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A reduced-complexity near-ML detection for broadband single-carrier transmission

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Abstract: In our recently proposed reduced-complexity near-maximum likelihood block signal detection employing QR decomposition and M-algorithm (QRM-MLBD) for broadband single-carrier (SC) block transmission, unreliable symbol candidates are removed from the symbol tree based on the hard-decision minimum mean square error based frequency-domain equalization (MMSE-FDE). In this paper, to further remove the unreliable symbol candidates and consequently reduce the computational complexity, we introduce a soft-decision MMSE-FDE based selection of reliable symbol candidates into reduced-complexity QRM-MLBD. It is shown that the soft-decision MMSE-FDE based selection significantly reduces the computational complexity while achieving a bit error rate (BER) performance similar to the original QRM-MLBD.

Keywords: single carrier, maximum likelihood detection, QR decomposition, M-algorithm, MMSE-FDE

Classification: Wireless Communication Technologies

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1 Introduction

The BER performance of broadband SC transmission severely degrades due to strong inter-symbol interference (ISI). Although it has been shown that the QRM-MLBD can greatly improve the BER performance of SC block transmission [1, 2], it requires high computational complexity. To remedy this problem, we proposed the complexity-reduced QRM-MLBD [3]. However, its complexity reduction is limited.

In this paper, we introduce a soft-decision MMSE-FDE based selection of the reliable symbol candidates [4] into our reduced-complexity QRM-MLBD for further complexity reduction. We evaluate, by computer simulation, the average BER performance of SC transmissions achievable with the reduced-complexity QRM-MLBD utilizing the soft-decision MMSE-FDE and discuss its computational complexity.

2 Principle of reduced-complexity QRM-MLBD [3]

This section summarizes the principle of reduced-complexity QRM-MLBD proposed in Ref. [3]. Throughout the paper, the T_s symbol-spaced discrete time representation is used. Assuming the single-input single-output cyclic prefix (CP) inserted SC block transmission same as Ref. [3], the frequency-domain received signal $\mathbf{Y} = [Y(0), \ldots, Y(k), \ldots, Y(N_c - 1)]^T$, in which N_c is the size of discrete Fourier transform (DFT), is given as

$$\mathbf{Y} = \sqrt{\frac{2E_s}{T_s}}\mathbf{HFd} + \mathbf{N}, \tag{1}$$

where E_s is the symbol energy, $\mathbf{H} = \mathrm{diag}[H(0), \ldots, H(k), \ldots, H(N_c - 1)]$ is the $N_c \times N_c$ frequency-domain channel matrix, $\mathbf{d} = [d(0), \ldots, d(t), \ldots, d(N_c - 1)]^T$ is the data symbol block, $\mathbf{F}$ is the $N_c \times N_c$ DFT matrix whose (i, j) component is $(1/\sqrt{N_c})\exp\{-j2\pi(i \times j)/N_c\}$, and $\mathbf{N} = [N(0), \ldots, N(k), \ldots, N(N_c - 1)]^T$ is the noise vector whose elements are the independent zero-mean Gaussian variables having the variance $2N_0/T_s$ with N_0 being the one-sided power spectrum density of the additive white Gaussian noise (AWGN).

In the reduced-complexity QRM-MLBD, MMSE-FDE is performed before QRM-MLBD. The MMSE-FDE output $\tilde{\mathbf{d}} = [\tilde{d}(0), \ldots, \tilde{d}(t), \ldots, \tilde{d}(N_c - 1)]^T$ and its t-th component can be respectively represented as

$$\tilde{\mathbf{d}} = \mathbf{F}^H\mathbf{WY}, \tag{2}$$

$$\tilde{d}(t) = \sqrt{\frac{2E_s}{T_s}}\left(\frac{1}{N_c}\sum_{k=0}^{N_c-1}\tilde{H}(k)\right)d(t) + \mu_{ISI}(t) + \mu_{noise}(t), \tag{3}$$

where $(.)^H$ is the Hermitian transpose operation, $\tilde{H}(k) = W(k)H(k)$, and $\mathbf{W} = \mathrm{diag}[W(0), \ldots, W(k), \ldots, W(N_c - 1)]$ is the MMSE weight matrix given as [5]

$$\mathbf{W} = \mathrm{diag}\left[\frac{H^*(0)}{|H(0)|^2 + N_0/E_s}, \ldots, \frac{H^*(k)}{|H(k)|^2 + N_0/E_s}, \ldots, \frac{H^*(N_c - 1)}{|H(N_c - 1)|^2 + N_0/E_s}\right] \tag{4}$$

In Eq. (3), the first, second, and third terms are the desired signal, residual ISI, and noise, respectively. $\tilde{d}(t)$ is normalized by $\sqrt{2E_s/T_s}(1/N_c)\sum_{k=0}^{N_c-1}\tilde{H}(k) = P_{norm}$ as

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$$\tilde{d}'(t) = d(t) + \mu'_{ISI}(t) + \mu'_{noise}(t), \tag{5}$$

where

$$\begin{cases} \mu'_{ISI}(t) = P_{norm}^{-1}\mu_{ISI} = P_{norm}^{-1}\sqrt{\dfrac{2E_s}{T_s}}\dfrac{1}{N_c}\sum_{k=0}^{N_c-1}\tilde{H}(k)\left\{\sum_{\substack{n=0,\\ n\neq t}}^{N_c-1} d(n)\exp\left(j2\pi k\dfrac{t-n}{N_c}\right)\right\} \\ \mu'_{noise}(t) = P_{norm}^{-1}\mu_{noise} = P_{norm}^{-1}\dfrac{1}{N_c}\sum_{k=0}^{N_c-1} W(k)N(k)\exp\left(j2\pi k\dfrac{t}{N_c}\right) \end{cases} \tag{6}$$

Approximating $\mu'_{ISI}(t)$ as a zero-mean complex Gaussian variable from the central limit theorem, $\mu'_{ISI}(t) + \mu'_{noise}(t) = \mu(t)$ can be treated as a new zero-mean complex Gaussian variable with the variance σ_μ^2 being

$$\sigma_\mu^2 = \frac{1}{2}E[|\mu(t)|^2] = \sigma_{ISI}^2 + \sigma_{noise}^2, \tag{7}$$

where

$$\begin{cases} \sigma_{ISI}^2 = P_{norm}^{-2}\dfrac{E_s}{T_s}\left\{\dfrac{1}{N_c}\sum_{k=0}^{N_c-1}|\tilde{H}(k)|^2 + \left|\dfrac{1}{N_c}\sum_{k=0}^{N_c-1}\tilde{H}(k)\right|^2\right\} \\ \sigma_{noise}^2 = P_{norm}^{-2}\dfrac{1}{N_c}\dfrac{N_0}{T_s}\sum_{k=0}^{N_c-1}|W(k)|^2 \end{cases} \tag{8}$$

Assuming that all symbols are transmitted with equal probability, the *a posteriori* probability (APP) of symbol candidate c_i ($i = 0 \sim X-1$ with X being the modulation level) for the given $\tilde{d}'(t)$ can be expressed using Bayes' theorem as

$$P(c_i|\tilde{d}'(t)) = \frac{p(\tilde{d}'(t)|c_i)}{\sum_{n=0}^{X-1} p(\tilde{d}'(t)|c_n)} = \frac{\dfrac{1}{2\pi\sigma_\mu^2}\exp(-|\tilde{d}'(t)-c_i|^2/2\sigma_\mu^2)}{\sum_{n=0}^{X-1}\left\{\dfrac{1}{2\pi\sigma_\mu^2}\exp(-|\tilde{d}'(t)-c_n|^2/2\sigma_\mu^2)\right\}}, \tag{9}$$

where $p(\tilde{d}'(t)|c_i)$ is the conditional probability of $\tilde{d}'(t)$. In the reduced-complexity QRM-MLBD, the selection of reliable symbol candidate based on the APP distribution is performed. Removing the unreliable symbol candidates from the symbol tree before QRM-MLBD, the complexity for tree search reduces.

In Ref. [3], the APP distribution after hard-decision MMSE-FDE is used. However, the symbol candidates which are located at the same distance from hard-decision MMSE-FDE results have the same APP and therefore, the selection of reliable symbol candidates is fixed according to hard-decision MMSE-FDE results and the complexity reduction is limited.

3 Soft-decision MMSE-FDE based selection of reliable symbol candidates for reduced-complexity QRM-MLBD

In this paper, the soft-decision MMSE-FDE based selection of reliable symbol candidates [4] is used in the reduced-complexity QRM-MLBD. Differently from the hard-decision MMSE-FDE based selection, the soft-decision MMSE-FDE based selection can variably select the reliable symbol candidates by adapting to

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the APP distribution after soft-decision MMSE-FDE. As shown in Fig. 1(a), the symbol candidates are selected in descending order of APP until the accumulated AAP (AAPP) exceeds the threshold α ($0 \leq \alpha \leq 1$). The number of remaining symbol candidates associated with the t-th symbol $d(t)$ is denoted by $N_{cand}(t)$. In Sect. 4, we show the effect of α on the symbol error rate (SER).

After the reliable symbol candidate selection, only one symbol candidate may remain for $d(t)$, i.e., $N_{cand}(t) = 1$. In this case, $d(t)$ having $N_{cand}(t) = 1$ is detected by hard decision after MMSE-FDE. Such symbols can be removed from the symbol tree and the corresponding column components are removed from an $N_c \times N_c$ equivalent channel matrix $\mathbf{HF} = \bar{\mathbf{H}}$ as shown in Fig. 1(b-2). Denoting C as the number of $d(t)$ having $N_{cand}(t) = 1$ in $\mathbf{d}$, the size of equivalent channel matrix reduces from $N_c \times N_c$ to $N_c \times (N_c - C)$, thereby the computational complexity of QR decomposition reduces. The depth of the symbol tree also reduces from N_c to $N_c - C$. Note that in Ref. [3], $N_{cand}(t)$ is always more than 1 and the computational complexity of QR decomposition cannot be reduced.

In each stage of the M-algorithm, M paths having the smallest accumulated path metrics (i.e., accumulated squared Euclidean distance between the received signal and the symbol candidates) survive [1, 2]. When M is small, the BER performance of QRM-MLBD degrades. In the QRM-MLBD, the equivalent channel is transformed to an upper triangular matrix. The lower-right elements of the upper triangular matrix, which are associated with the symbols closer to the end of symbol block, get weaker and the accuracy of path metrics in earlier stages lowers [2]. Consequently, when M is small, pruning the correct path in earlier stages occurs with a high probability. To remedy this problem, the $N_{cand}(t)$-based ordering [4] is performed. The symbols are reordered in descending order of $N_{cand}(t)$ and the symbol tree is reconstructed as shown in Fig. 1(b-3). Since the symbols having smaller $N_{cand}(t)$ are gathered in earlier stages, the number of paths in earlier stages becomes smaller. Hence, the BER performance improves even if small M is used.

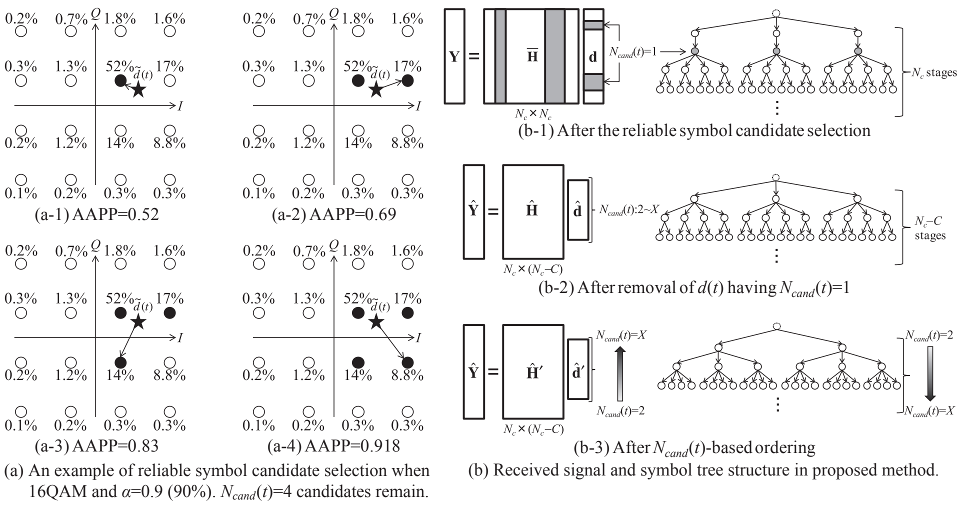

Fig. 1. Complexity reduction operations in reduced-complexity QRM-MLBD using the soft-decision MMSE-FDE based selection of reliable symbol candidates.

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Next, QRM-MLBD is performed. After removing $d(t)$ having $N_{cand}(t) = 1$ from the symbol tree and performing the $N_{cand}(t)$-based ordering, the frequency-domain received signal is expressed as

$$\hat{\mathbf{Y}} = \sqrt{\frac{2E_s}{T_s}}\hat{\mathbf{H}}'\hat{\mathbf{d}}' + \mathbf{N}, \tag{10}$$

where $\hat{\mathbf{H}}'$ is the $N_c \times (N_c - C)$ transformed equivalent channel matrix and $\hat{\mathbf{d}}'$ is the $(N_c - C) \times 1$ symbol vector. QR decomposition is applied to $\hat{\mathbf{H}}'$ as

$$\hat{\mathbf{H}}' = \hat{\mathbf{Q}}'\hat{\mathbf{R}}', \tag{11}$$

where $\hat{\mathbf{Q}}'$ is the $N_c \times (N_c - C)$ matrix satisfying $\hat{\mathbf{Q}}'^H\hat{\mathbf{Q}}' = \mathbf{I}_{N_c-C}$ with $\mathbf{I}_{N_c-C}$ being the $(N_c - C) \times (N_c - C)$ identity matrix and $\hat{\mathbf{R}}'$ is the $(N_c - C) \times (N_c - C)$ upper triangular matrix. Then, the transformed frequency-domain received signal $\hat{\mathbf{Z}} = [\hat{Z}(0), \ldots, \hat{Z}(k), \ldots, \hat{Z}(N_c - 1 - C)]^T$ is obtained as

$$\hat{\mathbf{Z}} = \hat{\mathbf{Q}}'^H\hat{\mathbf{Y}} = \sqrt{\frac{2E_s}{T_s}}\hat{\mathbf{R}}'\hat{\mathbf{d}}' + \hat{\mathbf{Q}}'^H\mathbf{N}, \tag{12}$$

Finally, the tree search using the M-algorithm is performed. The tree search is significantly simplified by selecting the reliable symbol candidates only, removing $d(t)$ having $N_{cand}(t) = 1$ from the symbol tree, and performing the $N_{cand}(t)$-based ordering.

4 Computer simulation

We assume a SC block transmission with 16QAM, $N_c = 64$, and the CP length is $N_g = 16$. The channel is assumed to be a frequency-selective block Rayleigh fading channel having symbol-spaced $L = 16$-path uniform power delay profile (PDP). Ideal channel estimation is assumed.

First, we discuss the effect of α on the error probability. The value of α is an important design parameter which affects the trade-off between the error probability and the computational complexity. Fig. 2(a) shows the average SER of QRM-MLBDs with $M = 256$ as a function of α at the average received $E_b/N_0 = 14$ dB ($E_b/N_0 = (E_s/N_0)(1 + N_g/N_c)/4$ for 16QAM). The original QRM-MLBD requires $M = 256$ for achieving the average BER $= 10^{-5}$ with a less than 1 dB E_b/N_0 degradation from the matched-filter (MF) bound [5] as seen from Fig. 2(c). Note that removal of $d(t)$ having $N_{cand}(t) = 1$ from the symbol tree and $N_{cand}(t)$-based ordering are not applied in Fig. 2(a) (our proposed QRM-MLBD consists of the soft-decision MMSE-FDE based selection of reliable symbol candidates, removal of $d(t)$ having $N_{cand}(t) = 1$ from the symbol tree and $N_{cand}(t)$-based ordering). In the following simulation, for each average received E_b/N_0, α is set to satisfy that the SER increase caused by the symbol candidate selection is less than 10% from the original QRM-MLBD with $M = 256$. For example, $\alpha = 0.9992$ is set when the average received $E_b/N_0 = 14$ dB.

Fig. 2(b) plots average $\overline{N_{cand}} = (1/N_c)\sum_{t=0}^{N_c-1} N_{cand}(t)$ and C as a function of the average received E_b/N_0, obtained by 1 million simulation runs. The value of α for each average received E_b/N_0 are also shown. It is seen from Fig. 2(b) that average $\overline{N_{cand}}$ reduces from $\overline{N_{cand}} = X = 16$ for original QRM-MLBD. Fig. 2(b) also shows that many stages can be removed from the symbol tree and hence, the

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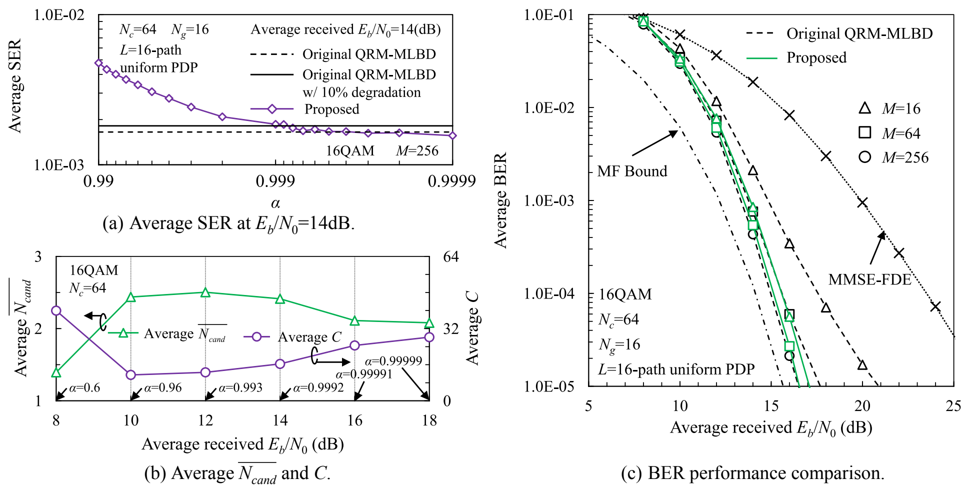

(a) Average SER at E_b/N_0=14dB.

(b) Average $\overline{N_{cand}}$ and C.

(c) BER performance comparison.

Fig. 2. Computer simulation results.

complexity of QR decomposition and the tree search can be reduced. Finally, Fig. 2(c) shows the average BER performance comparison. It is seen from Fig. 2(c) that the proposed QRM-MLBD with $M = 64$ achieves almost the same BER performance as the original QRM-MLBD with $M = 256$. Note that in Ref. [3], the average $\overline{N_{cand}} = 6.25$ and always $C = 0$, and $M = 256$ is required to achieve almost the same BER performance as the original QRM-MLBD.

In Table I, the average number of complex multiply operations is compared. The proposed QRM-MLBD requires computations of MMSE-FDE and the var-

Table I. The average number of complex multiply operations

	Original QRM-MLBD	Proposed QRM-MLBD
Fast Fourier transform (FFT)	$(N_c/2)\log_2 N_c$	
MMSE-FDE and inverse FFT (IFFT) to obtain $\tilde{\mathbf{d}}' = [\tilde{d}'(0),\ldots,\tilde{d}'(t),\ldots,\tilde{d}'(N_c-1)]^T$	-	$4N_c + (N_c/2)\log_2 N_c$
Computation of $\mathbf{HF} = \bar{\mathbf{H}}$	N_c^2	
Computation of σ_μ^2	-	$3N_c + 1$
Selection of reliable symbol candidates	-	XN_c
Removal of $d(t)$ having $N_{cand}(t) = 1$ from the symbol tree	-	CN_c
QR decomposition of equivalent channel matrix	N_c^3	$N_c(N_c - C)^2$
Transformation of equivalent channel matrix into upper triangular matrix	N_c^2	$N_c(N_c - C)$
Path metric computation ($M = 256$ for original QRM-MLBD and $M = 64$ for proposed QRM-MLBD)	$(N_c+1)X + \sum_{i=2}^{N_c}\left[\begin{array}{l}(i-1+X)\\ \times\min(X^{i-1}, M)\end{array}\right]$	$(N_c+1-C)\overline{N_{cand}} + \sum_{i=2}^{N_c-C}\left[\begin{array}{l}(i-1+\overline{N_{cand}})\\ \times\min(\overline{N_{cand}}^{\,i-1}, M)\end{array}\right]$

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iance of residual ISI plus noise, selection of reliable symbol candidates, and removal of $d(t)$ having $N_{cand}(t) = 1$ from the symbol tree, as well as QR decomposition. However, the complexity of QR decomposition and the path metric computation in the proposed QRM-MLBD can be reduced. Therefore, the computational complexity of proposed QRM-MLBD is greatly reduced compared to the original QRM-MLBD. For example, the proposed QRM-MLBD requires only 22% (26%) of computational complexity of original QRM-MLBD (complexity-reduced QRM-MLBD in Ref. [3]) when the average received $E_b/N_0 = 14$ dB.

5 Conclusion

The soft-decision MMSE-FDE based selection of reliable symbol candidate for reduced-complexity QRM-MLBD was proposed. Applying the soft-decision MMSE-FDE based symbol selection, the more number of unreliable symbol candidates can be removed from the symbol tree compared to Ref. [3]. Additionally applying removal of $d(t)$ having $N_{cand}(t) = 1$ from the symbol tree and $N_{cand}(t)$-based ordering, our proposed reduced-complexity QRM-MLBD greatly reduces the computational complexity from original QRM-MLBD while achieving almost the same BER performance. In this paper, the parameter α which affects the trade-off between the SER/BER and the computational complexity was predetermined for each average received E_b/N_0 by the preliminary computer simulation. The adaptive setting of α is left as an important future study.

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