PAPER

Transmit Multi-Block FDE for Space-Time Block Coded Joint Transmit/Receive Diversity in a Quasi-Static Fading Channel

Hiroyuki MIYAZAKI^{†a)}, Student Member and Fumiyuki ADACHI[†], Fellow

SUMMARY In this paper, we propose a transmit multi-block frequency-domain equalization (MB-FDE) for frequency-domain spacetime block coded joint transmit/receive diversity (FD-STBC-JTRD). Noting that a STBC codeword consists of multiple coded blocks, the transmit MB-FDE uses the multiple transmit FDE weight matrices, each associated with each coded block. Both single-carrier (SC) transmission and orthogonal frequency-division multiplexing (OFDM) transmission are considered. For SC transmission, the transmit MB-FDE weight matrices are jointly optimized so as to minimize the mean square error (MSE) between the transmit signal before STBC encoding and the received signal after STBC decoding. For OFDM transmission, they are jointly optimized so as to maximize the received signal-to-noise power ratio (SNR) after STBC decoding. We show by theoretical analysis that the proposed transmit MB-FDE can achieve $1/R_{STBC}$ times higher received SNR than the conventional transmit single-block FDE (SB-FDE), where R_{STBC} represents the code rate of STBC. It is confirmed by computer simulation that, when more than 2 receive antennas are used, MB-FDE can always achieve better BER performance than SB-FDE irrespective of the number of transmit antennas, and the channel frequency-selectivity.

key words: space-time block coding, transmit frequency-domain equalization

1. Introduction

In the next generation mobile communication systems, broadband data services around 1 Gbps are demanded. However, bit error rate (BER) performance of broadband transmission is severely degraded due to frequencyselective fading [1]. Orthogonal frequency-division multiplexing (OFDM) [2], [3] and single-carrier with frequencydomain equalization (SC-FDE) [4]-[6] are attractive broadband transmission schemes that can overcome the severe frequency-selectivity problem. Further performance improvement can be achieved by the use of spatial diversity. Space-time block coding (STBC) transmit diversity is an effective scheme to further improve the BER performance [7]–[13]. There are two types of STBC transmit diversity for broadband transmission: frequency-domain space-time transmit diversity (FD-STTD) [9] and frequency-domain space-time block coded joint transmit receive diversity (FD-STBC-JTRD) [10], [11]. FD-STTD is a combination of STBC transmit diversity and receive FDE. It requires the channel estimation at the receiver and simple conjugate/block exchange operation at the transmitter. On the other hand, FD-STBC-JTRD is a combination of STBC transmit diversity and transmit FDE. It requires the channel estimation at the transmitter and simple conjugate/sum operation at the receiver. Therefore, by applying FD-STTD to the uplink (mobile terminal-to-base station) and FD-STBC-JTRD to the downlink (base station-to-mobile terminal), the computational complexity required for mobile terminal can be reduced. In this paper, orthogonal STBC (OS-TBC) code [13] is considered. When more than two receive antennas are used, although the code rate of STBC decreases, better BER performance can be obtained. On the other hand, quasi-orthogonal STBC (QOSTBC) [14] achieves higher code rate of STBC than OSTBC. However, it requires the maximum likelihood detection (MLD) and therefore, it may increases the complexity of the receive signal processing. Therefore, in this paper, we consider FD-STBC-JTRD using OSTBC as our previous study of FD-STBC-JTRD [10], [11].

In the previous study of FD-STBC-JTRD [10], [11], the single transmit FDE weight matrix is assumed over a STBC codeword. However, noting that a STBC codeword consists of multiple coded blocks, multiple transmit FDE weight matrices can be used, each associated with each coded block. The previously derived transmit FDE [10], [11] is a suboptimal solution.

In this paper, we propose a transmit multi-block FDE (MB-FDE) for FD-STBC-JTRD in a quasi-static fading channel. Noting that a STBC codeword consists of multiple coded blocks, the transmit MB-FDE uses multiple transmit FDE weight matrices, each associated with each coded block. In this paper, we consider both SC and OFDM transmissions since the optimization criterion of MB-FDE weight is different with SC transmission and OFDM transmission. In the case of SC transmission, the transmission performance suffers from not only the noise but also the residual inter-symbol interference (ISI). Therefore, the transmit MB-FDE weight needs to be determined so as to minimize mean square error (MSE) between the transmit signal before STBC encoding and the received signal after STBC decoding. On the other hand, in the case of OFDM transmission, ISI does not occur and the transmission performance depends on the received signal-to-noise power ratio (SNR) only. Therefore, the transmit MB-FDE weight can be determined so as to maximize the block average received SNR after STBC decoding. The proposed transmit MB-FDE can achieve $1/R_{STBC}$ times higher received SNR than the conventional transmit single-block FDE (SB-FDE), where R_{STBC} is the code rate of STBC.

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[†]The authors are with the Department of Communications Engineering, Tohoku University, Sendai-shi, 980-8579 Japan.

a) E-mail: miyazaki@mobile.ecei.tohoku.ac.jp

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The rest of the paper is organized as follows. Section 2 presents the transmitter and receiver structures and signal representation in FD-STBC-JTRD. The transmit MB-FDE is proposed in Sect. 3. Section 4 derives the theoretical analysis of the signal-to-interference plus noise power ratio (SINR) in SC transmission, SNR in OFDM transmission and BER. The computer simulation results are discussed in Sect. 5. Section 6 offers conclusions.

Notations: E[.], $[.]^T$ and $[.]^H$ denote the ensemble average operation, the transpose operation, Hermitian transpose operation, respectively. $\|\mathbf{x}\|$ is the Euclidean norm of vector \mathbf{x} .

2. FD-STBC-JTRD Transmission

2.1 Transmitter/Receiver Structures

Figure 1 illustrates the transmitter and receiver structures. The transmitter has N_t antennas and the receiver equips with N_r antennas, respectively. In the transmitter, $J \times N_c$ data modulated symbols are divided into J blocks of N_c symbols each, where J denotes the number of transmit blocks before STBC encoding and N_c is the fast Fourier transform (FFT) block size. Then, in SC transmission, the time-domain transmit signal is transformed into the frequency-domain transmit signal by N_c -point FFT. J frequency-domain transmit signal blocks are encoded into N_r streams of Q coded frequency-domain signal blocks each by STBC encoding, where Q denotes the number of signal blocks in a STBC codeword. In this paper, OSTBC code [13] is considered. The relationship among the number J of transmit blocks, the number Q of coded signal blocks in a STBC codeword,





(b) Receiver

Fig. 1 Transmitter/receiver structures.

Table 1 Relationship among N_r , J, Q and R_{STBC} .

N_r	J	Q	R_{STBC}
2	2	2	1
3	3	4	3/4
4	3	4	3/4
5	10	15	2/3

and the code rate R_{STBC} (= J/Q) of STBC is shown in Table 1. In FD-STBC-JTRD, the combination of J, Q, and R_{STBC} depends on the number N_r of the receive antennas [10], [11]. After STBC encoding, N_t streams of Q coded frequency-domain signal blocks each are generated by applying the transmit MB-FDE to the STBC codeword. After transmit MB-FDE, the frequency-domain transmit signal is transformed back to the time-domain received signal by N_c point inverse FFT (IFFT). After inserting cyclic prefix (CP) into the beginning of each block, the transmitter transmits the STBC codeword to the receiver during Q time-slot.

At the receiver, after CP removal, the time-domain received signal is transformed into the frequency-domain received signal by N_c -point FFT. After STBC decoding, the frequency-domain received signal is transformed back to the time-domain received signal by N_c -point IFFT in SC transmission. Finally, data demodulation is carried out.

2.2 Signal Representation

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Throughout this paper, the symbol-spaced discrete-time signal representation is used.

In the transmitter, $J \times N_c$ data modulated symbols are divided into J blocks of N_c symbols each. Then, in SC transmission, the time-domain transmit signal is transformed into the frequency-domain transmit signal by N_c -point FFT. Representing the *j*th data modulated symbol block as $\{d_j(m) :$ $m = 0, ..., N_c - 1, j = 0, ..., J - 1\}$, the *j*th frequencydomain transmit signal $\{D_j(k) : k = 0, ..., N_c - 1, j = 0, ..., J - 1\}$ is given as

$$D_j(k) = \begin{cases} \frac{1}{\sqrt{N_c}} \sum_{m=0}^{N_c-1} d_j(m) \exp\left(-j2\pi km/N_c\right) & \text{for SC} \\ d_j(k) & \text{for OFDM} \end{cases},$$
(1)

where k is the frequency (subcarrier) index. J frequencydomain transmit signal blocks are encoded into N_r streams of Q coded frequency-domain signal blocks each by STBC encoding. This encoding is done for each frequency (subcarrier) k. The qth block of coded frequency-domain signal is represented by $\{X_q(n_r,k) : k = 0, ..., N_c - 1, n_r = 0, ..., N_r - 1, q = 0, ..., Q - 1\}$. Then, the qth coded transmit signal vector, $\mathbf{X}_q(k) = [X_q(0,k), ..., X_q(N_r - 1,k)]^T$, can be expressed as

$$\begin{pmatrix} \mathbf{X}_{1}^{0}(k) \\ \mathbf{X}_{1}^{T}(k) \end{pmatrix} = \begin{pmatrix} D_{0}(k) & D_{1}(k) \\ -D_{1}^{*}(k) & D_{0}^{*}(k) \end{pmatrix}, \dots \text{ for } N_{r} = 2,$$
(2a)
$$\begin{pmatrix} \mathbf{X}_{1}^{T}(k) \\ \mathbf{X}_{1}^{T}(k) \\ \mathbf{X}_{2}^{T}(k) \\ \mathbf{X}_{3}^{T}(k) \end{pmatrix} = \begin{pmatrix} D_{0}(k) & D_{1}(k) & D_{2}(k) \\ -D_{1}^{*}(k) & D_{0}^{*}(k) & 0 \\ -D_{2}^{*}(k) & 0 & D_{0}^{*}(k) \\ 0 & D_{2}(k) & -D_{1}(k) \end{pmatrix}, \dots \text{ for } N_{r} = 3,$$
(2b)

 $(\mathbf{X}_{0}^{T}(k))$ $D_0(k)$ $D_1(k)$ $D_2(k)$ 0 $\begin{array}{c} 0\\ D_0^*(k) \end{array}$ $D_0^*(k)$ $D_2(k)$ $\mathbf{X}_{1}^{T}(k)$ $-D_{1}^{*}(k)$ $-D_{2}^{*}(k)$ 0 $\mathbf{X}_{2}^{T}(k)$ $-D_1(k)$ $D_2(k)$ 0 ... for $N_r = 4$, (2c)

$$\begin{pmatrix} \mathbf{X}_{0}^{T}(k) \\ \mathbf{X}_{1}^{T}(k) \\ \mathbf{X}_{2}^{T}(k) \\ \mathbf{X}_{3}^{T}(k) \\ \mathbf{X}_{3}^{T}(k) \\ \mathbf{X}_{4}^{T}(k) \\ \mathbf{X}_{4}^{T}(k) \\ \mathbf{X}_{4}^{T}(k) \\ \mathbf{X}_{5}^{T}(k) \\ \mathbf{X}_{6}^{T}(k) \\ \mathbf{X}_{7}^{T}(k) \\ \mathbf{X}_{7}^{T}(k) \\ \mathbf{X}_{7}^{T}(k) \\ \mathbf{X}_{7}^{T}(k) \\ \mathbf{X}_{10}^{T}(k) \\ \mathbf{X}_{11}^{T}(k) \\ \mathbf{X}_{12}^{T}(k) \\ \mathbf{X}_{11}^{T}(k) \\ \mathbf{X$$

After STBC encoding, the transmit MB-FDE is performed. The *q*th transmit signal vector after the transmit MB-FDE, $\mathbf{S}_q(k) = [S_q(0,k), \dots, S_q(N_t - 1, k)]^T$ is given as

$$\mathbf{S}_{q}(k) = A\mathbf{W}_{q}(k)\mathbf{X}_{q}(k),\tag{3}$$

where $\mathbf{W}_q(k) = [\mathbf{W}_q(0, k), \dots, \mathbf{W}_q(N_r - 1, k)]$ with $\mathbf{W}_q(n_r, k) = [W_q(0, n_r, k), \dots, W_q(N_t - 1, n_r, k)]^T$ is the $N_t \times N_r$ transmit MB-FDE weight matrix for the *q*th coded transmit signal block. *A* is the power normalization factor for keeping average transmit power constant given as

$$A = \frac{1}{\sqrt{\frac{1}{N_c} \frac{1}{Q} \sum_{q=0}^{Q-1} \sum_{n_r=0}^{N_r-1} \sum_{k=0}^{N_c-1} \left\| \mathbf{W}_q(n_r, k) \right\|^2}}.$$
(4)

The STBC codeword after the transmit MB-FDE is transformed back to the time-domain transmit signal by N_c -point IFFT. After CP insertion, the transmitter transmits the STBC codeword to the receiver during Q time-slot.

At the receiver, after CP removal, the time-domain receive signal is transformed into the frequency-domain received signal by N_c -point FFT. Representing the *q*th frequency-domain received signal at the n_r th receive antenna by $\{R_q(n_r,k): k = 0, ..., N_c-1, q = 0, ..., Q-1, n_r = 0, ..., N_r-1\}$, the *q*th frequency-domain received signal vector, $\mathbf{R}_q(k) = [R_q(0,k), ..., R_q(N_r-1)]^T$, can be expressed as

$$\mathbf{R}_{q}(k) = \sqrt{2P\mathbf{H}(k)\mathbf{S}_{q}(k)} + \mathbf{N}_{q}(k), \qquad (5)$$

where $\mathbf{H}(k) = [\mathbf{H}^T(0, k), \dots, \mathbf{H}^T(N_r - 1, k)]^T$ with $\mathbf{H}(n_r, k) = [H(n_r, 0, k), \dots, H(n_r, N_t - 1, k)]$ is the $N_r \times N_t$ channel transfer function matrix. *P* denotes the transmit power. $\mathbf{N}_q(k) = [N_q(0, k), \dots, N_q(N_r - 1, k)]^T$ is the noise vector and $N_q(n_r, k)$ is the zero mean complex-valued additive white Gaussian noise (AWGN) having variance $2N_0/T_s$ with N_0 and T_s being the single-sided power spectrum density of AWGN and the symbol duration, respectively.

The STBC decoding is performed to obtain the decoded frequency-domain signal. The *j*th decoded frequency-domain signal $\{\hat{D}_j(k) : k = 0, ..., N_c - 1, j = 0, ..., J - 1\}$ is given as

$$\begin{pmatrix} \hat{D}_0(k) \\ \hat{D}_1(k) \end{pmatrix} = \begin{pmatrix} R_0(0,k) + R_1^*(1,k) \\ R_0(1,k) - R_1^*(0,k) \end{pmatrix}, \dots \text{ for } N_r = 2,$$
(6a)
$$\begin{pmatrix} \hat{D}_0(k) \\ \hat{D}_1(k) \\ \hat{D}_2(k) \end{pmatrix} = \begin{pmatrix} R_0(0,k) + R_1^*(1,k) + R_2^*(2,k) \\ R_0(1,k) - R_1^*(0,k) + R_3^*(2,k) \\ R_0(2,k) - R_2^*(0,k) - R_3^*(1,k) \end{pmatrix}, \dots \text{ for } N_r = 3,$$
(6b)

$$\hat{D}_{0}(k) \\ \hat{D}_{1}(k) \\ \hat{D}_{2}(k) \end{pmatrix} = \begin{pmatrix} R_{0}(0,k) + R_{1}^{*}(1,k) + R_{2}^{*}(2,k) + R_{3}^{*}(3,k) \\ R_{0}(1,k) - R_{1}^{*}(0,k) - R_{2}(3,k) + R_{3}^{*}(2,k) \\ R_{0}(2,k) + R_{1}(3,k) - R_{2}^{*}(0,k) - R_{3}^{*}(1,k) \end{pmatrix},$$
... for $N_{r} = 4$, (6c)

and

$$\begin{split} & \begin{pmatrix} \hat{D}_{0}(k) \\ \hat{D}_{1}(k) \\ \hat{D}_{2}(k) \\ \hat{D}_{3}(k) \\ \hat{D}_{4}(k) \\ \hat{D}_{6}(k) \\ \hat{D}_{6}(k) \\ \hat{D}_{6}(k) \\ \hat{D}_{6}(k) \\ \hat{D}_{9}(k) \\ \end{pmatrix} = \begin{pmatrix} R_{0}(0,k) - R_{1}^{*}(1,k) - R_{2}^{*}(2,k) - R_{4}^{*}(3,k) + R_{8}(4,k) \\ R_{1}(0,k) + R_{0}^{*}(1,k) - R_{3}(2,k) + R_{5}(3,k) - R_{7}^{*}(4,k) \\ R_{2}(0,k) + R_{3}(1,k) + R_{0}^{*}(2,k) + R_{6}(3,k) + R_{9}^{*}(4,k) \\ R_{4}(0,k) - R_{5}(1,k) - R_{6}(2,k) + R_{0}^{*}(3,k) - R_{11}^{*}(4,k) \\ R_{4}(0,k) - R_{5}(1,k) - R_{6}(2,k) + R_{12}^{*}(3,k) - R_{11}^{*}(4,k) \\ R_{7}(0,k) + R_{8}(1,k) + R_{10}^{*}(2,k) + R_{13}^{*}(3,k) - R_{2}^{*}(4,k) \\ R_{9}(0,k) + R_{10}^{*}(1,k) - R_{7}^{*}(2,k) + R_{13}(3,k) - R_{2}^{*}(4,k) \\ R_{10}(0,k) - R_{9}^{*}(1,k) - R_{7}^{*}(2,k) + R_{14}(3,k) + R_{3}(4,k) \\ R_{11}(0,k) - R_{12}^{*}(1,k) - R_{14}(2,k) - R_{7}^{*}(3,k) - R_{5}^{*}(4,k) \\ R_{12}(0,k) + R_{11}^{*}(1,k) - R_{14}(2,k) - R_{7}^{*}(3,k) - R_{5}^{*}(4,k) \\ R_{13}(0,k) - R_{14}(1,k) - R_{11}^{*}(2,k) - R_{9}^{*}(3,k) + R_{6}(4,k) \\ \end{split}$$

In SC transmission, the decoded frequency-domain signal is transformed back to the time-domain signal. Finally, the data demodulation is carried out.

3. MB-FDE for FD-STBC-JTRD

In this paper, we derive the transmit MB-FDE weight matrices for FD-STBC-JTRD. In the case of SC transmission, the transmission performance suffers from not only noise but also residual ISI after FDE and STBC decoding. Therefore, MB-FDE weight needs to be determined so as to minimize MSE between the transmit signal block before STBC encoding and the received signal block after STBC decoding. On the other hand, in the case of OFDM transmission, ISI does not occur and the transmission performance depends on the

2070

received SNR only. Therefore, MB-FDE weight can be determined so as to maximize the receive SNR after STBC decoding. In this section, the overview of derivation of the transmit MB-FDE is only presented. The detailed derivation of the transmit MB-FDE is referred in Appendix.

3.1 SC Transmission

For SC transmission, the transmit MB-FDE weight matrices are jointly optimized so as to minimize MSE between the transmit signal before STBC encoding and the received signal after STBC decoding. However, since the transmit FDE alters the transmitted signal spectrum shape, the received SNR is unproportional to the MSE. Therefore, in this paper, we introduce the relative MSE, *e*, defined as [10]

$$e = \sum_{j=0}^{J-1} \sum_{k=0}^{N_c-1} E\left[\left| \frac{\hat{D}_j(k) - \sqrt{2P}AD_j(k)}{\sqrt{2P}A\sqrt{E\left[\left| D_j(k) \right|^2 \right]}} \right|^2 \right].$$
 (7)

(a) Single-block FDE for SC transmission

The conventional transmit single-block FDE (SB-FDE) weight matrix is derived based on minimum mean square error (MMSE) criterion under a constraint that the single transmit FDE weight matrix is used over a STBC codeword. Assuming $\mathbf{W}_0(n_r, k) = \ldots = \mathbf{W}_{Q-1}(n_r, k) = \mathbf{W}(n_r, k)$ for $n_r = 0, \ldots, N_r - 1$, the transmit SB-FDE weight for SC transmission is derived from $\partial e / \partial \mathbf{W}(0, k) = 0, \ldots, \partial e / \partial \mathbf{W}(N_r - 1, k) = 0$ as

$$\mathbf{W}(n_r,k) = \mathbf{H}^H(n_r,k)C_{SB}^{-1}(k), \tag{8}$$

where

$$C_{SB}(k) = \sum_{n_r=0}^{N_r-1} \|\mathbf{H}(n_r, k)\|^2 + N_r \left(\frac{P}{N}\right)^{-1},$$
(9)

and $N = N_0/T_s$ is the noise power. This FDE weight is a suboptimal solution in the MMSE sense.

(b) Multi-block FDE for SC transmission

The transmit MB-FDE weight matrices are derived based on MMSE criterion considering that multiple transmit FDE weight matrices can be used, each associated with each coded block. By solving $\partial e/\partial \mathbf{W}_0(0, k) =$ $0, \ldots, \partial e/\partial \mathbf{W}_{Q-1}(N_r - 1, k) = 0$, the transmit MB-FDE weight for SC transmission is derived as

$$\begin{pmatrix} \mathbf{W}_{0}(k) \\ \mathbf{W}_{1}(k) \end{pmatrix} = \begin{pmatrix} \mathbf{H}^{H}(0,k) & \mathbf{H}^{H}(1,k) \\ \mathbf{H}^{H}(0,k) & \mathbf{H}^{H}(1,k) \end{pmatrix} C_{MB}^{-1}(k), \\ \dots \text{ for } N_{r} = 2, \quad (10a) \\ \begin{pmatrix} \mathbf{W}_{0}(k) \\ \mathbf{W}_{1}(k) \\ \mathbf{W}_{2}(k) \\ \mathbf{W}_{3}(k) \end{pmatrix} = \begin{pmatrix} \mathbf{H}^{H}(0,k) & \mathbf{H}^{H}(1,k) & \mathbf{H}^{H}(2,k) \\ \mathbf{H}^{H}(0,k) & \mathbf{H}^{H}(1,k) & \mathbf{0} \\ \mathbf{H}^{H}(0,k) & \mathbf{0} & \mathbf{H}^{H}(2,k) \\ \mathbf{0} & \mathbf{H}^{H}(1,k) & \mathbf{H}^{H}(2,k) \end{pmatrix} C_{MB}^{-1}(k), \\ \dots \text{ for } N_{r} = 3, \quad (10b)$$

$$\begin{aligned} \mathbf{W}_{0}(k) \\ \mathbf{W}_{1}(k) \\ \mathbf{W}_{2}(k) \\ \mathbf{W}_{3}(k) \end{aligned} = \begin{pmatrix} \mathbf{H}^{H}(0,k) \ \mathbf{H}^{H}(1,k) \ \mathbf{H}^{H}(2,k) \ \mathbf{0} \\ \mathbf{H}^{H}(0,k) \ \mathbf{H}^{H}(1,k) \ \mathbf{0} \ \mathbf{H}^{H}(3,k) \\ \mathbf{H}^{H}(0,k) \ \mathbf{0} \ \mathbf{H}^{H}(2,k) \ \mathbf{H}^{H}(3,k) \\ \mathbf{0} \ \mathbf{H}^{H}(1,k) \ \mathbf{H}^{H}(2,k) \ \mathbf{H}^{H}(3,k) \\ \end{aligned} \right\} C_{MB}^{-1}(k),$$
for $N_{x} = 4$ (10c)

and

$$\begin{split} \mathbf{W}_{0}(k) \\ \mathbf{W}_{1}(k) \\ \mathbf{W}_{1}(k) \\ \mathbf{W}_{2}(k) \\ \mathbf{W}_{3}(k) \\ \mathbf{W}_{3}(k) \\ \mathbf{W}_{3}(k) \\ \mathbf{W}_{4}(k) \\ \mathbf{W}_{5}(k) \\ \mathbf{W}_{6}(k) \\ \mathbf{W}_{7}(k) \\ \mathbf{W}_{1}(k) \\ \mathbf{W}_{1}(k)$$

where

$$C_{MB}^{-1}(k) = \sum_{n_r=0}^{N_r-1} ||\mathbf{H}(n_r,k)||^2 + N_r \left(\frac{J}{Q}\right) \left(\frac{P}{N}\right)^{-1}.$$
 (11)

3.2 OFDM Transmission

For OFDM transmission, the transmit MB-FDE weight matrices are jointly optimized so as to maximize the block average received SNR after STBC decoding. The block average received SNR, Γ_{OFDM} , are given as

$$\Gamma_{OFDM} = \frac{1}{JN_c} \sum_{k=0}^{N_c - 1} \sum_{j=0}^{J-1} \gamma_{OFDM,j}(k),$$
(12)

where $\gamma_{OFDM,j}(k)$ is the received SNR at the *k*th subcarrier in the *j*th decoded block.

(a) Single-block FDE for OFDM transmission

The transmit SB-FDE weight matrix is derived so as to maximize the block average received SNR under a constraint that the single transmit FDE weight matrix is used over a STBC codeword. Assuming $\mathbf{W}_0(n_r, k) = \ldots =$ $\mathbf{W}_{Q-1}(n_r, k) = \mathbf{W}(n_r, k)$ for $n_r = 0, \ldots, N_r - 1$, the transmit SB-FDE weight is derived from $\partial\Gamma_{OFDM}/\partial\mathbf{W}(0, k) =$ $0, \ldots, \partial\Gamma_{OFDM}/\partial\mathbf{W}(N_r - 1, k) = 0$ as

$$\mathbf{W}(n_r,k) = \mathbf{H}^H(n_r,k).$$
(13)

(b) Multi-block FDE for OFDM transmission

The transmit MB-FDE weight matrices are derived so as to maximize the block average received SNR considering that multiple transmit FDE weight matrices can be used, each associated with each coded block. By solving $\partial\Gamma_{OFDM}/\partial\mathbf{W}_0(0,k) = 0, \dots, \partial\Gamma_{OFDM}/\partial\mathbf{W}_{Q-1}(N_r - 1,k) =$ 0, the transmit MB-FDE weight for OFDM transmission is derived as

$$\begin{pmatrix} \mathbf{W}_{0}(k) \\ \mathbf{W}_{1}(k) \end{pmatrix} = \begin{pmatrix} \mathbf{H}^{H}(0,k) & \mathbf{H}^{H}(1,k) \\ \mathbf{H}^{H}(0,k) & \mathbf{H}^{H}(1,k) \end{pmatrix}, \dots \text{ for } N_{r} = 2, \quad (14a)$$

$$\begin{pmatrix} \mathbf{W}_{0}(k) \\ \mathbf{W}_{1}(k) \\ \mathbf{W}_{2}(k) \\ \mathbf{W}_{3}(k) \end{pmatrix} = \begin{pmatrix} \mathbf{H}^{H}(0,k) & \mathbf{H}^{H}(1,k) & \mathbf{H}^{H}(2,k) \\ \mathbf{H}^{H}(0,k) & \mathbf{0} & \mathbf{H}^{H}(2,k) \\ \mathbf{0} & \mathbf{H}^{H}(1,k) & \mathbf{H}^{H}(2,k) \end{pmatrix}, \\ \dots \text{ for } N_{r} = 3, \quad (14b)$$

$$\begin{pmatrix} \mathbf{W}_{0}(k) \\ \mathbf{W}_{1}(k) \\ \mathbf{W}_{2}(k) \\ \mathbf{W}_{3}(k) \end{pmatrix} = \begin{pmatrix} \mathbf{H}^{H}(0,k) & \mathbf{H}^{H}(1,k) & \mathbf{H}^{H}(2,k) & \mathbf{0} \\ \mathbf{H}^{H}(0,k) & \mathbf{H}^{H}(1,k) & \mathbf{0} & \mathbf{H}^{H}(3,k) \\ \mathbf{H}^{H}(0,k) & \mathbf{H}^{H}(1,k) & \mathbf{0} & \mathbf{H}^{H}(3,k) \\ \mathbf{H}^{H}(0,k) & \mathbf{0} & \mathbf{H}^{H}(2,k) & \mathbf{H}^{H}(3,k) \\ \mathbf{0} & \mathbf{H}^{H}(1,k) & \mathbf{H}^{H}(2,k) & \mathbf{H}^{H}(3,k) \end{pmatrix}, \\ \dots \text{ for } N_{r} = 4, \quad (14c)$$

and

$$\begin{pmatrix} \mathbf{W}_{0}(k) \\ \mathbf{W}_{1}(k) \\ \mathbf{W}_{2}(k) \\ \mathbf{W}_{3}(k) \\ \mathbf{W}_{3}(k) \\ \mathbf{W}_{4}(k) \\ \mathbf{W}_{6}(k) \\ \mathbf{W}_{6}(k) \\ \mathbf{W}_{6}(k) \\ \mathbf{W}_{6}(k) \\ \mathbf{W}_{1}(k) \\ \mathbf{W}_{1}(k)$$

It can be understood from Eqs. (8), (10), (13) and (14) that the weight matrix of proposed transmit MB-FDE is sparse while the weight matrix of SB-FDE is dense. In general, as the transmit FDE weight matrix becomes more sparse, the norm of transmit FDE weight matrix reduces and accordingly the received SNR improves [15] (the proof for this is presented in Appendix). Therefore, the transmit MB-FDE provides higher received SNR than the transmit SB-FDE.

4. Theoretical Analysis of SINR and BER

In this section, we derive the received SINRs (the received

SNRs) when using the transmit MB-FDE and SB-FDE theoretically and then, the conditional BERs.

(a) SC transmission

Assuming the sum of the residual inter-symbol interference (ISI) and noise as a new zero-mean complex-valued Gaussian variable, the received SINRs, γ_{SC_MB} and γ_{SC_SB} , after STBC decoding when using the transmit MB-FDE and SB-FDE can be respectively derived as

$$\begin{cases} \gamma_{SC_MB} = \frac{2\left(\frac{P}{N}\right) \left(\frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{H}_{MB}(k)\right)^2}{\left[\left(\frac{P}{N}\right) \left\{\frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{H}_{MB}^2(k) - \left(\frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{H}_{MB}(k)\right)^2\right\}\right] \\ + N_r \frac{J}{Q} \frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{H}_{MB}(k) \\ \gamma_{SC_SB} = \frac{2\left(\frac{P}{N}\right) \left(\frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{H}_{SB}(k)\right)^2}{\left[\left(\frac{P}{N}\right) \left\{\frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{H}_{SB}^2(k) - \left(\frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{H}_{SB}(k)\right)^2\right\}\right] \\ + N_r \frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{H}_{SB}(k) \end{cases}$$
(15)

where

$$\begin{cases} \tilde{H}_{MB}(k) = \sum_{\substack{n_r=0\\N_r-1}}^{N_r-1} \|\mathbf{H}(n_r,k)\|^2 C_{MB}^{-1}(k) \\ \tilde{H}_{SB}(k) = \sum_{\substack{n_r=0\\n_r=0}}^{N_r-1} \|\mathbf{H}(n_r,k)\|^2 C_{SB}^{-1}(k) \end{cases}$$
(16)

The first term is the contribution of the residual ISI after STBC decoding and the second term is the contribution of the noise. When the sufficiently high SNR is obtained by FD-STBC-JTRD, i.e., $\frac{1}{N_r} \left(\frac{p}{N}\right) \sum_{n_r=0}^{N_r-1} ||\mathbf{H}(n_r, k)||^2 \gg 1$, Eq. (15) can be approximated as

$$\begin{cases} \gamma_{SC_MB} \approx \frac{2}{N_r} \frac{1}{J/Q} \left(\frac{P}{N}\right) = \frac{1}{R_{STBC}} \gamma_{SC_SB} \\ \gamma_{SC_SB} \approx \frac{2}{N_r} \left(\frac{P}{N}\right) \end{cases}$$
(17)

Therefore, the transmit MB-FDE can obtain $1/R_{STBC}$ times higher SINR than the transmit SB-FDE. Furthermore, assuming QPSK data modulation, the conditional BER, p_{e,SC_MB} and p_{e,SC_SB} , for the given channel transfer function when using the transmit MB-FDE and SB-FDE can be respectively given as

$$p_{e,SC_MB} = \frac{1}{2} erfc \left[\sqrt{\frac{\gamma_{SC_MB}}{4}} \right], \tag{18a}$$

$$p_{e,SC_SB} = \frac{1}{2} erfc \left[\sqrt{\frac{\gamma_{SC_SB}}{4}} \right], \tag{18b}$$

where $erfc(x) = (2/\sqrt{\pi}) \int_{x}^{\infty} \exp(-t^2) dt$ is the complementary error function.

(b) OFDM transmission

From Eqs. (2), (3), (4), (5), (6), (13) and (14), the received SNRs, $\gamma_{OFDM_MB}(k)$ and $\gamma_{OFDM_SB}(k)$, after STBC decoding at the *k*th subcarrier when using the transmit MB-FDE and SB-FDE are derived as

$$\begin{cases} \gamma_{OFDM_MB}(k) = \frac{2}{N_r(J/Q)} \left(\frac{P}{N}\right) \sum_{n_r=0}^{N_r-1} \|\mathbf{H}(n_r,k)\|^2 \\ = \frac{1}{R_{STBC}} \gamma_{OFDM_SB}(k) \\ \gamma_{OFDM_SB}(k) = \frac{2}{N_r} \left(\frac{P}{N}\right) \sum_{n_r=0}^{N_r-1} \|\mathbf{H}(n_r,k)\|^2 \end{cases}$$
(19)

It is obviously seen from Eq. (19) that the transmit MB-FDE can obtain $1/R_{STBC}$ times higher received SNR than the transmit SB-FDE. Furthermore, assuming QPSK data modulation, the conditional BER, $p_{e,OFDM_MB}$ and $p_{e,OFDM_SB}$, for the given channel transfer function when using the transmit MB-FDE and SB-FDE can be respectively given as

$$p_{e,OFDM_MB} = \frac{1}{2N_c} \sum_{k=0}^{N_c-1} erfc \left[\sqrt{\frac{\gamma_{OFDM_MB}(k)}{4}} \right], \quad (20a)$$

$$p_{e,OFDM_SB} = \frac{1}{2N_c} \sum_{k=0}^{N_c - 1} erfc \left[\sqrt{\frac{\gamma_{OFDM_SB}(k)}{4}} \right].$$
(20b)

5. Computer Simulation

We evaluate, by computer simulation, the BER performance when using FD-STBC-JTRD with the transmit MB-FDE. Computer simulation conditions are summarized in Table 2. QPSK data modulation is considered. FFT block size N_c and CP length N_g are set to $N_c = 128$ symbols and $N_g = 16$ samples, respectively. The channel is assumed to be a quasistatic frequency-selective block Rayleigh fading channel having symbol spaced L = 16 path exponential power delay profile with decay factor α . In this paper, we assume that perfect channel state information can be available at the transmitter.

5.1 BER Performance

Figure 2 shows the BER performances when using the transmit MB-FDE as a function of transmit signal energy per bit-to-AWGN power spectrum density ratio E_b/N_0 . The number of the transmit antennas is set to $N_t = 1$ and the decay factor α is assumed to be $\alpha = 0$ dB, respectively as an example. For the comparison, the BER performances when using the transmit SB-FDE are also plotted in Fig. 2. Furthermore, the markers show the computer simulation results and the lines show the theoretical BER curves, respectively. It is seen from Fig. 2 that when $N_r = 2$, the transmit MB-FDE achieves the same BER performance as the transmit SB-FDE. This is because, when $N_r = 2$, the transmit

 Table 2
 Computer simulation conditions

	Data modulation	QPSK	
Tuonanittan	FFT block size	$N_c = 128$	
/receiver	CP size	$N_g=16$	
	Channel state information	Perfect	
	Fadina trus	Quasi-static Block	
	rading type	Rayleigh fading	
		Symbol-spaced	
Channel	Power delay profile	L=16-path	
		exponential	
	Delay time of the	$\tau_l = lT_s, l = 0,, L - 1$	
	<i>l</i> th path τ_l		



(b) OFDM transmission

Fig. 2 BER performance in a strong frequency-selective fading channel.

MB-FDE matrix is the same as the transmit SB-FDE weight matrix. On the other hand, when $N_r > 2$, the transmit MB-FDE can achieve better BER performance than the transmit SB-FDE. When the number of the receive antennas is $N_r = 3, 4$ ($N_r = 5$), the transmit MB-FDE can reduce the required transmit E_b/N_0 for BER = 10^{-4} by about 1.2 dB 2074

(1.6 dB) compared to the transmit SB-FDE. As shown in Eqs. (8), (10), (13) and (14), the transmit MB-FDE weight matrix is sparse while the transmit SB-FDE weight matrix is dense. Therefore, the transmit MB-FDE reduces the norm of the transmit FDE weight matrix and accordingly, provides higher received SNR than the transmit SB-FDE. It is also seen from Fig. 2(a) and 2(b) that the transmit MB-FDE achieves BER performance superior to the transmit SB-FDE in both SC and OFDM transmission. Furthermore, we can see from Fig. 2 that the computer simulation results correspond to the theoretical BER curves derived in the previous section.

5.2 Impact of the Number of the Transmit Antennas

Figure 3 shows the BER performance when using the transmit MB-FDE as a function of the transmit E_b/N_0 . The number of the receive antenna is set to $N_r = 3$. The decay factor α is assumed to be $\alpha = 0$ dB. For the comparison, the BER performances when using the transmit SB-FDE are also plotted in Fig. 3. Furthermore, the markers show the computer simulation results and the lines show the theoreti-



(b) OFDM transmission



cal curves. It is seen from Fig. 3 that the transmit MB-FDE achieves BER performance superior to the transmit SB-FDE irrespective of the number of the transmit antennas. The reason for this is explained below. As shown in the first equation of Eq. (17) and that of Eq. (19), the performance improvement depends only on the code rate of STBC. In FD-STBC-JTRD transmission, the code rate of STBC does not depend on the number of transmit antennas but only the number of receive antennas as shown in Table 1. Therefore, the performance improvement is independent of the number of transmit antennas. For example, when $N_t = 5$, the transmit MB-FDE can reduce the required E_b/N_0 for BER = 10^{-4} by about 1.2 dB than compared to the transmit SB-FDE. This result also corresponds to the theoretical analysis presented in the previous section.

5.3 Impact of the Channel Frequency-Selectivity

Figure 4 shows the BER performance when the decay factor of channel is assumed to be $\alpha = 6 \text{ dB}$ as a function of the transmit E_b/N_0 . The number of the transmit antennas



BER performance in a weak frequency-selective fading channel.

Fig. 4

is set to $N_t = 1$. It is seen from Fig. 4 that, when $N_r > 2$, the transmit MB-FDE can achieve better BER performance than the transmit SB-FDE even if $\alpha = 6 \text{ dB}$. For example, the number of the receive antennas is $N_r = 3$, the transmit MB-FDE can reduce the required E_b/N_0 for BER = 10^{-4} by about 1.2 dB compared to the transmit SB-FDE. This result corresponds to the previous discussion. This is because the performance improvement depends only on the code rate of STBC and therefore, it is independent of the channel frequency-selectivity.

5.4 Throughput Performance

The throughput, S (bps/Hz), when using FD-STBC-JTRD is defined as

$$S = R_{STBC} \cdot Z \cdot (1 - PER) \cdot \frac{N_c}{N_c + N_g},$$
(21)

where Z is the number of bits per symbol and *PER* denotes packet error rate. In this paper, we assume one packet is composed of 5120 bits.

Figure 5 shows the throughput performances when using FD-STBC-JTRD with MB-FDE as a function of the transmit signal energy per symbol-to-AWGN power spectrum density ratio E_s/N_0 . The number of transmit antennas is set to $N_t = 2$ and the decay factor of channel is assumed to be $\alpha = 0$ dB. For comparison, the throughput performance when using FD-STBC-JTRD with SB-FDE is also shown in Fig. 5.



Fig. 5 Throughput performance.

It is seen from Fig. 5(a) that SB-FDE can hardly improve the throughput of SC transmission in low transmit E_s/N_0 region by increasing the number N_r of receive antennas. The reason for this is explained below. In general, the received SNR improves as the diversity order increases. In the case of SC transmission, both spatial and frequency diversity effects can be obtained. Because the frequencyselectivity of the channel having L = 16-path uniform power delay profile is sufficiently strong, a large diversity order (i.e., 16th order) is obtained by the frequency diversity effect only and the received SNR improvement is already saturated. Therefore, the throughput can hardly be improved by increasing N_r . On the other hand, MB-FDE improves the throughput in low transmit E_s/N_0 region by increasing N_r more than 2, i.e., $N_r > 2$. The reason for this is explained below. As described in Sect. 3, as the transmit FDE weight matrix becomes more sparse, the norm of transmit FDE weight matrix reduces and accordingly the received SNR improves [15]. It can be understood from Eqs. (8) (10), (13) and (14) that by increasing N_r , the transmit MB-FDE weight matrix gets more sparse while the transmit SB-FDE weight matrix remains dense. Therefore, by increasing N_r , the MB-FDE provides higher throughput than SB-FDE.

It is seen from Fig. 5(b) that by increasing N_r , both SB-FDE and MB-FDE improve the throughput of OFDM transmission in low transmit E_s/N_0 region. In the case of OFDM transmission, the frequency diversity effect cannot be obtained and only the spatial diversity effect can be obtained. Therefore, increasing N_r provides higher received SNR and accordingly higher throughput due to increased spatial diversity order. It is also seen from Fig. 5(b) that MB-FDE provides higher throughput than SB-FDE in low E_s/N_0 region. This is because the transmit MB-FDE weight matrix is sparse while the transmit SB-FDE weight matrix is dense.

6. Conclusion

In this paper, we proposed a transmit MB-FDE for FD-STBC-JTRD for OFDM and SC transmissions in a quasistatic fading channel. Noting that a STBC codeword consists of multiple coded blocks, the transmit MB-FDE uses multiple transmit FDE weight matrices, each associated with each coded block. For SC transmission, the transmit MB-FDE weight matrices are jointly optimized so as to minimize MSE between the transmit signal before STBC encoding and the received signal after STBC decoding. On the other hand, for OFDM transmission, they are jointly optimized so as to maximize the received SNR after STBC decoding. It was shown by the theoretical analysis and computer simulation that the proposed transmit MB-FDE can achieve $1/R_{STBC}$ times higher received SNR than the transmit SB-FDE. It was confirmed by computer simulation that, when the number of the receive antennas is $N_r > 2$, the transmit MB-FDE always achieves better BER performance than the transmit SB-FDE irrespective of the number of the transmit antennas and the channel frequency-selectivity. It was also shown by computer simulation that MB-FDE improves the throughput in low transmit E_s/N_0 region by increasing N_r more than 2.

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Appendix: Derivation of MB-FDE Weight, SINR and SNR

In this appendix, a detailed derivation of MB-FDE weight

and the received SINR (the received SNR) in SC transmission (OFDM transmission) when $N_r = 3$ is only presented below due to page limitation. However, the transmit MB-FDE and the received SINR (the received SNR) when $N_r = 2, 4, 5$ can be also derived similar to when $N_r = 3$.

From Eqs. (2), (3), (4), (5) and (6), the *j*th decoded frequency-domain received signal $\{\hat{D}_j(k) : k = 0, ..., N_c - 1\}$ can be written as

$$\hat{D}_j(k) = \sqrt{2P}A\hat{H}_j(k)D_j(k) + \Pi_j(k), \qquad (A \cdot 1)$$

where

$$\begin{cases} H_0(k) = \mathbf{H}(0,k)\mathbf{W}_0(0,k) + \mathbf{W}_1^H(1,k)\mathbf{H}^H(1,k) \\ + \mathbf{W}_2^H(2,k)\mathbf{H}^H(2,k) \\ \hat{H}_1(k) = \mathbf{H}(1,k)\mathbf{W}_0(1,k) + \mathbf{W}_1^H(0,k)\mathbf{H}^H(0,k) \\ + \mathbf{W}_3^H(2,k)\mathbf{H}^H(2,k) \\ \hat{H}_2(k) = \mathbf{H}(2,k)\mathbf{W}_0(2,k) + \mathbf{W}_2^H(0,k)\mathbf{H}^H(0,k) \\ + \mathbf{W}_3^H(1,k)\mathbf{H}^H(1,k) \end{cases}, \quad (\mathbf{A} \cdot 2)$$

and

$$\begin{cases} \Pi_0(k) = N_0(0,k) + N_1^*(1,k) + N_2^*(2,k) \\ \Pi_1(k) = N_0(1,k) - N_1^*(0,k) + N_3^*(2,k) \\ \Pi_2(k) = N_0(2,k) - N_2^*(1,k) - N_3^*(1,k) \end{cases}$$
(A·3)

Below, we will derive the transmit MB-FDE and the received SINR (the received SNR) for SC (OFDM) transmission, respectively.

(a) SC transmission

From Eqs. (4) and $(A \cdot 1)$, Eq. (7) can be rewritten as

$$e = \sum_{k=0}^{N_c-1} \sum_{j=0}^{J-1} \left| \hat{H}_j(k) - 1 \right|^2 + N_r \frac{1}{Q} \left(\frac{P}{N} \right)^{-1} \sum_{k=0}^{N_c-1} \sum_{q=0}^{Q-1} \sum_{n_r=0}^{N_r-1} \left\| \mathbf{W}_q(n_r, k) \right\|^2.$$
(A·4)

The first term is the contribution of the residual ISI after STBC decoding and the second term is the contribution of the noise. It is seen from Eqs. (A·2) and (A·4) that the transmit FDE weights, $\mathbf{W}_1(2,k)$, $\mathbf{W}_2(1,k)$ and $\mathbf{W}_3(0,k)$ do not affect the residual ISI but only the noise. This is because the STBC encoding matrix is sparse. Therefore, by solving $\partial e/\partial \mathbf{W}_1(2,k) = 0$, $\partial e/\partial \mathbf{W}_2(1,k) = 0$ and $\partial e/\partial \mathbf{W}_3(0,k) = 0$, we can obtain $\mathbf{W}_1(2,k) = \mathbf{W}_2(1,k) = \mathbf{W}_3(0,k) = \mathbf{0}$. Furthermore, by solving $\partial e/\partial \mathbf{W}_0(0,k) = 0$, ..., $\partial e/\partial \mathbf{W}_{Q-1}(N_r - 1,k) = 0$, we can obtain the transmit MB-FDE weight matrices given as Eq. (10b).

From Eq. (A·1), the *j*th decoded time-domain received signal $\{\hat{d}_j(m) : m = 0, ..., N_c - 1\}$ can be expressed as

$$\hat{d}_{j}(m) = \frac{1}{\sqrt{N_{c}}} \sum_{k=0}^{N_{c}-1} D_{j}(k) \exp\left(j2\pi km/N_{c}\right)$$
$$= \sqrt{2PA} \left(\frac{1}{N_{c}} \sum_{k=0}^{N_{c}-1} \hat{H}_{j}(k)\right) d_{j}(m) + \mu_{j}^{ISI}(m) + \mu_{j}^{noise}(m),$$
(A·5)

where $\mu_i^{ISI}(m)$ and $\mu_i^{noise}(m)$ are the contributions of the residual ISI and noise, respectively. They are given as

$$\begin{cases} \mu_j^{ISI}(m) = \sqrt{2P}A \frac{1}{N_c} \sum_{k=0}^{N_c-1} \hat{H}_j(k) \left[\sum_{m'=0 \atop \neq m}^{N_c-1} d_j(m') \exp\left(j2\pi k \frac{m-m'}{N_c}\right) \right],\\ \mu_j^{noise}(m) = \frac{1}{\sqrt{N_c}} \sum_{k=0}^{N_c-1} \Pi_j(k) \exp\left(j2\pi k \frac{m}{N_c}\right) \end{cases}$$
(A·6)

Assuming the sum of the residual ISI and noise as a new zero-mean complex-valued Gaussian variable, $\mu_i(m) =$ $\mu_i^{ISI}(m) + \mu_i^{noise}(m)$, the received SINR after STBC decoding, $\gamma_{SC,i}$, can be derived as

$$\gamma_{SC,j} = \frac{2PA^2 \left| \frac{1}{N_c} \sum_{k=0}^{N_c - 1} \hat{H}_j(k) \right|^2}{\frac{1}{2} E \left[\left| \mu_j(m) \right|^2 \right]} \\ = \frac{\frac{2P}{N} \left| \frac{1}{N_c} \sum_{k=0}^{N_c - 1} \hat{H}_j(k) \right|^2}{\left[\frac{P}{N} \left\{ \frac{1}{N_c} \sum_{k=0}^{N_c - 1} \left| \hat{H}_j(k) \right|^2 - \left| \frac{1}{N_c} \sum_{k=0}^{N_c - 1} \hat{H}_j(k) \right|^2 \right]} \\ + N_r \frac{1}{Q} \sum_{k=0}^{N_c - 1} \sum_{q=0}^{Q - 1} \sum_{n_r = 0}^{N_r - 1} \left\| \mathbf{W}_q(n_r, k) \right\|^2 \right]}.$$
(A·7)

It is seen from Eq. $(A \cdot 7)$ that the received SINR after STBC decoding increases the norm of transmit FDE weight matrix, $\sum_{k=0}^{N_c-1} \sum_{q=0}^{Q-1} \sum_{n_r=0}^{N_r-1} \|\mathbf{W}_q(n_r,k)\|^2$, decreases. As shown in Eqs. (8) and (10b), the transmit MB-FDE weight matrix is sparse while the transmit SB-FDE weight matrix is dense. Therefore, MB-FDE reduces the norm of the transmit FDE weight matrix and accordingly, provides higher received SINR than SB-FDE. Finally, the received SINR after STBC decoding, $\gamma_{SC_MB}(\gamma_{SC_SB})$, can be obtained by substituting Eq. (10b) (Eq. (8)) and Eq. $(A \cdot 2)$ to Eq. $(A \cdot 7)$.

(b) OFDM transmission

From Eqs. (4) and (A · 1), the received SNR, $\gamma_{OFDM,i}(k)$, at the kth subcarrier in the *i*th decoded signal block is given as

$$\gamma_{OFDM,j}(k) = \frac{2P}{N} \frac{\left|\hat{H}_{j}(k)\right|^{2}}{N_{r} \frac{1}{Q} \sum_{k=0}^{N_{c}-1} \sum_{q=0}^{Q-1} \sum_{n_{r}=0}^{N_{r}-1} \left\|\mathbf{W}_{q}\left(n_{r},k\right)\right\|^{2}}.$$
(A·8)

Therefore, the block average received SNR, Γ_{OFDM} , after STBC decoding can be rewritten as

$$\Gamma_{OFDM} = \frac{2P}{N} \frac{\frac{1}{JN_c} \sum_{k=0}^{N_c-1} \sum_{j=0}^{J-1} |\hat{H}_j(k)|^2}{N_r \frac{1}{Q} \sum_{k=0}^{N_c-1} \sum_{q=0}^{Q-1} \sum_{n_r=0}^{N_r-1} \left\| \mathbf{W}_q(n_r,k) \right\|^2}.$$
 (A·9)

It is seen from Eqs. $(A \cdot 2)$ and $(A \cdot 9)$ that the transmit FDE weights, $W_1(2, k)$, $W_2(1, k)$ and $W_3(0, k)$, do not affect the desired signal power but only the noise. Therefore, by solving $\partial \Gamma_{OFDM} / \partial \mathbf{W}_1(2,k) = 0$, $\partial \Gamma_{OFDM} / \partial \mathbf{W}_2(1,k) = 0$ and $\partial \Gamma_{OFDM} / \partial \mathbf{W}_3(0,k) = 0$, we can obtain $\mathbf{W}_1(2,k) =$ $\mathbf{W}_2(1,k) = \mathbf{W}_3(0,k) = \mathbf{0}$. Furthermore, by solving $\partial \Gamma_{OFDM} / \partial \mathbf{W}_0(0,k) = 0, \dots, \partial \Gamma_{OFDM} / \partial \mathbf{W}_{O-1} (N_r - 1,k) =$ 0, we obtain

12)

$$\begin{cases} \left| \mathbf{H}(0,k)\mathbf{W}_{0}(0,k) + \mathbf{W}_{1}^{H}(1,k)\mathbf{H}^{H}(1,k) \right|^{2} \\ + \mathbf{W}_{2}^{H}(2,k)\mathbf{H}^{H}(2,k) \\ + \left| \mathbf{H}(1,k)\mathbf{W}_{0}(1,k) + \mathbf{W}_{1}^{H}(0,k)\mathbf{H}^{H}(0,k) \right|^{2} \\ + \left| \mathbf{H}(2,k)\mathbf{W}_{0}(2,k) + \mathbf{W}_{2}^{H}(0,k)\mathbf{H}^{H}(0,k) \right|^{2} \\ + \left| \mathbf{H}(2,k)\mathbf{W}_{0}(2,k)\mathbf{H}^{H}(1,k) \right|^{2} \\ + \left| \mathbf{W}_{0}(1,k)\mathbf{W}_{0}^{2} + \left| \mathbf{W}_{1}(0,k)\mathbf{W}_{0}^{2} + \left| \mathbf{W}_{2}(2,k)\mathbf{W}_{0}^{2} \right|^{2} \\ + \left| \mathbf{W}_{0}(2,k)\mathbf{W}_{0}^{2} + \left| \mathbf{W}_{2}(0,k)\mathbf{W}_{0}^{2} + \left| \mathbf{W}_{3}(1,k)\mathbf{W}_{0}^{2} \right|^{2} \\ + \left| \mathbf{W}_{0}(2,k)\mathbf{W}_{0}^{2} + \left| \mathbf{W}_{2}(0,k)\mathbf{W}_{0}^{2} + \left| \mathbf{W}_{3}(1,k)\mathbf{W}_{0}^{2} \right|^{2} \\ + \left| \mathbf{W}_{0}(1,k)\mathbf{W}_{0}^{2} + \left| \mathbf{W}_{2}(0,k)\mathbf{W}_{0}^{2} + \left| \mathbf{W}_{3}(1,k)\mathbf{W}_{0}^{2} \right|^{2} \\ + \left| \mathbf{W}_{0}(1,k)\mathbf{W}_{0}^{2} + \left| \mathbf{W}_{0}(0,k)\mathbf{W}_{0}^{2} + \left| \mathbf{W}_{3}(1,k)\mathbf{W}_{0}^{2} \right|^{2} \\ + \left| \mathbf{W}_{0}(1,k)\mathbf{W}_{0}^{2} + \left| \mathbf{W}_{0}(0,k)\mathbf{W}_{0}^{2} + \left| \mathbf{W}_{3}(1,k)\mathbf{W}_{0}^{2} \right|^{2} \\ + \left| \mathbf{W}_{0}(1,k)\mathbf{W}_{0}^{2} + \left| \mathbf{W}_{0}(0,k)\mathbf{W}_{0}^{2} + \left| \mathbf{W}_{3}(1,k)\mathbf{W}_{0}^{2} \right|^{2} \\ + \left| \mathbf{W}_{0}(1,k)\mathbf{W}_{0}^{2} + \left| \mathbf{W}_{0}(0,k)\mathbf{W}_{0}^{2} + \left| \mathbf{W}_{3}(1,k)\mathbf{W}_{0}^{2} \right|^{2} \\ + \left| \mathbf{W}_{0}(1,k)\mathbf{W}_{0}^{2} + \left| \mathbf{W}_{0}(0,k)\mathbf{W}_{0}^{2} + \left| \mathbf{W}_{0}(1,k)\mathbf{W}_{0}^{2} \right|^{2} \\ + \left| \mathbf{W}_{0}(1,k)\mathbf{W}_{0}^{2} + \left| \mathbf{W}_{0}(1,k)\mathbf{W}_{0}^{2} + \left| \mathbf{W}_{0}(1,k)\mathbf{W}_{0}^{2} \right|^{2} \\ + \left| \mathbf{W}_{0}(1,k)\mathbf{W}_{0}^{2} \right|^{$$

Equation (A · 10) corresponds to Caushy-Schwarz inequality attaining the equality [16]. Therefore, from the condition that the equality is attained, i.e., $\mathbf{W}_i(n_r, k) = \mathbf{H}^H(n_r, k)$, we can derive the transmit MB-FDE weight matrices for OFDM transmission given as Eq. (14b).

It is seen from Eq. $(A \cdot 8)$ that the received SNR after STBC decoding depends on the norm of transmit FDE weight matrix. As shown in Eqs. (13) and (14b), the transmit MB-FDE weight matrix is sparse while the transmit SB-FDE weight matrix is dense. Therefore, MB-FDE reduces the norm of the transmit FDE weight matrix and accordingly, provides higher received SNR than SB-FDE in not only SC transmission but also OFDM transmission. Finally, the received SNR after STBC decoding, γ_{OFDM_MB} (γ_{OFDM_SB}), can be obtained by substituting Eq. (14b) (Eq. (13)) and Eq. (A \cdot 2) to Eq. (A \cdot 8).



Hiroyuki Miyazaki received his B.S. degree in Information and Intelligent Systems in 2011 and M.S. degree in Electrical and Communications Engineering, in 2013, respectively, from Tohoku University, Sendai Japan. Currently he is a Japan Society for the Promotion of Science (JSPS) research fellow, studying toward his PhD degree at the Department of Communications Engineering, Graduate School of Engineering, Tohoku University. His research interests include space-time block coding for mobile

communication systems. He was a recipient of 2013 IEICE RCS (Radio Communication Systems) Active Research Award.



Fumiyuki Adachi received the B.S. and Dr. Eng. degrees in electrical engineering from Tohoku University, Sendai, Japan, in 1973 and 1984, respectively. In April 1973, he joined the Electrical Communications Laboratories of Nippon Telegraph & Telephone Corporation (now NTT) and conducted various types of research related to digital cellular mobile communications. From July 1992 to December 1999, he was with NTT Mobile Communications Network, Inc. (now NTT DoCoMo, Inc.), where

he led a research group on wideband/broadband CDMA wireless access for IMT-2000 and beyond. Since January 2000, he has been with Tohoku University, Sendai, Japan, where he is a distinguished Professor of Communications Engineering at the Graduate School of Engineering. His research interest is in the areas of wireless signal processing and networking including broadband wireless access, equalization, transmit/receive antenna diversity, MIMO, adaptive transmission, channel coding, etc. He was a program leader of the 5-year Global COE Program "Center of Education and Research for Information Electronics Systems" (2007-2011), awarded by the Ministry of Education, Culture, Sports, Science and Technology of Japan. From October 1984 to September 1985, he was a United Kingdom SERC Visiting Research Fellow in the Department of Electrical Engineering and Electronics at Liverpool University. He is an IEICE Fellow and was a co-recipient of the IEICE Transactions best paper of the year award 1996, 1998, and 2009 and also a recipient of Achievement award 2003. He is an IEEE Fellow and a VTS Distinguished Lecturer for 2011 to 2013. He was a co-recipient of the IEEE Vehicular Technology Transactions best paper of the year award 1980 and again 1990 and also a recipient of Avant Garde award 2000. He was a recipient of Thomson Scientific Research Front Award 2004, Ericsson Telecommunications Award 2008, Telecom System Technology Award 2009, and Prime Minister Invention Prize 2010.