence $\tilde{s}(n)$.

3   NMSE analysis

The following NMSE is used to evaluate the transmission performance.

$$\text{NMSE} \equiv E[|\tilde{s}(n) - s(n)|^2]/E[|s(n)|^2]. \quad (15)$$

Without loss of generality, the transmit signal is assumed to have unit average power. Using Eq. (11), NMSE is rewritten as

$$\text{NMSE} = E[|\text{Re}\{\mu_{\text{ISI}}(n) + \mu_{\text{noise}}(n)\}|^2]. \quad (16)$$

Since $\mu_{\text{ISI}}(n)$ and $\mu_{\text{noise}}(n)$ are statistically independent, the variance of $\mu(n) = \mu_{\text{ISI}}(n) + \mu_{\text{noise}}(n)$ is given as

$$2\sigma^2 = E[|\mu(n)|^2] = 2\sigma_{\text{ISI}}^2 + 2\sigma_{\text{noise}}^2, \quad (17)$$

where $2\sigma_{\text{ISI}}^2$ and $2\sigma_{\text{noise}}^2$ are the variance of $\mu_{\text{ISI}}(n)$ and $\mu_{\text{noise}}(n)$, respectively. From Eq. (14), it should be noted that $\mu_{\text{ISI}}(n)$ is real-valued signal while $\mu_{\text{noise}}(n)$ approximates real value. The reason is that the USB and LSB of residual inter-sample interference after spectrum copy process have a complex conjugate relation,

and so do those of noise. After some manipulations as in [1], the conditional NMSE of analog SC-FDE using SSB technique for the given set of channel gains $\{H(k); k = 0 \sim N_c - 1\}$ is obtained as (the derivations is omitted for the brevity)

$$\text{NMSE}_{\text{SSB}}(\Gamma, \{H(k)\}) = 2\sigma^2$$

$$= \frac{\frac{1}{M}\left\{\sum_{k=0}^{M/2}|\tilde{H}(k)|^2 + \sum_{k=1}^{M/2-1}|\tilde{H}(k)|^2\right\} + \frac{1}{Q\Gamma}\cdot\frac{1}{M}\left\{\sum_{k=0}^{M/2}|W(k)|^2 + \sum_{k=1}^{M/2-1}|W(k)|^2\right\}}{\left|\frac{1}{M}\left\{\sum_{k=0}^{M/2}\tilde{H}(k) + \sum_{k=1}^{M/2-1}\tilde{H}(k)\right\}\right|^2} - 1.$$

$$(18)$$

Similarly, the conditional NMSE of analog SC-FDE, which transmits double sideband (DSB) of analog signal, is obtained as [1]

$$\text{NMSE}_{\text{DSB}}(\Gamma, \{H(k)\}) = \frac{1}{2}\left\{\frac{\frac{1}{M}\sum_{k=0}^{M-1}|\tilde{H}(k)|^2 + \frac{1}{\Gamma}\cdot\frac{1}{M}\sum_{k=0}^{M-1}|W(k)|^2}{\left|\frac{1}{M}\sum_{k=0}^{M-1}\tilde{H}(k)\right|^2} - 1\right\}. \quad (19)$$

The average NMSE is numerically evaluated by averaging Eqs. (18) and (19) over all possible realizations of $\{H(k); k = 0 \sim N_c - 1\}$.

4   Simulation and theoretical results

A bandwidth-limited (4 kHz) voice (a news announcer voice) transmission is considered with the sampling rate of $1/T = 8$ kHz. In analog SC-FDE using SSB technique, a block of $M = 64$ analog signal samples is modulated into a signal block of $N_c = 8192$ samples and is transmitted over a bandwidth (1.024 MHz) of $N_c = 8192$ subcarriers (subcarrier interval is 125 Hz). For comparison, the conventional SSB transmission [5] is also considered. Both analog SC-FDE using SSB technique and the conventional SSB signal go through the same Rayleigh fading channel having a T_s-spaced ($T_s = T \times M/N_c \approx 1$ msec) $L = 16$-path with uniform power delay profile. A very slow (or block) fading model is assumed, where fading stays unchanged over each block of $N_c = 8192$ samples for the analog SC-FDE using SSB technique (also over a period of $64T(= 8192T_s = 8$ msec) for the conventional SSB). In the conventional SSB, the channel becomes a frequency-nonselective fading channel over the transmit signal bandwidth of 4 kHz. Ideal channel estimation and coherent SSB demodulation are assumed.

The simulated NMSE performance of analog SC-FDE using SSB technique, conventional SSB, and analog SC-FDE are plotted in Fig. 3. A fairly good agreement is seen between the simulated and numerical results. By exploiting the SSB technique to transmit LSB components only, analog SC-FDE using SSB achieves a twofold increase in spectrum efficiency compared to analog SC-FDE. Fig. 3 shows that the proposed scheme achieves better NMSE performance than analog SC-FDE. Because of transmitting LSB components only, the noise power in the proposed scheme is half of analog SC-FDE scheme. Consequently, the proposed scheme achieves two times higher received signal-to-noise power ratio (SNR) than analog
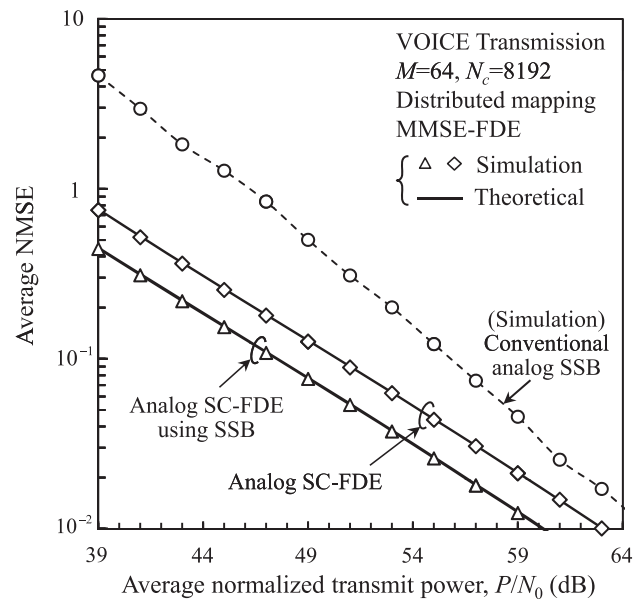
Fig. 3. NMSE performance.

SC-FDE for the same average normalized transmit power, thereby improving the NMSE performance. In comparison with the conventional SSB transmission, analog SC-FDE using SSB technique achieves much better performance, because a combined use of distributed mapping and MMSE-FDE obtains the significant frequency diversity gain.

5 Conclusion

In this paper, analog SC-FDE using SSB technique was proposed. By exploiting the SSB technique to transmit LSB components only, analog SC-FDE using SSB achieves a twofold increase in spectrum efficiency compared to analog SC-FDE. It was confirmed by theoretical analysis and computer simulation that the proposed scheme achieves better NMSE performance than both conventional SSB and analog SC-FDE. Transmission of single analog signal stream was presented and ideal channel estimation was assumed. Multi-access using the proposed analog SC-FDE using SSB technique is possible based on FDMA principle. Channel estimation is left as an interesting future study.