RESEARCH ARTICLE

Sparse LMS/F algorithms with application to adaptive system identification

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ABSTRACT

Standard least mean square/fourth (LMS/F) is a classical adaptive algorithm that combined the advantages of both least mean square (LMS) and least mean fourth (LMF). The advantage of LMS is fast convergence speed while its shortcoming is suboptimal solution in low signal-to-noise ratio (SNR) environment. On the contrary, the advantage of LMF algorithm is robust in low SNR while its drawback is slow convergence speed in high SNR case. Many finite impulse response systems are modeled as sparse rather than traditionally dense. To take advantage of system sparsity, different sparse LMS algorithms with l_p -LMS and l_0 -LMS have been proposed to improve adaptive identification performance. However, sparse LMS algorithms have the same drawback as standard LMS. Different from LMS filter, standard LMS/F filter can achieve better performance. Hence, the aim of this paper is to introduce sparse penalties to the LMS/F algorithm so that it can further improve identification performance. We propose two sparse LMS/F algorithms using two sparse constraints to improve adaptive identification performance. Two experiments are performed to show the effectiveness of the proposed algorithms by computer simulation. In the first experiment, the number of nonzero coefficients is changing, and the proposed algorithms can achieve better mean square deviation performance than sparse LMS algorithms. In the second experiment, the number of nonzero coefficient is fixed, and mean square deviation performance of sparse LMS/F algorithms is still better than that of sparse LMS algorithms. Copyright © 2013 John Wiley & Sons, Ltd.

KEYWORDS

least mean square; least mean fourth; least mean square/fourth (LMS/F); *I*_p-norm LMS/F; *I*₀-norm LMS/F; sparse penalty; adaptive system identification

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1. INTRODUCTION

1.1. Background and motivation

Adaptive system identification includes many applications such as interference cancelation [1], adaptive beamforming [2], and channel estimation in different systems [3–6]. One of the classical algorithms is called least mean square (LMS), which is first proposed by Widrow and Hoff [7]. In the last decades, LMS filter is widely used in many applications [8]. In most of these scenarios, finite impulse responses (FIRs) of unknown systems can be modeled sparsely [9–15]. The FIR coefficients vector is supported only by very few dominant coefficients. A typical example of sparse system is shown in Figure 1, where length of FIR is N = 16 while number of dominant coefficients is K = 2. As we know, using such sparse prior information can improve the filtering performance. However, standard LMS filter never takes advantage of such information. In the past years, many sparse LMS algorithms have been proposed to exploit sparsity. Motivated by the compressive sensing (CS) [16,17], Chen and his collaborators proposed zero-attracting LMS (ZA-LMS) and reweighted ZA-LMS (RZA-LMS) algorithms using l_1 -norm sparse penalty [18]. Based on this work, Taheri and Vorobyov proposed an improved sparse LMS algorithm using l_p -norm sparse penalty [19], which is termed as LP-LMS. Gu and his collaborators also proposed an improved sparse penalty [20], which is termed as L0-LMS. According to



Figure 1. Example of a 16-length sparse system where the number of dominant coefficients is K = 2.

CS [16,17], it is well known that a stronger sparse constraint can exploit more accurate sparse structure information. Hence, performance comparison between the four aforementioned sparse LMS algorithms can be sorted from good to poor: L0-LMS, LP-LMS, RZA-LMS, and ZA-LMS. Interested readers can also refer to the overall discussions and simulation results in [21,22].

From the preceding introduction of sparse LMS algorithms, we deduce that their adaptive updating equations are based on updating the equation of standard LMS algorithm. Unfortunately, the common drawback of these algorithms is that LMS is sensitive to scaling of input signal and noise interference, especially in low signal-to-noise ratio (SNR) regime [13,23]. To mitigate the two hostile effects, adaptive algorithms using higher-order moments of the error signal have been shown to perform better mean square estimation than LMS in some important applications. The typical algorithm is that least mean fourth (LMF) algorithm, developed by Walach and Widrow in [23], applied a *fourth-order power* optimization criterion instead of the square power used for LMS. Their idea came from the fact that higher-order power filters can mitigate noise interference effectively [24]. However, standard LMF filter does not exploit sparsity on system identification. To take advantage of such sparsity, we proposed sparse LMF algorithms to improve identification performance [13]. In this research, sparse LMF filters can achieve much better performance than sparse LMS. According to theoretical analysis and computer simulations, it was found that sparse LMF algorithm can achieve much better performance than sparse LMS algorithms in low SNR environment without sacrificing high computational complexity. In high SNR regime, unfortunately, sparse LMF algorithms cannot work well because of its slow convergence speed.

To full take advantage of obvious merits of LMS and LMF, it is logical to combine two algorithms with application to adaptive system identification. The combined LMS/F algorithm has been first proposed in [25] and further developed in [26], as a method to improve the performance of the LMS adaptive filter without sacrificing the simplicity and stability properties of LMS. However, they have never considered its application to adaptive sparse system identification.

1.2. Main contribution

In this paper, we propose sparse LMS/F algorithms to exploit system sparsity using two sparse penalties, that is, l_p -norm and l_0 -norm. They are termed as LP-LMS/F and L0-LMS/F, respectively. As we know, both LP-LMS and L0-LMS filters have achieved better performance than ZA-LMS and RZA-LMS [19,21]. Hence, two ZA-LMS/F and RZA-LMS/F algorithms are omitted because of space limitation.

The main contribution of this paper is to first propose sparse LMS/F algorithms with application to adaptive sparse system identification. Sparse penalized cost functions are constructed to implement sparse LMS/F algorithms. At last, two experiments are given to confirm the effectiveness of our proposed methods. In the first experiment, the mean square deviation (MSD) performance of sparse LMS/F algorithms is evaluated according to different numbers of dominant FIR coefficients. In the second experiment, when the number of dominant FIR coefficients are invariant, the MSD performance of the proposed algorithms is evaluated in different SNRs.

1.3. Relations to other works

In our previous work [13], sparse LMF algorithm using fourth-order power optimization criterion was proposed to improve system identification performance. The main drawback of this proposed algorithm is its instability in high SNR regime (SNR \geq 10 dB). Hence, the proposed algorithm can only be applied in low SNR regime. In a previous work [22], we proposed an improved sparse LMS algorithm using second-order power optimization criterion. In addition, several normalized sparse LMS algorithms were proposed. To improve the performance of sparse LMS algorithm, an improved μ -law proportionate normalized LMS algorithm was also proposed in [27]. Unlike the proposed methods using either fourth-order or second-order power optimization criterion, the proposed sparse LMS/F uses a hybrid power optimization criterion, which combines fourth-order and second-order power optimization criteria.

1.4. Notations

The rest of the paper is organized as follows. Section 2 reviews the LMS and LMS/F algorithms. In Section 3, we construct sparse penalized LMS/F cost functions and propose two adaptive sparse algorithms. In Section 4, Monte

Carlo simulation results for MSD standard are presented to confirm the effectiveness of sparse LMS/F algorithms. Concluding remarks are presented in Section 5.

2. REVIEW OF STANDARD LMS AND LMS/F ALGORITHM

Assume an unknown system as shown in Figure 2, input signal is $\mathbf{x}(t)$ at time t, and N length FIR filter coefficients vector is $\mathbf{w} = [w_0, w_1, \dots, w_{N-1}]^T$, and then the observed signal y(t) is given by

$$y(t) = \mathbf{w}^{\mathrm{T}} \mathbf{x}(t) + z(t) \tag{1}$$

where $\mathbf{x}(t) = [x(t), x(t-1), \dots, x(t-N+1)]^{T}$ denotes the vector of input signal x(t), and z(t) is the observation noise assumed to be independent with x(t). The goal of LMF-type filters is to sequentially estimate the unknown coefficient vector using the input signal x(t)and the desired output y(t). Let $\mathbf{w}(n)$ be the estimated coefficient vector of the adaptive filter at iteration *n*. The instantaneous error is defined as $e(n) = y(n) - \mathbf{w}^{T}(n)\mathbf{x}(n)$.

In the standard LMS [9], its cost function $L_{lms}(n)$ is defined as

$$L_{\rm s}(n) = \frac{1}{2}e^2(n)$$
 (2)

Then the corresponding updating equation of LMS can be written as

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu_{s} \frac{\partial L_{\text{lms}}(n)}{\partial \mathbf{w}(n)} = \mathbf{w}(n) + \mu_{s} e(n) \mathbf{x}(n)$$
(3)

where μ_s is the update step-size constant that controls stability and rate of convergence of two algorithms. In the standard LMF, the cost function L_{lms} is defined as

$$L_{\rm lmf}(n) = \frac{1}{4}e^4(n)$$
 (4)



Figure 2. LMS/F filter-based adaptive identification system.

The filter coefficient vector is then updated by

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu_{\text{lmf}} \frac{\partial L_{\text{lmf}}(n)}{\partial \mathbf{w}(n)} = \mathbf{w}(n) + \mu_{\text{lmf}} e^3(n) \mathbf{x}(n)$$
(5)

where μ_{lmf} is the step size that controls stability and rate of convergence of the LMF algorithm.

In the standard LMS/F algorithm, the cost functions $L_{\text{lmsf}}(n)$ is constructed as follows:

$$L_{\rm lmsf}(n) = \frac{1}{2} \varepsilon \ln \left(e^2(n) + \varepsilon \right) - \frac{1}{2} e^2(n)$$
 (6)

where ε is a positive parameter that control convergence speed and steady-state performance. Then the corresponding updating equation of LMS/F can be given as

$$\mathbf{w}_{\text{Imsf}}(n+1) = \mathbf{w}_{\text{Imsf}}(n) - \mu_{\text{f}} \frac{\partial L_{\text{Imsf}}(n)}{\partial \mathbf{w}_{\text{Imsf}}(n)}$$
$$= \mathbf{w}_{\text{Imsf}}(n) + \mu_{\text{f}} \frac{e^{3}(n)}{e^{2}(n) + \varepsilon} \mathbf{x}(n),$$
$$= \mathbf{w}_{\text{Imsf}}(n) + \underbrace{\frac{\mu_{\text{f}}}{1 + \varepsilon/e^{2}(n)}}_{\mu_{\text{f}}(n)} e(n) \mathbf{x}(n)$$
(7)

when $\varepsilon \gg e^2(n)$, LMS/F algorithm in Equation (7) behaves like the LMF with a step size of $\mu_{\text{lmsf}}/\varepsilon$; $\varepsilon \ll e^2(n)$, LMS/F algorithm in Equation (7) reduces to the standard LMS algorithm with a step size of μ_{lmsf} . Based on preceding discussion, the range of $\mu_f(n)$ is $(0, \mu_f)$. Hence, this gives the combined benefits of a large step-size LMS for fast convergence and small step-size LMF for steady-state performance.

3. SPARSE LMS/F ALGORITHMS

Adaptive system identification applies standard LMS/F algorithm, which combined both advantages of LMS and LMF. However, for an unknown sparse system, LMS/F may neglect the sparse structure information, which can be considered as prior information to improve identification performance. In this paper, we propose two sparse LMS/F algorithms for adaptive sparse system identification. Like the standard LMS/F algorithm, the common behavior of the two sparse LMS/F algorithms also applies fourth-order power optimization criterion. Hence, sparse LMS/F algorithms for adaptive system identification have two merits: (i) can mitigate noise interference effectively by using higher-order power filter and (ii) can exploit system sparsity by applying sparse penalty.

3.1. LP-LMS/F algorithm

By introducing l_p -norm sparse penalty to LMF/S-based adaptive sparse system identification, its cost function is given by

$$L_{\rm lp}(n) = \frac{1}{2} \varepsilon \ln \left(e^2(n) + \varepsilon \right) - \frac{1}{2} e^2(n) + \lambda_{\rm lp} \|w(n)\|_p$$
(8)

where $\lambda_{lp} > 0$ is a regularization parameter that balances the identification error and system sparsity; parameter ε > is a threshold that controls the convergence speed and identification error for adaptive updating. Here, please note that ε plays the same role as standard LMS/F algorithm in Equation (6). For easy understanding of the sparse constraint function in (8), geometrical interpretation is shown in Figure 3. By using l_p -norm sparse constraint function, one can obtain unique sparse solution in the solution plane. It is easy to deduce that adaptive sparse system identification using LP-LMS/F algorithm can also be achieved by constructing the cost function in (8). Hence, the corresponding update equation of LP-LMS/F is derived as

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu_{\rm f} \frac{\partial L_{\rm lp}(n)}{\partial \mathbf{w}(n)}$$

$$= \mathbf{w}(n) + \frac{\varepsilon \mu_{\rm f} e(n) \mathbf{x}(n)}{e^2(n) + \varepsilon} - \mu_{\rm f} e(n) \mathbf{x}(n)$$

$$- \rho_{\rm lp} \frac{\|\mathbf{w}(n)\|_p^{1-p} \operatorname{sgn} \{\mathbf{w}(n)\}}{\varepsilon_{\rm lp} + |\mathbf{w}(n)|^{1-p}} \qquad (9)$$

$$= \mathbf{w}(n) + \mu_{\rm f} \frac{e^3(n) \mathbf{x}(n)}{e^2(n) + \varepsilon}$$

$$- \rho_{\rm lp} \frac{\|\mathbf{w}(n)\|_p^{1-p} \operatorname{sgn} \{\mathbf{w}(n)\}}{\varepsilon_{\rm lp} + |\mathbf{w}(n)|^{1-p}}$$

where $\rho_{lp} = \mu_f \lambda_{lp}$ and $\varepsilon_{LP} > 0$. If we define the sparse penalty function of **w** (*n*) as

$$G_{\rm lp}(\mathbf{w}(n)) = \frac{\|\mathbf{w}(n)\|_p^{1-p} \operatorname{sgn} \{\mathbf{w}(n)\}}{\varepsilon_{\rm lp} + |\mathbf{w}(n)|^{1-p}}$$
(10)

then a geometrical figure can also be depicted as in Figure 4. To exploit the sparsity, indeed, neglect sparse penalty on dominant coefficients. It was well known that l_p -norm sparse constraint function is nonconvex and cannot exploit the sparsity efficiently. For example, $G_{lp}(\mathbf{w}(n))$ attracts all filter coefficients uniformly as zero in high probability as shown in Figure 4.

3.2. LO-LMS/F algorithm

Consider l_0 -norm penalty on LMS/F cost function to produce sparse solution because this penalty term forces the small nonzero filter coefficients of w(n) to approach zero. The cost function of L0-LMS/F is given by

$$L_{l0}(n) = \frac{1}{2}\varepsilon \ln\left(e^2\left(n\right) + \varepsilon\right) - \frac{1}{2}e^2(n) + \lambda_{l0} \|\mathbf{w}(n)\|_0$$
(11)

where λ_{l0} is a positive regularization parameter that trades off the identification error and system sparsity. For the geometrical perspective, the l_0 -norm sparse constraint function in (10) is depicted as geometrical Figure 5. Unlike (8), cost function $L_{l0}(n)$ using l_0 -norm sparse constraint function can achieve optimal sparse solution. As solving



Figure 3. Sparse solution is obtained using *l*_p-norm sparse constraint.



Figure 4. Sparse penalty function $G_{lp}(w(n))$.

the l_0 -norm minimization is a non-polynomial hard problem, we replace it with an approximate continuous function [28] as $\|\mathbf{w}\|_0 \approx \sum_{i=0}^{N-1} (1 - e^{-\beta |w_i|})$. According to the approximate function, L0-LMS/F cost function can be rewritten as

$$L_{l0}(n) = \frac{1}{2} \varepsilon \ln(e^2(n) + \varepsilon) - \frac{1}{2} e^2(n) + \lambda_{l0} \sum_{i=1}^{N} (1 - e^{-\beta |w_i|})$$
(12)

Then the update equation of L0-LMS/F-based adaptive sparse system identification is given by

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_{\rm f} \frac{e^3(n)\mathbf{x}(n)}{e^2(n) + \varepsilon} - \rho_{l0}\beta \operatorname{sgn}\{\mathbf{w}(n)\}e^{-\beta|w_i|}$$
(13)



Figure 5. Sparse solution is obtained using I0-norm sparse constraint.

where $\rho_{l0} = \mu_f \lambda_{l0}$. It is worth mentioning that the exponential function in Equation (13) will cause high computational complexity. To reduce the computational complexity, the first-order Taylor series expansion of exponential function is taken into consideration as

$$e^{-\beta |w_i(n)|} \approx \begin{cases} 1-\beta |w_i(n)|, \text{ when } |w_i(n)| \le \frac{1}{\beta} \\ 0, \text{ others} \end{cases}$$
 (1)

where $e^{-\beta |\mathbf{w}(n)|} = \left[e^{-\beta |w_0(n)|}, \dots, e^{-\beta |w_i(n)|}, \dots, e^{-\beta |w_N-1(n)|}\right]^{\mathrm{T}}$. It was worth noting that the positive parameter β controls the system sparseness and identification performance. Although the L0-LMS/F can exploit system sparsity on adaptive system identification, unsuitable parameter β will cause performance degradation, because if we choose bigger parameter β , it cannot exploit sparsity effectively; on the contrary, by choosing smaller parameter β , it will attract some active FIR coefficients as zero. The parameter of L0-LMS is suggested as $\beta = 10$ in [22]. In this paper, we set the parameter of L0-LMS/F as $\beta = 10$. We can also find that L0-LMS/F algorithm using $\beta = 10$ is very flexible in different SNRs in simulation results. According to preceding analysis, the modified update equation of L0-LMS/F can be rewritten as

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_{\rm f} \frac{e^3(n)\mathbf{x}(n)}{e^2(n) + \varepsilon} - \rho_{l0}G_{l0}\{\mathbf{w}(n)\}$$
(15)



Figure 6. Sparse penalty function $G_{l0}(w(n))$.

where $\rho_{l0} = \mu_f \lambda_{l0}$ and l_0 -norm approximation sparse penalty function $G_{l0} \{ \mathbf{w}(n) \}$ is defined as

$$G_{I0}\{w_i(n)\} = \begin{cases} 2\beta^2 w_i(n) - 2\beta \operatorname{sgn}\{w_i(n)\}, \text{ when } |w_i(n)| \le \frac{1}{\beta} \\ 0, & \text{others} \end{cases}$$
(16)

where $G_{l0} \{\mathbf{w}(n)\} = [G_{l0} \{w_0(n)\}, \dots, G_{l0} \{w_i(n)\}, \dots, G_{l0} \{w_{N-1}(n)\}]^{\mathrm{T}}$. Let us take $\beta = 10$ for example. The sparse penalty function $G_{l0} \{w_i(n)\}$ is depicted in Figure 6. As the figure shows, $G_{l0} (\mathbf{w}(n))$ replaces small filter coefficients (smaller than $1/\beta$) by zeros in high probability while neglecting sparse penalty on dominant coefficients (larger than $1/\beta$).

4. EXPERIMENTAL RESULTS

In this section, all of the filters are 1000 independent Monte Carlo runs for averaging. Performance comparisons between sparse LMS algorithms and sparse LMS/F algorithms are evaluated by MSD, which is defined as

$$\mathrm{MSD}\{\mathbf{w}(n)\} = E\left\{\|\mathbf{w} - \mathbf{w}(n)\|_{2}^{2}\right\}$$
(17)

where $E\{\cdot\}$ denotes expectation operator, and \mathbf{w} and $\mathbf{w}(n)$ denote actual FIR coefficient vector and its estimator, respectively. The FIR filter length is set as N = 16, and its number of nonzero coefficients is set as $K \in \{2, 4, 8\}$. The values of the nonzero FIR coefficients follow the Gaussian distribution, and the positions of coefficients are randomly allocated within the FIR filter \mathbf{w} , which is subjected to $E\{\|\mathbf{w}\|_2^2\} = 1$. The received SNR is defined as $SNR = 10 \log (E_0/\sigma_n^2)$, where $E_0 = 1$ is transmitted signal power. Then the noise power is given as $\sigma_n^2 = 10^{-SNR/10}$. All of the step sizes of gradient descent and

Input signal $\boldsymbol{x}(t)$	Gaussian distribution	CN(0,1)
	Normalized power	$E\left\{\ \boldsymbol{x}(t)\ ^{2}\right\} = E_{0} = 1$
Random additive noise	Gaussian distribution	$\mathcal{CN}(0,\sigma_n^2)$
FIR-based filter w	Filter length	<i>N</i> = 16
	Nonzero coefficient	$K \in \{2, 4, 8\}$
	Coefficients distribution	CN(0,1)
Sparse LMS algorithms	Step size	$\mu_{s} = 0.04$
	LP-LMS	$\lambda_{ m lp} = 0.002\sigma_n^2$ and $p = 0.5$
	L0-LMS	$\lambda_{I0}=0.02\sigma_n^2$ and $eta=10$
Sparse LMS/F algorithms	Step size	$\mu_{\rm f} = 0.04$
	LP-LMS/F	$\lambda_{ m lp} = 0.002\sigma_n^2$ and $p = 0.5$
	L0-LMS/F	$\lambda_{I0} = 0.02\sigma_n^2$ and $\beta = 10$





Figure 7. Performance evaluation (SNR = 5 dB and K = 2).

regularization parameters are listed in Table I. Two experiments have been designed to demonstrate their convergence speed and performance in different noise level, that is, $SNR \in \{5 \text{ dB}, 10 \text{ dB}\}$.

In the first experiment, as shown in Figures 7-10, are comparisons of MSD performance with different numbers of nonzero FIR coefficients K in two SNR regimes, that is, $SNR \in \{5 \text{ dB}, 10 \text{ dB}\}$. First of all, Figures 7–10 show that LMS/F-type algorithms achieved much better MSD performance than LMS-type algorithms in different nonzero filter coefficients, K. It is easy to deduce that the performance advantage of LMS/F-type algorithms is benefited from hybrid power optimization criterion. Furthermore, our proposed sparse LMS/F algorithms have the same stability as sparse LMS ones in two different SNR regimes. Hence, proposed sparse LMS/F algorithms combine performance advantage when comparing with sparse LMS algorithms [21,22] and stability when comparing with sparse LMF algorithms [13]. Additionally, let us take the K = 2 and K = 8 for example. When K = 2 in Figures 7 and 8, the performance gap of sparse LMS/F algorithms and standard LMS/F algorithm is bigger than that of the case of K = 8as shown in Figures 11 and 12, respectively. One can find



Figure 8. Performance evaluation (SNR = 10 dB and K = 2).



Figure 9. Performance evaluation (SNR = 5 dB and K = 4).

that the sparse LMS/F algorithms can achieve better performance for sparser FIR filter. This also coincided with sparse signal recovery theory in the framework of CS



Figure 10. Performance evaluation (SNR = 10 dB and K = 4).



Figure 11. Performance evaluation (SNR = 5 dB and K = 8).

[16,17]. At the same time, all performance curves of sparse LMS/F algorithms are lower than the performance curves of sparse LMS algorithms and the LMS/F one.

In the second experiment, as we can see from Figures 13 and 14, MSD performance of sparse LMS/F algorithms at different threshold parameters, for example, $\varepsilon \in \{0.4, 0.6, 0.8\}$ is evaluated in two SNR regimes. When the FIR filter works in a very low SNR regime (*SNR* = 5 dB), both LMS and sparse LMS/F algorithms yield faster convergence than the LMS/F. However, LMS/F can achieve better identification performance than LMS and LMS/F. In practical system identification, it is necessary to trade off the performance and convergence speed. In the high noise case, for example, *SNR* = 5 dB, we suggest that the parameter is chosen as $\varepsilon = 0.8$, because in LMS/F of up to 800 iterations, steady-state performance of LMS/F is much better than that of standard LMS. In the higher



Figure 12. Performance evaluation (SNR = 10 dB and K = 8).



Figure 13. Performance evaluation (SNR = 5 dB and K = 2).



Figure 14. Performance evaluation (SNR = 10 dB and K = 2).

noise level regime, for example, SNR = 10 dB as shown in Figure 14, if we set parameter $\varepsilon = 0.4$, LMS/F algorithm can keep higher convergence speed than $\varepsilon \in \{0.6, 0.8\}$ while its steady-state performance is better than that of LMS algorithm.

5. CONCLUSION AND FUTURE WORK

We have investigated adaptive sparse identification approaches using several classical standard algorithms. Sparse LMS algorithms, for example, LP-LMS and L0-LMS, do exploit sparsity on unknown sparse systems. However, their performance is easily degraded because they are sensitive to scaling of input signal. Motivated by the background that standard LMS/F algorithm can achieve better performance than LMS, cost function of LMS/F algorithm can be penalized by sparse constraints. In this paper, we proposed two sparse LMS/F algorithms using two sparse constraints to improve adaptive identification performance. Computer simulation results confirmed the effectiveness of the propose algorithms, which have achieved better MSD performance than sparse LMS algorithms. In future work, the proposed algorithms will be applied in sparse channel estimation for different practical systems, such as multi-input multi-output (MIMO) systems [29,30], cooperative MIMO systems [31], and MIMO two-way relay networks [32].

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