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RESEARCH ARTICLE

Sub-Nyquist rate ADC sampling-based compressive channel estimation

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ABSTRACT

To realize high-speed communication, broadband transmission has become an indispensable technique in the next-generation wireless communication systems. Broadband channel is often characterized by the sparse multipath channel model, and significant taps are widely separated in time, and thereby, a large delay spread exists. Accurate channel state information is required for coherent detection. Traditionally, accurate channel estimation can be achieved by sampling the received signal with large delay spread by analog-to-digital converter (ADC) at Nyquist rate and then estimate all of channel taps. However, as the transmission bandwidth increases, the demands of the Nyquist sampling rate already exceed the capabilities of current ADC. In addition, the high-speed ADC is very expensive for ordinary wireless communication. In this paper, we present a novel receiver, which utilizes a sub-Nyquist ADC that samples at much lower rate than the Nyquist one. On the basis of the sampling scheme, we propose a compressive channel estimation method using Dantzig selector algorithm. By comparing with the traditional least square channel estimation, our proposed method not only achieves robust channel estimation but also reduces the cost because low-speed ADC is much cheaper than high-speed one. Computer simulations confirm the effectiveness of our proposed method. Copyright © 2013 John Wiley & Sons, Ltd.

KEYWORDS
compressive channel estimation; analog-to-digital converter (ADC); sub-Nyquist rate sampling; compressive sensing

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1. INTRODUCTION

1.1. Background and motivation

With the increasing number of wireless subscribers, various portable wireless devices, for example, smartphones and laptops, have generated increasing massive data traffic. The demand for high-speed data services is getting more difficult to be satisfied. Broadband transmission is an indispensable technique in the next-generation communication systems [1] and sensor networks [2–5]. However, the broadband signal is susceptible to interference caused by frequency-selective fading. Consequently, accurate channel estimation is required at the receiver for coherent detection. Because the broadband channel is often described by the sparse channel model, sparse channel estimation methods have been proposed to take the advantage of channel sparsity [6–13]. By exploiting the sparse prior information, channel estimation performance or spectral efficiency can be improved.

Nyquist rate analog-to-digital converter (ADC) sampling system based on traditional sparse channel estimation is shown in Figure 1. As the data rate increases, such a system poses two challenges for the next-generation broadband wireless communication systems. The first challenge is that the requirement of Nyquist sampling speed exceeds the capability of current ADC. Besides, high-speed ADC device becomes very expensive when the sampling rate increases. The second challenge is that Nyquist sampling-based system will reduce the spectrum efficiency because of the large number of training sequence used for channel estimation. Therefore, it is necessary to develop a novel technique to relax the requirement on high-speed ADC sampling in the broadband communication systems.

Shannon sampling theorem [14] is one of the fundamental theorems of modern signal processing. Given a

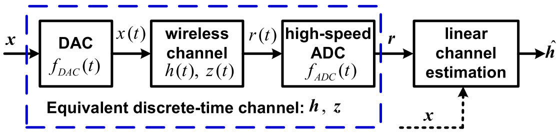

Figure 1. Communication system using Nyquist sampling rate ADC.

continuous signal $r(t), t \in [0, T)$ whose highest frequency is less than $W/2$ hertz, Shannon theorem suggests that the signal be sampled uniformly at a rate of W hertz. The signal can be recovered from the samples, given by

$$r(t) = \sum\nolimits_{n\in\Omega} r\left(\frac{n}{W}\right) \mathrm{sinc}(Wt - n), t \in [0, T) \qquad (1)$$

However, this well-known approach becomes impractical when the transmission rate or signal bandwidth is large because it is challenging to build a sampling device that operates at a sufficiently high speed. The demands of many modern applications already exceed the capabilities of current technology [15]. Even though recent developments in ADC have increased the sampling rate, state-of-the-art architectures are not yet adequate for the emerging applications. According to the Shannon sampling theorem [14], the number of taps in channel sampling has a linear relation with the transmission bandwidth. And the number of channel sampling taps easily reaches hundreds when its transmission bandwidth becomes large. A simple example of the number of channel sampling taps is depicted in Figure 2.

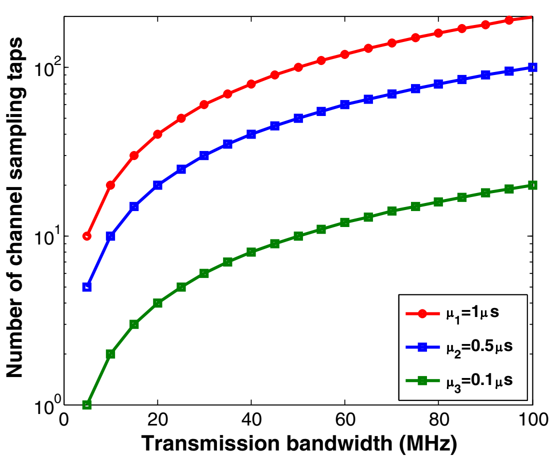

Figure 2. Relation between the number of channel sampling taps with Nyquist rate sampling and transmission bandwidth at different time delay spreads: $\mu_1 = 1\,\mu$s, $\mu_2 = 5\,\mu$s, and $\mu_3 = 0.1\,\mu$s.

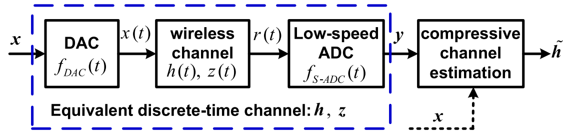

Figure 3. Broadband communication system using sub-Nyquist sampling rate ADC.

1.2. Main contribution

In this paper, we investigate compressive channel estimation (CCE) problem on the basis of low-speed ADC working at sub-Nyquist sampling rate. By using traditional channel estimation methods, low-speed ADC sampling will result in low-resolution channel estimation. As a result, the overall system performance will be significantly degraded. To realize high-resolution channel estimation with low costs, it is necessary to develop high-resolution CCE techniques. Different from the methods proposed in our previous work [10,12,13], in this paper, we assume that the receiver is equipped with low-speed ADC as shown in Figure 3. On the basis of the low-speed ADC sampling system, we propose a high-resolution CCE method. Channel estimation problem is formulated for sub-Nyquist ADC sampling-based system, and high-resolution CCE method is proposed to estimate the sparse channel. In addition, to obtain robust channel estimator, we design sub-Nyquist sampling rate-based training matrix to satisfy the restricted isometry property (RIP) [16] in the framework of compressive sensing (CS) [17,18].

1.3. Relations to other works

In our previous works [10,12,13], sparse channel estimation methods have been proposed for broadband communication system using Nyquist rate sampling ADC at the receiver. Although the proposed methods can take advantage of the channel sparsity, high-speed ADC sampling is required at the receiver to satisfy the Nyquist sampling rate. It will pose a big challenge on the high-speed ADC design for industry. In addition, high-speed ADC is also too expensive for ordinary wireless communication. Unlike the previous methods, we consider sub-Nyquist sampling ADC at the receiver in this study. The benefit of the system is to relax the requirement of high-speed ADC sampling and therefore reduce the communication cost. In [19], CS-based high-resolution channel estimation method has been proposed for orthogonal frequency division modulation systems. The authors utilized low-speed ADC to reduce the sampling speed by using random convolution, and high-resolution channel estimation was achieved by iterative support detection algorithm [20]. Different from this work, we use consecutive piecewise integration to realize high-resolution CCE when the sub-Nyquist sampling rate ADC is used.

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1.4. Outline and notations

The rest of this paper is organized as follows. Section 2 introduces the system model and problem formulation. Section 3 discusses CCE method from the sub-Nyquist rate sampling perspective. In Section 4, simulation results and discussion on the performance of the channel estimators are given. Finally, concluding remarks are presented in Section 5.

In this paper, boldface lower case letters $\mathbf{x}$ denote vectors, boldface capital letters $\mathbf{X}$ denote signal matrices, and lower case letters $x[n]$ and $x(n)$ represent the discrete-time signal and continuous-time signal, respectively. E[·] stands for the expectation operation. $\mathbf{X}^T$, $\mathbf{X}^*$, and $\mathbf{X}^\dagger$ denote the matrix transposition, conjugate, and conjugate transposition operations of $\mathbf{X}$, respectively. Considering the signal vector $\mathbf{x}$, $\|\mathbf{x}\|_0$ accounts its number of nonzero entries; $\|\mathbf{x}\|_1$ denotes L_1-norm and computes its absolute value; $\|\mathbf{x}\|_2$ is the Euclidean norm of $\mathbf{x}$; and $\|\mathbf{x}\|_\infty$ denotes infinity norm and selects its maximal absolute value. $\mathbb{C}$ denotes complex-valued representation of signals.

2. SYSTEM MODEL AND PROBLEM FORMULATION

A sparse multipath broadband communication system is considered in this paper. We assume that the discrete transmit data sequence is $\mathbf{x}_d = \{x_d[n], n = 0, 1, \ldots, N_d - 1\}$ and the training sequence is $\mathbf{x} = \{x[n], n=0, 1, \ldots, N-1\}$. Hereby, the transmit data block is composed orthogonally of the data sequence $\mathbf{x}_d$ and training sequence $\mathbf{x}$. The focus of this study is channel estimation, and only the transmission of the training sequence is considered.

2.1. Nyquist rate sampling-based system model

According to the Shannon sampling theorem, utilizing digital-to-analog converter (DAC) with impulse response $f_{DAC}(t)$, which uniformly works at W hertz, the discrete-time signal $\mathbf{x}$ can be converted into the continuous-time transmit waveform

$$x(t) = \sum_{n=0}^{N-1} x[n] f_{DAC}(t - n/W) \tag{2}$$

where n/W is the nth up-sampling period and $f_{DAC}(t) = \text{sinc}(t) = \sin(\pi t)/\pi t$ is the normalized sinc function. The waveform $x(t)$ is transmitted over a frequency-selective fading channel $h(t)$ with additive Gaussian noise interference $z(t)$; the receive continuous-time signal waveform is obtained as

$$r(t) = \int_0^{\tau_{max}} h(\tau) x(t - \tau) d\tau + z(t) \tag{3}$$

where τ_{max} denotes the maximum time-delay spread of the channel. According to the Shannon sampling theorem, discrete channel vector consists of a maximal number of sampling taps with

$$L = \lceil W \tau_{max} \rceil + 1 \tag{4}$$

at the Nyquist rate sampling period $1/W$. Hence, the physical channel impulse response $h(t)$ can be approximately by [21]

$$h(t) = \sum_{l=0}^{L-1} h(l) \delta(l - l/W) \tag{5}$$

Here, we assume that the L-length discrete channel vector $\mathbf{h} = [h[0], h[1], \ldots, h[L-1]]^T$ is supported by only K, $(K \ll L)$ dominant channel taps, which is often termed as K-sparse multipath channel. A simple example is shown in Figure 4, where $K = 5$ denotes the number of dominant taps and $L = 100$ denotes the channel sampling length. Combining Equations (2-3), we obtain a discrete-time channel that is described by the following relation between the discrete-time signals $x[n]$ and $r[n]$

$$r[n] = \sum_{l=0}^{L-1} h[l] x[n - l] + z[n] \tag{6}$$

where the equivalent discrete-time channel impulse response

$$h[l] = \int_{-\infty}^{\infty} \int_0^{\tau_{max}} h(\tau) f_{DAC}(t - \tau + n/W) f_{ADC}(-t) dt d\tau \tag{7}$$

with $l = 0, 1, \ldots, L-1$, and the discrete-time noise $z[n] = \int_{-\infty}^{\infty} z(t) f_{ADC}(n/W - t) dt$. From Equation (7), it can be found that the equivalent system channel vector $\mathbf{h}$ has a linear relationship with Nyquist sampling rate ADC. Assuming a fixed time-delay spread, the number of sampling channel taps increases with the transmission bandwidth, as shown in Figure 1.

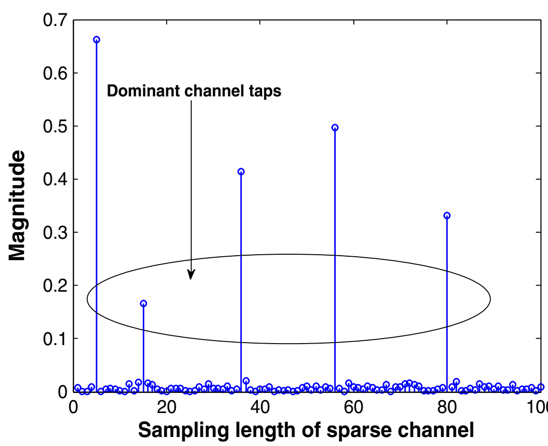

Figure 4. An example of sparse multipath channel with the overall sampling length 100 and the number of dominant channel taps is 5.

If the number of sampling channel taps is L, Shannon sampling theorem suggests that the length of sampling on the received signal should be N ($N \geq L$). Even though ADC sampling speed have been increased, state-of-the-art architectures are not yet adequate for the emerging broadband communication systems [15,22–24]. Fortunately, wireless channel is often sparse and is supported by only a few dominant channel taps. Hence, only the dominant taps are necessary to be estimated. If the channel sparsity can be exploited, then the length of sampling on the received signal reduces to $M \geq \mathrm{O}(K \log(N/K))$ rather than N. Hence, in the next-generation broadband communication systems, CCE is one of the key technologies to reduce the burden of high-speed sampling ADC at the receiver.

2.2. Sub-Nyquist rate sampling-based system model

Different from the receiver with Nyquist sampling rate ADC, here, received signal waveform $r(t)$ is sampled by low-speed ADC with impulse response $f_{\mathrm{S\_DAC}}(t)$ working at sub-Nyquist rate R, ($R \ll W$). As is shown in Figure 5, the components of the sub-Nyquist rate sampling ADC include a pseudorandom number (PN) generator, a mixer, a consecutive integrator, and a low-speed sampling ADC. When the received waveform $r(t)$ is input to the sub-Nyquist ADC, it is multiplied by the PN sequence $p(t)$ and then segmented by consecutive integrator. Then, at the mth integral window, the output discrete-time signal is given by

$$\begin{aligned} y[m] &= R\int_{m/R}^{m+1/R} y(t)\mathrm{d}t \\ &= R\int_{m/R}^{m+1/R} r(t)p(t)\mathrm{d}t \\ &= R\int_{m/R}^{m+1/R}\int_{0}^{\tau_{\max}} h(t)\,x(t-\tau)\,p(t)\mathrm{d}\tau\mathrm{d}t \\ &\quad + R\int_{m/R}^{m+1/R} z(t)p(t)\mathrm{d}t \end{aligned} \tag{8}$$

for $m = 0, 1, \ldots, M-1$, where $M = RN/W$ denotes the length of sub-Nyquist rate sampling; if $W = R$, then the sub-Nyquist sampling length M equals to the Nyquist sampling length N. According to previous Equations (2–8),

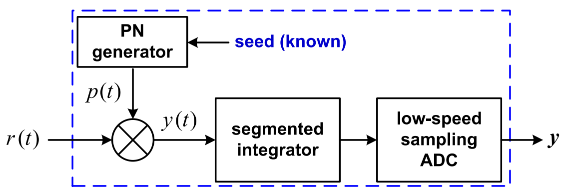

Figure 5. The components of the sub-Nyquist rate sampling ADC include a PN generator, a mixer, a segmented integrator, and a low-speed sampling ADC.

we can obtain the matrix–vector form input–output system relation, given by

$$\mathbf{y} = \mathbf{QPXh} + \mathbf{z} = \mathbf{\Theta h} + \mathbf{z} \tag{9}$$

where $\mathbf{y} = [y[0], y[1], \ldots, y[N-1]]^{\mathrm{T}} \in \mathbb{C}^{M\times 1}$ denotes the received signal vector, $\mathbf{z} = [z[0], z[1], \ldots z[N-1]]^{\mathrm{T}} \in \mathbb{C}^{M\times 1}$ is an observed additive white Gaussian noise vector that is distributed as CN $\left(\mathbf{0}\sigma_n^2\mathbf{I}_M\right)$; $\mathbf{h}$ is a discrete-time channel vector; $\mathbf{\Theta} = \mathbf{QPX}$ denotes a sub-Nyquist rate sampling training matrix with its column vector $\theta_l = \sum_{n=1}^{N}\mathbf{p}_n x_{nl}\mathbf{q}_n$, $l = 1, 2, \ldots, L$, x_{nl} is the (nl)-th entry of $\mathbf{X}$; $\mathbf{X}$ is the $N \times L$ equivalent discrete circulant training matrix,

$$\begin{bmatrix} x[L] & \ldots & x[2] & x[1] \\ x[L+1] & \ldots & x[3] & x[2] \\ \vdots & & \vdots & \vdots \\ x[N+L-1] & \ldots & x[N+1] & x[N] \end{bmatrix} \tag{10}$$

and

$$\mathbf{P} = \mathrm{diag}\{\mathbf{p}\} = \begin{bmatrix} p_1 & & \\ & \ddots & \\ & & p_N \end{bmatrix} \in \mathbb{C}^{N\times N} \tag{11}$$

is the random diagonal demodulation matrix whose diagonal entries $\mathbf{p} = [p_0 p_1, \ldots, p_N]$ are generated by PN binary sequences $(+1/-1)$ with equal probability; the matrix $\mathbf{Q}$ is given by

$$\mathbf{Q} = \begin{bmatrix} \mathbf{q}_1 & & & & \\ & \ddots & & & \\ & & \mathbf{q}_m & & \\ & & & \ddots & \\ & & & & \mathbf{q}_M \end{bmatrix} \in C^{N\times N} \tag{12}$$

It is an equivalent sub-Nyquist sampling matrix. $\mathbf{q} \in C^{1\times N/M}$ denotes each segmented N/M consecutive unit entries in mth row and starts in each column $mN/M+1$ for each $m = 0, 1, \ldots, M-1$. In the next section, we focus on CCE methods for the sub-Nyquist rate sampling-based communication systems.

3. COMPRESSIVE CHANNEL ESTIMATION

3.1. Review of the compressive sensing theory

The CS theory [17,18] states that a K-sparse channel vector $\mathbf{h}$ can be robustly estimated from Equation (10), where $\mathbf{\Theta}$ is equivalent sub-Nyquist matrix with M rows and L

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columns, $M < L$, and $\mathbf{z}$ is the additive noise. Mathematically, the optimal compressive channel estimator $\mathbf{h}_{\text{opt}}$ can be achieved by

$$\mathbf{h}_{\text{opt}} = \arg\min_{h} \left\{ \frac{1}{2} \|\mathbf{y} - \mathbf{\Theta h}\|_2^2 + \lambda_0 \|\mathbf{h}\|_0 \right\} \tag{13}$$

where λ_0 is a regularization parameter, which balances the estimation error and channel sparsity. Unfortunately, Equation (13) is non-deterministic polynomial-time hard problem [18]. In other words, optimal compressive channel estimators are unlikely to be calculated efficiently even in noiseless environment. Fortunately, numerous practical alternative CS algorithms can acquire a suboptimal solution for the channel $\mathbf{h}$ if the $\mathbf{\Theta}$ satisfies the RIP [16] in Equation (10). For any K-sparse channel vector $\mathbf{h}$ in the noise observation model in Equation (13), if the training matrix $\mathbf{\Theta}$ satisfies

$$(1 - \delta_K)\|\mathbf{h}\|_2^2 \le \|\mathbf{\Theta h}\|_2^2 \le (1 + \delta_K)\|\mathbf{h}\|_2^2 \tag{14}$$

with high probability with a parameter $\delta_K \in (0, 1)$, then accurate channel estimation can be obtained. It is worth noting that reasonable sub-Nyquist rate R should be chosen to ensure $\mathbf{\Theta}$ to satisfy RIP with a high probability. In other words, the sub-Nyquist sampling rate R should be appropriate for the following theorem so that $\mathbf{\Theta}$ satisfies RIP with high probability.

3.2. Restricted isometry property for equivalent training matrix

For the sub-Nyquist sampling receiver, accurate channel estimation can be acquired if we can choose appropriate sampling rate R so that the equivalent training matrix $\mathbf{\Theta}$ satisfies RIP. The theorem of RIP is introduced next.

Theorem 1. *(The RIP for equivalent training matrix* $\mathbf{\Theta}$ *based on the sub-Nyquist rate sampling* [22]*): If the matrix* $\mathbf{\Theta}$ *satisfies RIP with constant* $\delta_K \in (0, 1)$ *with a high probability* $1 - \mathrm{O}(\mathrm{W}^{-1})$*, then sub-Nyquist sampling rate* R *should be satisfied.*

$$R \ge C\delta_K^{-2} K \log^6 W \tag{15}$$

Proof. The RIP of the matrix $\mathbf{\Theta}$ has been given in Equation (15), which can also be rewritten as

$$\left| \frac{\|\mathbf{\Theta h}\|_2^2 - \|\mathbf{h}\|_2^2}{\|\mathbf{h}\|_2^2} \right| \le \delta_K \tag{16}$$

for $\|\mathbf{h}\|_0 \le K$. It is not easy to find the relationship between RIP and $\mathbf{\Theta}$ in Equation (16). For this reason, we resort to its equivalent equation as

$$\left| \frac{\mathbf{h}^*(\mathbf{\Theta}^*\mathbf{\Theta} - \mathbf{I})\mathbf{h}}{\mathbf{h}^*\mathbf{h}} \right| \le \delta_K \tag{17}$$

According to Equation (17), if $\mathbf{\Theta}$ satisfies the RIP, then the most significant eigenvalue of any $K \times K$ principal sub-matrix of $(\mathbf{\Theta}^*\mathbf{\Theta} - \mathbf{I})$ should be satisfied.

$$\epsilon = E\left\{ \left\| \left| (\mathbf{\Theta}^*\mathbf{\Theta} - \mathbf{I}) \right| \right\| \right\} \le \delta_K \tag{18}$$

where $|||\mathbf{\Theta}||| = \sup_{|\Omega| \le K} \|(\mathbf{\Theta}|_{\Omega\times\Omega})\|$ and sup denotes supreme function. Assuming that $\theta_m^*, m = 1, 2, \ldots, M$ denotes the mth row of $\mathbf{\Theta}$, then the Gram matrix of $\mathbf{\Theta}$ can be expressed by

$$\mathbf{\Theta}^*\mathbf{\Theta} = \sum_{m=1}^{M} \theta_m \otimes \theta_m \tag{19}$$

Following the definition of the RIP, we can get

$$\begin{aligned} \epsilon &= \mathrm{E}\left\{ \left\| \left| \mathbf{\Theta}^*\mathbf{\Theta} - \mathbf{I} \right| \right\| \right\} \\ &= \mathrm{E}\left\{ \left\| \left| \sum\nolimits_{m=1}^{M} \theta_m \otimes \theta_m - \mathbf{I} \right| \right\| \right\} \\ &= \mathrm{E}\left\{ \left\| \left| \sum\nolimits_{m=1}^{M} \theta_m \otimes \theta_m - \mathbf{E}\{\phi_m \otimes \phi_m\} \right| \right\| \right\} \end{aligned} \tag{20}$$

where $\{\phi_m m = 1, 2, \ldots, M\}$ is an independent copy of $\{\theta_m m = 1, 2, \ldots, M\}$. In other words, each $\phi_m m = 1, 2, \ldots, M$ is chosen randomly, and it can keep orthogonal to other vectors with high probability.

For this aim, it is necessary to analyze the Gram matrix of $\mathbf{\Theta} = \mathbf{QPX}$. Let n and n' be the column indices in $\mathbf{Q}$, $\mathbf{P}$, and $\mathbf{X}$; it is easy to find that column vectors $\mathbf{x}_n, n = 1, 2, \ldots, N$ are orthogonal to each other. It can be obtained that

$$\begin{aligned} \langle \theta_m, \theta_{m'} \rangle &= \sum\nolimits_{n,n'=1}^{N} \mathbf{p}_n \mathbf{p}_{n'} \langle \mathbf{q}_m, \mathbf{q}_{m'} \rangle \mathbf{x}_{nm}^* \mathbf{x}_{n'm'} \\ &= \delta_{mm'} + \sum\nolimits_{n \ne n'}^{N} \mathbf{p}_n \mathbf{p}_{n'} \langle \mathbf{q}_m, \mathbf{q}_{m'} \rangle \mathbf{x}_{nm}^* \mathbf{x}_{n'm'} \end{aligned} \tag{21}$$

where $\delta_{mm'}$ denotes the Kronecker delta function, that is, $\delta_{mm'} = 1$ if $m = m'$ and $\delta_{mm'} \ne 0$ if $m \ne m'$. Hence, the Gram matrix of $\mathbf{\Theta}$ can be divided as

$$\mathbf{\Theta}^*\mathbf{\Theta} = \mathbf{I} + \mathbf{D} \tag{22}$$

where $d_{mm'} = \sum_{n \ne n'}^{N} \mathbf{p}_n \mathbf{p}_{n'} \langle \mathbf{q}_m, \mathbf{q}_{m'} \rangle \mathbf{x}_{nm}^* \mathbf{x}_{n'm'}$. It is clear that $\mathrm{E}\{\mathbf{D}\} = 0$ so that

$$\mathrm{E}\left\{ \mathbf{\Theta}^*\mathbf{\Theta} \right\} = \mathbf{I} \tag{23}$$

According to the symmetrization theorem in [22], Equation (20) can be rewritten as

$$\begin{aligned}\epsilon &\le 2\mathrm{E}\left\{\left\|\left|\sum_{m=1}^{M} \mathbf{p}_m \theta_m \otimes \theta_m\right|\right\|\right\} \\ &\le 2\mathrm{E}\left\{C_1 B \sqrt{K} \log^2 W \cdot \left\|\left|\sum_{m=1}^{M} \theta_m \otimes \theta_m\right|\right\|^{1/2}\right\} \\ &\le 2\sqrt{6} C_1 \sqrt{\frac{K \log^5 W \cdot}{R}} \mathrm{E}\left\{\left\|\left|\sum_{m=1}^{M} \theta_m \otimes \theta_m\right|\right\|^{1/2}\right\} \\ &\le \sqrt{\frac{C_2 K \log^5 W \cdot}{R}} \left(\epsilon^2 + 1\right)^{1/2} \\ &\le \sqrt{\frac{C_2 K \log^5 W \cdot}{R}} \end{aligned} \tag{24}$$

where $B = \max\limits_{m,n} \{x_{mn}\}$ is a random entry of $\mathbf{X}$. To guarantee that $\boldsymbol{\Theta}$ satisfies RIP,

$$\sqrt{\frac{C_2 K \log^5 W}{R}} \le \delta_K \tag{25}$$

if $R \ge C_2 \delta^{-2} K \log^5 W$, then $\boldsymbol{\Theta}$ satisfies RIP with the probability

$$\mathrm{P}\left\{\left\|\left|\boldsymbol{\Theta}^* \boldsymbol{\Theta} - \mathbf{I}\right|\right\| < \delta_K\right\} = 1 - \mathrm{O}\left(W^{-1}\right) \tag{26}$$

□

3.3. Compressive channel estimation

We formulate the sub-Nyquist rate sampling-based channel estimation as a CS problem [17,18] and also name the estimation methods as CCE. Sparse channel estimation methods [6–12] have been intensively studied in recent years. However, all of the proposed methods are based on the Nyquist rate sampling. The estimation methods can be classified by two types: one is mixed-norm-based convex relation algorithm, for example, least absolute shrinkage and selection operator [25] and Dantzig selector (DS) [26], and the other is greedy iterative algorithm, for example, orthogonal matching pursuit [27] and compressive sampling matching pursuit [28]. Considering the sub-Nyquist sampling, CCE method is implemented by DS algorithm, which is termed as CCE-DS. The flowchart of CCE-DS is shown in Table I.

3.4. Lower bound of compressive channel estimator

To evaluate the performance of the proposed methods, we give lower bound on the basis of sub-Nyquist ADC sampling. Consider the sub-Nyquist ADC sampling, which is shown in Figure 3; the equivalent training matrix is $\boldsymbol{\Theta} \in \mathbb{C}^{M \times L}$, and the discrete received signal is $\mathbf{y} \in \mathbb{C}^{M \times 1}$.

Table I. Sub-Nyquist rate sampling-based compressive channel estimation Dantzig selector.

Input	An $N \times L$ complex training matrix $\mathbf{X}$ An $M \times N$ sub-Nyquist sampling matrix $\mathbf{Q}$ An $N \times N$ diagonal modulation matrix $\mathbf{P}$ An M-dimensional received vector $\mathbf{y}$ A regularized parameter $\lambda = \sigma\sqrt{2\log \mathbf{K}}$
Run	cvx_begin variable $\mathbf{h}(L)$ minimize $\{\|\mathbf{y} - \boldsymbol{\Theta}\mathbf{h}\|_\infty + \lambda \|\mathbf{h}\|_1\}$ cvx_end
Output	An M-dimensional channel estimator $\mathbf{h}_{DS}$

The tap position set of channel $\mathbf{h}$ is defined as Ω, and we assume the position set of its dominant taps denoted by Ω_K is known. By using the prior information, the lower bound of channel estimator $\mathbf{h}_{\mathrm{S\_ADC}}$ can be given by

$$\mathbf{h}_{\mathrm{S\_ADC}} = \begin{cases} \arg\min_{\Omega_K} \left\|\mathbf{r} - \boldsymbol{\Theta}_{\Omega_K} \mathbf{h}_{\Omega_K}\right\|_2, & \Omega_K \\ 0, & \Omega / \Omega_K \end{cases} \tag{27}$$

where $\mathbf{h}_{\Omega_K}$ contains the Ω_K dominant channel taps of $\boldsymbol{h}$ and $\boldsymbol{\Theta}_{\Omega_K}$ is the sub-matrix constructed from the Ω_K columns of $\mathbf{X}$. In the following, the lower bound of channel estimator $\mathbf{h}_{\mathrm{S\_ADC}}$ is obtained by average mean square error (MSE), given by

$$\begin{aligned}\mathrm{E}\left\{\|\mathbf{h} - \mathbf{h}_{\mathrm{S\_ADC}}\|_2^2\right\} &= \mathrm{E}\left\{\left(\boldsymbol{\Theta}_{\Omega_K}^H \boldsymbol{\Theta}_{\Omega_K}\right)^{-1} \boldsymbol{\Theta}_{\Omega_K}^H \mathbf{z}\right\} \\ &= \mathrm{Trace}\left\{\left(\boldsymbol{\Theta}_{\Omega_K}^H \boldsymbol{\Theta}_{\Omega_K}\right)^{-1} \boldsymbol{\Theta}_{\Omega_K}^H \mathbf{z}\mathbf{z}^H \right. \\ &\quad \left. \boldsymbol{\Theta}_{\Omega_K} \left(\boldsymbol{\Theta}_{\Omega_K}^H \boldsymbol{\Theta}_{\Omega_K}\right)^{-1}\right\} \\ &= \sigma_n^2 \mathrm{Trace}\left\{\left(\boldsymbol{\Theta}_{\Omega_K}^H \boldsymbol{\Theta}_{\Omega_K}\right)^{-1}\right\} \\ &\ge \frac{\sigma_n^2 K^2}{\mathrm{Trace}\left\{\left(\boldsymbol{\Theta}_{\Omega_K}^H \boldsymbol{\Theta}_{\Omega_K}\right)\right\}} \\ &= \sigma_n^2 K / N \end{aligned} \tag{28}$$

where $\mathrm{Trace}\left\{\left(\boldsymbol{\Theta}_{\Omega_K}^H \boldsymbol{\Theta}_{\Omega_K}\right)\right\} = N/K$.

4. NUMERICAL SIMULATIONS

In this section, we will compare the performance of the proposed estimator with least square (LS)-based linear estimator by numerical results on the basis of 1000 independent Monte Carlo runs. The broadband bandwidth is set to be $W/2 = 100\,\mathrm{MHz}$ and channel delay spread $\tau_{\max} = 0.5\,\mu\mathrm{s}$. Hereby, the sampling length of channel vector $\mathbf{h}$ is set to be $L = 100$, and the number of dominant channel taps of $\mathbf{h}$ is $K = 4$ where the positions of

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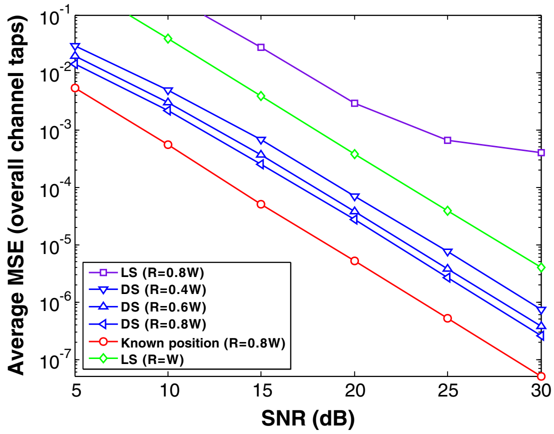

Figure 6. Average MSE performance of overall channel taps.

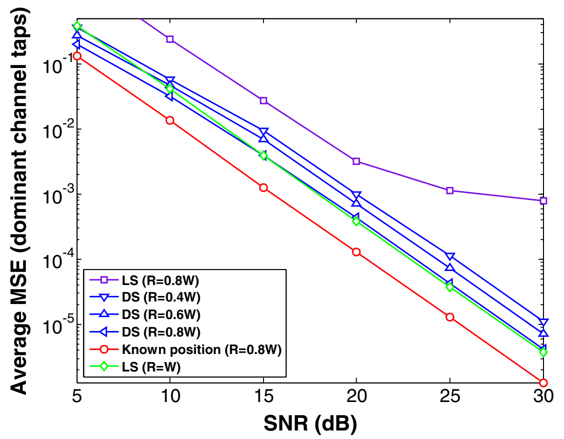

Figure 7. Average MSE performance of dominant channel taps.

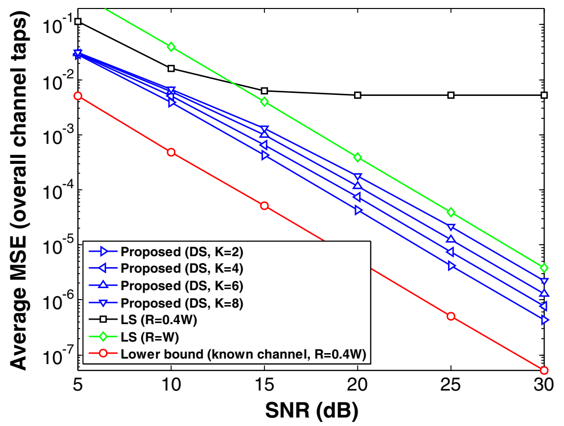

Figure 8. Average MSE performance of overall channel taps.

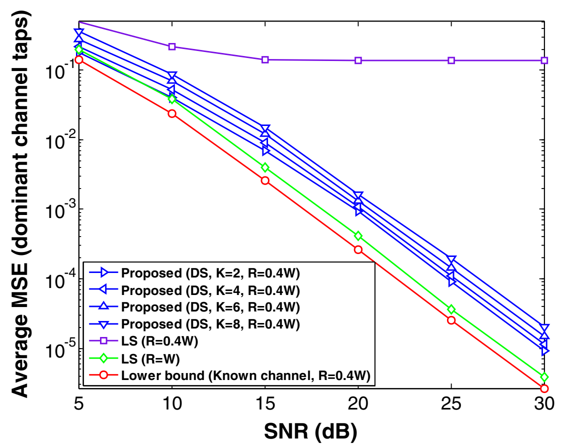

Figure 9. Average MSE performance of dominant channel taps.

dominant taps are generated following Gaussian distribution and subject to $\|\mathbf{h}\|_2^2 = 1$. The Nyquist sampling rate of high-speed ADC is $W = 200\,\text{MHz}$, and sub-Nyquist sampling rate of low-speed ADC is $R = \alpha W$, where α is a sub-Nyquist sampling factor. In this simulation, we consider the sub-Nyquist sampling factor to be $\alpha = 0.4, 0.6$, and 0.8. The length of training sequence is $N = 120$. The received signal-to-noise ratio is defined as $10\log(P/\sigma_n^2)$. Channel estimators $\hat{\mathbf{h}}$ are evaluated by average MSE, which is defined by

$$\text{Average MSE}\left(\hat{\mathbf{h}}\right) = \text{E}\left\{\left\|\mathbf{h} - \hat{\mathbf{h}}\right\|_2^2\right\}/L \tag{29}$$

where $\mathbf{h}$ and $\hat{\mathbf{h}}$ denote the channel vector and its estimator, respectively, and L is the sampling length.

In Figure 6, we compare the average MSE performance of the proposed channel estimator and the LS channel estimator on the overall channel taps. It is found that the sub-Nyquist sampling rate-based compressive channel estimators are better than Nyquist rate sampling-based LS channel estimator. For the sparse multipath channel, the proposed method can estimate the dominant channel taps robustly, whereas the non-dominant channel taps are forced to zeros. Unlike the proposed method, LS-based channel estimator has to estimate all the channel taps uniformly even if the channel taps are equal to zero. It is worth mentioning that the LS channel estimator cannot work well on sub-Nyquist rate sampling, as shown in the purple curve in Figure 6; when sub-Nyquist rate $R = 0.8\,\text{W}$ is used for LS channel estimator, the average MSE performance has been significantly degraded.

In Figure 7, the comparison of average MSE performance is taken on dominant channel taps. It is found that when the sub-Nyquist sampling rate $R = 0.8\,\text{W}$, the proposed channel estimator can achieve the same average MSE performance as Nyquist sampling rate LS channel estimator. Even for the cases when $R = 0.4\,\text{W}$ and $R = 0.6\,\text{W}$, the performances of the proposed estimator are also close to LS-based one. From Figures 6 and 7, it can be concluded that the proposed channel estimator can achieve almost the same high-resolution channel estimation as the Nyquist sampling rate LS channel estimator.

To further confirm the advantage of the proposed method, we vary the channel sparsity, and the average MSE performances are evaluated and shown in Figures 8 and 9. The number of nonzero channel taps is set to be $K = 2, 4, 6,$ and 8, respectively. And the sub-Nyquist sampling rate is set to be $R = 0.4\,\mathrm{W}$. Figure 8 depicts the average MSE performance of the overall channel taps. It is observed that sub-Nyquist sampling-based CCE method can achieve better estimation than Nyquist sampling-based LS method. In addition, as shown in the blue curves in Figure 8, the proposed method can robustly estimate the channels with different channel sparsity under the sub-Nyquist sampling rate. It is worth mentioning that the sparser the channel is, the better estimation performance can be achieved by the proposed method. However, the LS channel estimator can obtain robust performance only under the Nyquist sampling rate; when sub-Nyquist sampling rate is used, the performance is significantly degraded. Figure 9 shows the average MSE performance of the dominant channel taps. It is shown that the proposed sub-Nyquist sampling rate-based channel estimator can achieve close performance as the Nyquist sampling rate-based LS channel estimator.

5. CONCLUSION

Broadband transmission poses a big challenge on ADC sampling capability in the next-generation wireless communication systems. To avoid the high-speed ADC sampling at receiver, we consider low-speed sampling ADC at the receiver in this paper. The channel estimation has been formulated as a CS problem, and a CCE method using DS algorithm has been proposed. By comparing with traditional Nyquist sampling rate LS channel estimation, the proposed method can reduce sampling rate while achieving similarly good estimation performance. The effectiveness of the proposed method has been verified by the simulation results.

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