

The Application of MIMO to Non-Orthogonal Multiple Access

Zhiguo Ding, *Member, IEEE*, Fumiyuki Adachi, *Fellow, IEEE*, and H. Vincent Poor, *Fellow, IEEE*

Abstract—This paper considers the application of multiple-input multiple-output (MIMO) techniques to nonorthogonal multiple access (NOMA) systems. A new design of precoding and detection matrices for MIMO-NOMA is proposed and its performance is analyzed for the case with a fixed set of power allocation coefficients. To further improve the performance gap between MIMO-NOMA and conventional orthogonal multiple access schemes, user pairing is applied to NOMA and its impact on the system performance is characterized. More sophisticated choices of power allocation coefficients are also proposed to meet various quality-of-service requirements. Finally, computer simulation results are provided to facilitate the performance evaluation of MIMO-NOMA and also demonstrate the accuracy of the developed analytical results.

Index Terms—Cognitive radio, multiple-input multiple-output (MIMO), non-orthogonal multiple access (NOMA), precoding design.

I. INTRODUCTION

NON-ORTHOGONAL multiple access (NOMA) has recently received considerable attention as a promising enabling technique in fifth generation (5G) mobile networks because of its superior spectral efficiency [1] and [2]. The key idea of NOMA is to explore the power domain for realizing multiple access (MA) in mobile networks. Specifically NOMA users in one cell are served by a base station (BS) on the same time/code/frequency channel, and their signals are multiplexed by using different power allocation coefficients. The novelty of NOMA is to opportunistically allocate the transmit power to different users by exploiting the difference among the users' channel conditions, e.g., users with poorer channel conditions are allocated more transmission power. In this way, these users are able to decode their own messages by treating the others' information as noise, since the power level of their messages is higher. On the other hand, the users with better channel conditions will use the successive interference

cancellation (SIC) strategy, i.e., they first decode the messages of the users with poorer channel conditions and then decode their own by removing the other users' information.

The concept of NOMA can be linked to many well-known methods used in previous communication systems. For example, NOMA downlink transmissions resemble Cover and Thomas's description of broadcast channels provided in [3]. Another example is that the use of SIC has been extensively investigated in conventional multiple-input multiple-output networks, particularly in V-BLAST systems [4]. The superimposed messages transmitted in NOMA systems also resemble the concept of hierarchical modulation widely used for digital video broadcasting [5]. But unlike these existing techniques, NOMA seeks to strike a balance between throughput and fairness. For example, the transmission power allocated to the users in NOMA systems is inversely proportional to their channel conditions, which is important to ensure that all the users are served simultaneously. On the other hand, conventional opportunistic schemes prefer to give more power to users with better channel conditions, which can improve the overall system throughput but deteriorate fairness.

The impact of path loss on the performance of NOMA has been characterized in [6] by assuming that users are randomly deployed in a cell, which has demonstrated that NOMA can outperform conventional orthogonal multiple access (OMA) schemes. In [7] the implementation of NOMA has been considered in a scenario with two base stations, and the design of uplink NOMA has been proposed in [8]. The user fairness of NOMA has been considered in [9] by studying the impact of different choices of power allocation coefficients. In [10] a cognitive radio inspired NOMA scheme was proposed, in which the power allocation coefficients are chosen to meet the predefined users' quality of service (QoS) requirements.

In this paper, we focus on the application of multiple-input multiple-output (MIMO) to NOMA downlink communication systems. The concept of MIMO-NOMA has been validated by using systematic implementation in [11]–[13], which demonstrates that the use of MIMO can outperform conventional MIMO-OMA. For example, MIMO-NOMA with random beamforming was proposed in [13], where a base station with multiple antennas will randomly form some transmit beams, and the users falling in these beams will be served in an opportunistic manner. Compared to these existing works, the contributions of this paper are as follows:

- We first consider a general NOMA downlink scenario, in which all users participate in NOMA with a fixed set of power allocation coefficients. A new design of precoding

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Z. Ding is with the Department of Electrical Engineering, Princeton University, Princeton, NJ 08544 USA, and also with the School of Computing and Communications, Lancaster University, Lancaster LA1 4WA, U.K. (e-mail: z.ding@lancaster.ac.uk).

F. Adachi is with the Graduate School of Engineering, Tohoku University, Sendai 980-8579, Japan (e-mail: adachi@ecei.tohoku.ac.jp).

H. V. Poor is with the Department of Electrical Engineering, Princeton University, Princeton, NJ 08544 USA (e-mail: poor@princeton.edu).

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and detection matrices is proposed, and the impact of this design on the performance of NOMA is characterized by using the criteria of outage probabilities and diversity orders. The provided analytical and numerical results demonstrate that MIMO-NOMA can achieve better outage performance than conventional MIMO-OMA, even for users that suffer strong co-channel interference.

- To enlarge the performance gap between MIMO-NOMA and MIMO-OMA, user pairing is applied to NOMA. Analytical results, such as an exact expression for the average sum-rate gap between MIMO-NOMA and MIMO-OMA and its high SNR approximation, are developed. These analytical results demonstrate that the design of user pairing in NOMA systems is very different from conventional user scheduling scenarios. Conventionally it is preferable to schedule the users whose channel conditions are superior, but in the context of NOMA, it is important to schedule users whose channel conditions are very distinct. This is consistent with the findings obtained for single-antenna NOMA cases in [1] and [10].
- Inspired by the concept of cognitive radio networks, more sophisticated choices for the power allocation coefficients are proposed. Particularly we consider two types of constraints for the power allocation coefficients. One is to meet a predefined QoS requirement, i.e., a user's rate supported by NOMA is larger than a targeted data rate. The other is to meet a more dynamic QoS requirement, where the user's rate supported by NOMA needs to be larger than that supported by conventional OMA. Analytical results are developed for both scenarios to facilitate performance evaluation.

II. SYSTEM MODEL WITH FIXED POWER ALLOCATION

Consider a downlink communication scenario, where a BS equipped with M antennas communicates with multiple users equipped with N antennas each. To make the NOMA principle applicable to this scenario, the users are randomly grouped into M clusters with K users in each cluster. In this paper $N \geq M$ is assumed, which can be justified as follows. It is envisioned that small cells will be ultra-densely deployed in 5G networks, in which low-cost and low-power small-cell base stations will be used [14]. Therefore it is very likely that such a low-cost small-cell base station has the same number of antennas as user handsets, or even less, particularly given the rapidly growing capabilities of smart phones and tablets. Cloud radio access networks (C-RANs) are another example of a 5G application of the proposed scheme, in which users are served by a small number of low cost remote radio heads (RRHs) in order to reduce the fronthaul overhead. The proposed scheme can also be applicable to aspects of Internet of Things, such as smart homes, in which the capability of a home base station is similar to that of laptops and other digital devices¹. The signals transmitted by

¹In the case that the base station has more antennas than the users, different approaches need to be used to implement MIMO-NOMA. For example, one possible way is to allocate different beamforming vector to different users individually, and the precoding matrices at the base station can be optimized by taking user fairness into consideration [15].

the BS are given by

$$\mathbf{x} = \mathbf{P}\tilde{\mathbf{s}}, \quad (1)$$

where the $M \times 1$ vector $\tilde{\mathbf{s}}$ is given by

$$\tilde{\mathbf{s}} = \begin{bmatrix} \alpha_{1,1}s_{1,1} + \cdots + \alpha_{1,K}s_{1,K} \\ \vdots \\ \alpha_{M,1}s_{M,1} + \cdots + \alpha_{M,K}s_{M,K} \end{bmatrix} \triangleq \begin{bmatrix} \tilde{s}_1 \\ \vdots \\ \tilde{s}_M \end{bmatrix} \quad (2)$$

where $s_{m,k}$ denotes the information bearing signal to be transmitted to the k -th user in the m -th cluster, $\alpha_{i,j}$ denotes the NOMA power allocation coefficient, and the design of the $M \times M$ precoding matrix \mathbf{P} will be discussed in the next section.

Without loss of generality, we focus on the users in the first cluster. The observation at the k -th user in the first cluster is given by

$$\mathbf{y}_{1,k} = \mathbf{H}_{1,k}\mathbf{P}\tilde{\mathbf{s}} + \mathbf{n}_{1,k}, \quad (3)$$

where $\mathbf{H}_{1,k}$ is the $N \times M$ Rayleigh fading channel matrix from the BS to the k -th user in the first cluster, and $\mathbf{n}_{1,k}$ is an additive Gaussian noise vector. Denote by $\mathbf{v}_{1,k}$ the detection vector used by this user. After applying this detection vector, the signal model can be rewritten as follows:

$$\mathbf{v}_{1,k}^H \mathbf{y}_{1,k} = \mathbf{v}_{1,k}^H \mathbf{H}_{1,k} \mathbf{P} \tilde{\mathbf{s}} + \mathbf{v}_{1,k}^H \mathbf{n}_{1,k}. \quad (4)$$

Denote the i -th column of \mathbf{P} by \mathbf{p}_i . The above signal model can be rewritten as follows:

$$\begin{aligned} \mathbf{v}_{1,k}^H \mathbf{y}_{1,k} &= \mathbf{v}_{1,k}^H \mathbf{H}_{1,k} \mathbf{p}_1 (\alpha_{1,1}s_{1,1} + \cdots + \alpha_{1,K}s_{1,K}) \\ &+ \sum_{m=2}^M \mathbf{v}_{1,k}^H \mathbf{H}_{1,k} \mathbf{p}_m \tilde{s}_m + \mathbf{v}_{1,k}^H \mathbf{n}_{1,k}. \end{aligned} \quad (5)$$

The channel conditions are crucial to the implementation of NOMA. Without loss of generality, we assume that the effective channel gains are ordered as follows:

$$|\mathbf{v}_{1,1}^H \mathbf{H}_{1,1} \mathbf{p}_1|^2 \geq \cdots \geq |\mathbf{v}_{1,K}^H \mathbf{H}_{1,K} \mathbf{p}_1|^2, \quad (6)$$

and following the principle of NOMA, the users' power allocation coefficients are ordered as follows:

$$\alpha_{1,1} \leq \cdots \leq \alpha_{1,K}.$$

In this section, constant power allocation coefficients will be considered, and more sophisticated choices will be used in Section V. It is worth pointing out that optimizing power allocation according to instantaneous channel conditions can be used to further improve the performance of MIMO-NOMA, which is beyond the scope of this paper.

Based on the above signal model, the signal-to-interference-plus-noise ratio (SINR) for the K -th ordered user in the first cluster is given by

$$\begin{aligned} \text{SINR}_{1,K} &= \frac{|\mathbf{v}_{1,K}^H \mathbf{H}_{1,K} \mathbf{p}_1|^2 \alpha_{1,K}^2}{\sum_{l=1}^{K-1} |\mathbf{v}_{1,K}^H \mathbf{H}_{1,K} \mathbf{p}_l|^2 \alpha_{1,l}^2 + \sum_{m=2}^M |\mathbf{v}_{1,K}^H \mathbf{H}_{1,K} \mathbf{p}_m|^2 + |\mathbf{v}_{1,K}^H \mathbf{n}_{1,K}|^2 \frac{1}{\rho}}, \end{aligned} \quad (7)$$

where ρ denotes the transmit signal to noise ratio (SNR).

The k -th user, $1 < k < K$, needs to decode the messages to the users with poorer channel conditions first², before detecting its own. The messages $s_{1,j}$, $K \geq j \geq (k+1)$, will be detected at the k -th user with the following SINR:

$$\begin{aligned} \text{SINR}_{1,k}^j &= \frac{|\mathbf{v}_{1,k}^H \mathbf{H}_{1,k} \mathbf{p}_1|^2 \alpha_{1,j}^2}{\sum_{l=1}^{j-1} |\mathbf{v}_{1,k}^H \mathbf{H}_{1,k} \mathbf{p}_l|^2 \alpha_{1,l}^2 + \sum_{m=2}^M |\mathbf{v}_{1,k}^H \mathbf{H}_{1,k} \mathbf{p}_m|^2 + |\mathbf{v}_{1,k}|^2 \frac{1}{\rho}}. \end{aligned} \quad (8)$$

If the message $s_{1,j}$ can be decoded successfully, i.e., $\log(1 + \text{SINR}_{1,k}^j) > R_{1,j}$, then it will be removed from the k -th user's observation, where $R_{1,j}$ denotes the j -th user's targeted data rate. This SIC will be carried out until the k -th user's own message is decoded with the SINR, $\text{SINR}_{1,k}^k$.

The first user in the first cluster needs to decode all the other users' messages with $\text{SINR}_{1,1}^j$, $K \geq j \geq 2$. If successful, it will decode its own message with the following SINR:

$$\text{SINR}_{1,1}^1 = \frac{|\mathbf{v}_{1,1}^H \mathbf{H}_{1,1} \mathbf{p}_1|^2 \alpha_{1,1}^2}{\sum_{m=2}^M |\mathbf{v}_{1,1}^H \mathbf{H}_{1,1} \mathbf{p}_m|^2 + |\mathbf{v}_{1,1}|^2 \frac{1}{\rho}}. \quad (9)$$

The design of the precoding and detection matrices will be discussed in the following section.

III. DESIGN OF PRECODING AND DETECTION MATRICES

To completely remove inter-cluster interference, the precoding and detection matrices need to satisfy the following constraints:

$$\mathbf{v}_{i,k}^H \mathbf{H}_{i,k} \mathbf{p}_m = 0, \quad (10)$$

for any $m \neq i$.

In order to reduce system overhead caused by acquiring channel state information (CSI) at the BS, it is assumed that the BS does not have the global CSI³, which leads to the following choice of \mathbf{P} :

$$\mathbf{P} = \mathbf{I}_M,$$

where \mathbf{I}_M is the $M \times M$ identity matrix. The above choice means that the BS broadcasts the users' messages without manipulating them. The advantage of this choice is that it avoids asking the users to feedback all their CSI to the BS, which consumes significant system overhead.

With this choice of \mathbf{P} , the constraints on the detection matrices in (10) become

$$\mathbf{v}_{i,k}^H \mathbf{h}_{m,ik} = 0, \quad (11)$$

²With a careful design of precoding and detection matrices, inter-cluster interference can be canceled completely, as can be seen in the next section. Therefore the MIMO-NOMA system can be reduced into simple single-input single-output (SISO) NOMA channels, in which the use of SIC can yield the optimal performance, the same as for non-linear approaches, such as non-linear SIC and dirty paper coding [1], [3], [16] and [17].

³It is worth pointing out that the BS still needs to know the order of the users' effective channel gains in order to implement NOMA as shown in (6). Since each user needs to feed back only a scalar effective channel gain, this imposes a much less demanding requirement compared to the case of knowing all the users' channel matrices at the BS.

where $\mathbf{h}_{m,ik}$ is the m -th column of $\mathbf{H}_{i,k}$. Therefore at the k -th user in the i -th cluster, the constraints can be rewritten as follows:

$$\mathbf{v}_{i,k}^H \underbrace{[\mathbf{h}_{1,ik} \cdots \mathbf{h}_{i-1,ik} \mathbf{h}_{i+1,ik} \cdots \mathbf{h}_{M,ik}]}_{\tilde{\mathbf{H}}_{i,k}} = 0. \quad (12)$$

Note that the dimension of $\tilde{\mathbf{H}}_{i,k}$ is $N \times (M-1)$ since it is a submatrix of $\mathbf{H}_{i,k}$ formed by removing one column. As a result, $\mathbf{v}_{i,k}$ can be obtained from the null space of $\tilde{\mathbf{H}}_{i,k}$, i.e.,

$$\mathbf{v}_{i,k} = \mathbf{U}_{i,k} \mathbf{z}_{i,k}, \quad (13)$$

where $\mathbf{U}_{i,k}$ contains all the left singular vectors of $\tilde{\mathbf{H}}_{i,k}$ corresponding to zero singular values, and $\mathbf{z}_{i,k}$ is a $(N-M+1) \times 1$ normalized vector to be optimized later. In order to ensure the existence of $\mathbf{v}_{i,k}$, $N \geq M$ is assumed. It is worth pointing out that the proposed MIMO-NOMA scheme also works for the case in which users have different numbers of antennas, as long as the number of users' antennas is larger than or equal to that of the base station, in order to ensure the existence of a solution in (12).

By using the above precoding and detection matrices, the SINR for the K -th user in the first cluster is given by

$$\text{SINR}_{1,K} = \frac{|\mathbf{v}_{1,K}^H \mathbf{h}_{1,1K}|^2 \alpha_{1,K}^2}{\sum_{l=1}^{K-1} |\mathbf{v}_{1,K}^H \mathbf{h}_{1,1K}|^2 \alpha_{1,l}^2 + |\mathbf{v}_{1,K}|^2 \frac{1}{\rho}}, \quad (14)$$

where inter-cluster interference has been removed.

At the k -th user, $1 < k < K$, the messages $s_{1,j}$, $K \geq j \geq (k+1)$, will be detected with the following SINR:

$$\text{SINR}_{1,k}^j = \frac{|\mathbf{v}_{1,k}^H \mathbf{h}_{1,1k}|^2 \alpha_{1,j}^2}{\sum_{l=1}^{j-1} |\mathbf{v}_{1,k}^H \mathbf{h}_{1,1k}|^2 \alpha_{1,l}^2 + |\mathbf{v}_{1,k}|^2 \frac{1}{\rho}}. \quad (15)$$

If successful, $s_{j,1}$ will be removed from the k -th user's observation, and SIC will be carried out until its own message is decoded with the SINR, $\text{SINR}_{1,k}^k$.

The first user in the first cluster will decode the other users' messages with $\text{SINR}_{1,1}^j$, $K \geq j \geq 2$. If successful, it will decode its own message with the following SINR:

$$\text{SINR}_{1,1}^1 = \rho \frac{|\mathbf{v}_{1,1}^H \mathbf{h}_{1,11}|^2 \alpha_{1,1}^2}{|\mathbf{v}_{1,1}|^2}. \quad (16)$$

As can be observed from the above SINR expressions, $\mathbf{z}_{i,k}$ determines the SINRs through $|\mathbf{v}_{i,k}^H \mathbf{h}_{i,ik}|^2$. Therefore, one possible choice of $\mathbf{z}_{i,k}$ can be obtained by using maximal ratio combining (MRC). Particularly, the choice of $\mathbf{z}_{i,k}$ based on MRC is given by

$$\mathbf{z}_{i,k} = \frac{\mathbf{U}_{i,k}^H \mathbf{h}_{i,ik}}{|\mathbf{U}_{i,k}^H \mathbf{h}_{i,ik}|}. \quad (17)$$

In this paper, the outage probability is used as the criterion for performance evaluation since the probability of detection error can be tightly bounded by the outage probability [18]. The benefit of using the outage probability is that the fundamental limits

of a communication system can be evaluated without using specific channel coding or modulation. (The design of channel coding and modulation for MIMO-NOMA is beyond the scope of this paper). The following theorem provides an exact expression for the outage probability achieved by MIMO-NOMA and its high SNR approximation.

Theorem 1: Assume that the users in each cluster are ordered as in (6). With MIMO-NOMA, the outage probability experienced by the k -th ordered user in the i -th cluster is given by

$$P_{i,k}^o = \sum_{p=0}^{k-1} \binom{k-1}{p} \frac{(-1)^p K! \left[\frac{\gamma(N-M+1, \epsilon_{i,k}^*)}{(N-M)!} \right]^{K-k+p+1}}{(K-k)!(k-1)!(K-k+p+1)}, \quad (18)$$

if $\alpha_{i,j}^2 > \beta_{i,j}$, for all $k \leq j \leq K$, otherwise $P_{i,k}^o = 1$, where $\epsilon_{i,k} = 2^{R_{i,k}} - 1$, $\beta_{i,k} = \epsilon_{i,k} \sum_{k=1}^{K-1} \alpha_{i,k}^2$, $\gamma(\cdot)$ denotes the incomplete gamma function, $\epsilon_{i,k}^* = \max\left\{\frac{\epsilon_{i,K}}{\rho(\alpha_{i,K}^2 - \beta_{i,K})}, \dots, \frac{\epsilon_{i,k}}{\rho(\alpha_{i,k}^2 - \beta_{i,k})}\right\}$, for $2 \leq k \leq K$ and $\epsilon_{i,1}^* = \max\left\{\frac{\epsilon_{i,K}}{\rho(\alpha_{i,K}^2 - \beta_{i,K})}, \dots, \frac{\epsilon_{i,2}}{\rho(\alpha_{i,2}^2 - \beta_{i,2})}, \frac{\epsilon_{i,1}}{\rho\alpha_{i,1}^2}\right\}$. A high SNR approximation for the outage probability is given by

$$P_{i,k}^o \approx \frac{K! \left[\frac{(\epsilon_{i,k}^*)^{N-M+1}}{(N-M+1)!} \right]^{K-k+1}}{(K-k)!(k-1)!(K-k+1)}. \quad (19)$$

Proof: Please refer to the appendix. ■

Regarding the achievable multiplexing gain, consider the use of a fixed set of power allocation coefficients, and assume the users are ordered as in (6). Recall that the achievable rate for the k -th user in the i -th cluster is $\log(1 + SINR_{i,k})$ for $1 \leq k \leq K$, since $\log(1 + SINR_{i,q}^k) > \log(1 + SINR_{i,p}^k)$ for $1 \leq q < p \leq k$, where we use the following notation: $SINR_{i,k}^k \triangleq SINR_{i,k}$. It is straightforward to show that $\lim_{\rho \rightarrow \infty} \frac{\log(1 + SINR_{i,k})}{\log \rho} = 0$, for $2 \leq k \leq K$, and $\lim_{\rho \rightarrow \infty} \frac{\log(1 + SINR_{i,1})}{\log \rho} = 1$, since only the first user, i.e., the one with the strongest CSI, can remove all the interference completely. Therefore in each cluster, the maximal multiplexing gain for the best user is one, and a multiplexing gain of zero is achieved for the remaining users.

The diversity-multiplexing tradeoff (DMT) for MIMO-NOMA is not trivial to obtain, since NOMA is a multi-user system by nature. Therefore the achievable rate region of NOMA needs to be used for the analysis of the DMT, where the relationship between the rate region of NOMA and the capacity region of broadcast channels has been illustrated in [17]. It is worth pointing out that the analysis of the DMT for multiple access channels is already very difficult as can be seen in [19] and [20], and therefore finding the DMT of NOMA will be more challenging since broadcast channels are more difficult to analyze in this context than multiple access channels.

A. Conventional MIMO-OMA

A benchmarking scheme based on conventional MIMO-OMA can be described as follows. The MIMO-OMA

transmission consists of K time slots. During each time slots, M users, one from each cluster, are served simultaneously based on the same manner as described for MIMO-NOMA. Without loss of generality, we focus on the first time slot, in which the first user of each cluster is selected. Again assume that the users in each cluster are ordered as in (6). As a result, during the first time slot, the 1-st user in the 1-st cluster will observe the following

$$\mathbf{y}_{1,1} = \mathbf{H}_{1,1} \mathbf{P} \bar{\mathbf{s}} + \mathbf{n}_{1,1}, \quad (20)$$

where $\bar{\mathbf{s}} = [s_{1,1} \dots s_{M,1}]^T$. Note that $\bar{\mathbf{s}}$ only contains M messages, instead of MK messages as in (2). Again denote by $\mathbf{v}_{1,k}$ the detection vector used by this user, and the SINR at the k -th user in the i -th cluster is given by

$$SINR_{i,k} = \frac{|\mathbf{v}_{i,k}^H \mathbf{H}_{i,k} \mathbf{p}_i|^2}{\sum_{m=1, m \neq i}^M |\mathbf{v}_{i,k}^H \mathbf{H}_{i,k} \mathbf{p}_m|^2 + |\mathbf{v}_{i,k}|^2 \frac{1}{\rho}}. \quad (21)$$

Note that the MRC detection vector used for MIMO-NOMA is also applicable to MIMO-OMA, which means that a zero-forcing type of detection is used. The outage probability achieved by this version of MIMO-OMA can be obtained in the following corollary straightforwardly by following the steps in the proof for Theorem 1.

Corollary 1: Assume that the users in each cluster are ordered as in (6). By applying conventional MIMO-OMA, the outage probability experienced by the k -th ordered user in the i -th cluster is given by

$$P_{i,k}^o = \sum_{p=0}^{k-1} \binom{k-1}{p} \frac{(-1)^p K! \left[\frac{\gamma(N-M+1, \phi_{i,k})}{(N-M)!} \right]^{K-k+p+1}}{(K-k)!(k-1)!(K-k+p+1)}, \quad (22)$$

where $\phi_{i,k} = \frac{2^{KR_{i,k}} - 1}{\rho}$. A high SNR approximation is given by

$$P_{i,k}^o \approx \frac{K! \left[\frac{(\phi_{i,k})^{N-M+1}}{(N-M+1)!} \right]^{K-k+1}}{(K-k)!(k-1)!(K-k+1)}. \quad (23)$$

As can be observed from Theorem 1 and Corollary 1, MIMO-NOMA can achieve a diversity gain of $(N - M + 1)(K - k + 1)$, the same as conventional MIMO-OMA. But this diversity gain is achieved by allowing all the K users from the same cluster to share the same bandwidth resource, which yields better spectral efficiency. For example, the simulation results provided in Section VI demonstrate that MIMO-NOMA can achieve a smaller outage probability compared to conventional NOMA. The superior spectral efficiency of MIMO-NOMA can also be demonstrated by the fact that it can realize a larger sum rate, as shown in the following section when the impact of user pairing is investigated.

IV. THE IMPACT OF USER PAIRING

User pairing has the potential to reduce the complexity of NOMA systems. Specifically the users in one cluster can be

divided into groups with fewer users in each group. A hybrid multiple access scheme can be used, where NOMA will be implemented among the users within each group, and conventional OMA can be used for inter-group multiple access. In addition to reducing system complexity, user pairing/grouping can also significantly increase the performance gain of NOMA over conventional MIMO-OMA, as shown in this section. It is important to point out that the focus of this section is not to develop a new user pairing/scheduling protocol, but rather to investigate the impact of the user pairing on the performance gain of MIMO-NOMA over MIMO-OMA.

In order to obtain some insightful analytical results, we focus on the case in which two users are paired together for performing NOMA in each cluster. Particularly the n -th and k -th ordered users from each cluster are scheduled to perform NOMA, where the n -th user has a better channel condition, i.e., $n < k$. By using the same choices of the precoding and detection matrices, the SNR for the k -th user in the first cluster is given by

$$SNR_{1,k} = \frac{|\mathbf{v}_{1,k}^H \mathbf{h}_{1,1k}|^2 \alpha_{1,k}^2}{|\mathbf{v}_{1,k}^H \mathbf{h}_{1,1k}|^2 \alpha_{1,n}^2 + |\mathbf{v}_{1,k}|^2 \frac{1}{\rho}}, \quad (24)$$

and the SNR at the n -th user is given by

$$SNR_{1,n} = \rho \frac{|\mathbf{v}_{1,n}^H \mathbf{h}_{1,1n}|^2 \alpha_{1,n}^2}{|\mathbf{v}_{1,n}|^2}, \quad (25)$$

conditioned on the event that the n -th user can decode the other user's information correctly. Note that the power allocation coefficients satisfy $\alpha_{1,n}^2 + \alpha_{1,k}^2 = 1$.

We are particularly interested in the sum-rate gap between MIMO-NOMA and conventional MIMO, which is given by

$$\begin{aligned} \Delta &\triangleq \sum_{i=1}^M [\log(1 + SNR_{i,k}) + \log(1 + SNR_{i,n})] \\ &\quad - \frac{1}{2} \sum_{i=1}^M \left[\log\left(1 + \rho |\mathbf{v}_{i,k}^H \mathbf{h}_{i,ik}|^2\right) + \log\left(1 + |\mathbf{v}_{i,n}^H \mathbf{h}_{i,in}|^2\right) \right]. \end{aligned} \quad (26)$$

Following the same definitions used in the proof for Theorem 1, the average sum rate gap can be expressed as follows:

$$\begin{aligned} \mathcal{E}\{\Delta\} &= M \mathcal{E}\{\log(1 + SNR_{1,k}) + \log(1 + SNR_{1,n})\} \\ &\quad - \frac{M}{2} \mathcal{E}\left\{\log\left(1 + \rho |\mathbf{v}_{1,k}^H \mathbf{h}_{1,1k}|^2\right) + \log\left(1 + |\mathbf{v}_{1,n}^H \mathbf{h}_{1,1n}|^2\right)\right\} \\ &= M \mathcal{E}\left\{\log\left(1 + \frac{x_k \alpha_{1,k}^2}{x_k \alpha_{1,n}^2 + \frac{1}{\rho}}\right) + \log\left(1 + x_n \alpha_{1,n}^2 \rho\right)\right\} \\ &\quad - \frac{M}{2} \mathcal{E}\{\log(1 + \rho x_k) + \log(1 + \rho x_n)\}, \end{aligned}$$

where $x_k = |\mathbf{v}_{1,k}^H \mathbf{h}_{1,1k}|^2$ for notational simplicity.

After some manipulations, we can write

$$\begin{aligned} \mathcal{E}\{\Delta\} &= \frac{M}{2} \mathcal{E}\{\log(1 + \rho x_k)\} + M \mathcal{E}\left\{\log\left(1 + \rho x_n \alpha_{1,n}^2\right)\right\} \\ &\quad - M \mathcal{E}\left\{\log\left(1 + \rho x_k \alpha_{1,n}^2\right)\right\} - \frac{M}{2} \mathcal{E}\{\log(1 + \rho x_n)\}. \end{aligned} \quad (27)$$

The key for evaluating the rate gap $\mathcal{E}\{\Delta\}$ is to characterize $\mathcal{E}\{\log(1 + x_n \phi)\}$ which can be calculated as follows:

$$\begin{aligned} \mathcal{E}\{\log(1 + x_n \phi)\} &= - \int_0^\infty \log(1 + x \phi) d(1 - F_{x_n}(x)) \\ &= \frac{\phi}{\ln 2} \int_0^\infty \frac{1 - F_{x_n}(x)}{1 + x \phi} dx. \end{aligned} \quad (28)$$

By applying the cumulative distribution function (CDF) of the channel gain, x_n , provided in (45) in the proof for Theorem 1, the sum rate gap can be expressed as follows:

$$\begin{aligned} \mathcal{E}\{\log(1 + x_n \phi)\} &= \frac{\phi}{\ln 2} \int_0^\infty \frac{1 - \gamma_n \int_0^x f_{\bar{x}_k}(x) [F_{\bar{x}_k}(x)]^{K-n} [1 - F_{\bar{x}_k}(x)]^{n-1} dy}{1 + x \phi} dx \\ &= \frac{\phi}{\ln 2} \int_0^\infty \frac{1 - \sum_{p=0}^{n-1} \binom{n-1}{p} \gamma_n (-1)^p \frac{[F_{\bar{x}_k}(x)]^{K-n+p+1}}{K-n+p+1}}{1 + x \phi} dx, \end{aligned} \quad (29)$$

where $\gamma_n = \frac{K!}{(K-n)!(n-1)!}$ and the CDF $F_{\bar{x}_k}(x)$ is obtained following the density function in (44). By using the above equation and with some straightforward manipulations, the ergodic rate gap can be obtained in the following lemma.

Lemma 1: Suppose that the n -th and k -th users are grouped to perform MIMO-NOMA. The average sum rate gap between MIMO-NOMA and conventional MIMO-OMA is given by

$$\begin{aligned} \mathcal{E}\{\Delta\} &= \frac{M}{2} \varphi(k, \rho) + M \varphi(n, \rho \alpha_{1,n}^2) \\ &\quad - M \varphi(k, \rho \alpha_{1,n}^2) - \frac{M}{2} \varphi(n, \rho), \end{aligned} \quad (30)$$

where

$$\begin{aligned} \varphi(n, \phi) &= \frac{\phi}{\ln 2} \int_0^\infty \frac{1}{1 + x \phi} \left(1 - \sum_{p=0}^{n-1} \binom{n-1}{p} \gamma_n (-1)^p \right. \\ &\quad \left. \frac{\left[\frac{\gamma(N-M+1, x)}{(N-M)!} \right]^{K-n+p+1}}{K-n+p+1} \right) dx. \end{aligned} \quad (31)$$

While the analytical result in Lemma 1 can be used to replace Monte-Carlo simulations for performance evaluation, this is still quite complicated due to the integrals and special functions. In the following, some case studies will be carried out in order to obtain some insight into MIMO-NOMA.

Case studies for the sum-rate gain of MIMO-NOMA

In this subsection, we focus on two extreme cases as described in the following:

- Case I: In each cluster, pair the user having the worst channel condition with the one having the best channel condition, i.e., $n = 1$ and $k = K$.
- Case II: In each cluster, pair the user having the best channel condition with the one having the second best channel condition, i.e., $n = 1$ and $k = 2$.

In conventional MA systems, scheduling users with better channel conditions is beneficial for improving system throughput, but we can show that NOMA has a behavior different from conventional MA.

Lemma 2: For the case with $N = M = 2$, $n = 1$ and $k = K$, the average sum-rate gap between MIMO-NOMA and MIMO-OMA is given by

$$\begin{aligned} \mathcal{E} \{ \Delta \} &= -\log(e) e^{\frac{K}{\rho}} \mathbf{E}_i \left(-\frac{K}{\rho} \right) + 2 \log(e) e^{\frac{K}{\rho \alpha_{1,1}^2}} \mathbf{E}_i \left(-\frac{K}{\rho \alpha_{1,1}^2} \right) \\ &+ \frac{2}{\ln 2} \left(\sum_{l=1}^K \binom{K}{l} (-1)^l e^{\frac{l}{\rho \alpha_{1,1}^2}} \mathbf{E}_i \left(-\frac{l}{\rho \alpha_{1,1}^2} \right) \right) \\ &- \frac{1}{\ln 2} \left(\sum_{l=1}^K \binom{K}{l} (-1)^l e^{\frac{l}{\rho}} \mathbf{E}_i \left(-\frac{l}{\rho} \right) \right), \end{aligned} \quad (32)$$

where $\mathbf{E}_i(\cdot)$ denotes the exponential integral function. At high SNR, the gap can be approximated as follows:

$$\mathcal{E} \{ \Delta \} \approx \log K + \sum_{l=1}^K \binom{K}{l} (-1)^l \log l. \quad (33)$$

For the case with $N = M = 2$, $n = 1$ and $k = 2$, the average sum-rate gap between MIMO-NOMA and MIMO-OMA is given by (34). At high SNR, the average gap can be approximated as follows:

$$\begin{aligned} \mathcal{E} \{ \Delta \} &= \frac{1}{\ln 2} \left(K \sum_{p=1}^{K-1} \binom{K-1}{p} (-1)^p e^{\frac{p}{\rho}} \mathbf{E}_i \left(-\frac{p}{\rho} \right) \right. \\ &\quad \left. - (K-1) \sum_{l=1}^K \binom{K}{l} (-1)^l e^{\frac{l}{\rho}} \mathbf{E}_i \left(-\frac{l}{\rho} \right) \right) \\ &+ \frac{2}{\ln 2} \left(\sum_{l=1}^K \binom{K}{l} (-1)^l e^{\frac{l}{\rho \alpha_{1,1}^2}} \mathbf{E}_i \left(-\frac{l}{\rho \alpha_{1,1}^2} \right) \right) \\ &- \frac{1}{\ln 2} \left(\sum_{l=1}^K \binom{K}{l} (-1)^l e^{\frac{l}{\rho}} \mathbf{E}_i \left(-\frac{l}{\rho} \right) \right) \\ &- \frac{2}{\ln 2} \left(K \sum_{p=1}^{K-1} \binom{K-1}{p} (-1)^p e^{\frac{p}{\rho \alpha_{1,1}^2}} \mathbf{E}_i \left(-\frac{p}{\rho \alpha_{1,1}^2} \right) \right. \\ &\quad \left. - (K-1) \sum_{l=1}^K \binom{K}{l} (-1)^l e^{\frac{l}{\rho \alpha_{1,1}^2}} \mathbf{E}_i \left(-\frac{l}{\rho \alpha_{1,1}^2} \right) \right). \end{aligned} \quad (34)$$

$$\begin{aligned} \mathcal{E} \{ \Delta \} &\approx K \left(-\sum_{p=1}^{K-1} \binom{K-1}{p} (-1)^p \log p \right. \\ &\quad \left. + \sum_{l=1}^K \binom{K}{l} (-1)^l \log l \right). \end{aligned} \quad (35)$$

Proof: Please refer to the appendix. ■

Define $\varpi(k) = \sum_{l=1}^k \binom{k}{l} (-1)^l \log l$ which is a mono-increasing function of k . Lemma 1 shows that, at high SNR, the sum-rate gap for Case I can be approximated

as $(\log K + \varpi(K))$, which means that the larger K is, the more gain MIMO-NOMA can offer compared to conventional MIMO-OMA. On the other hand, numerical results show that the value of $k(\varpi(k) - \varpi(k-1))$ quickly goes to zero by increasing k , which means the sum-rate gain offered by MIMO-NOMA for Case II is diminishing with increasing K . These two extreme cases demonstrate that careful user pairing is critical for MIMO-NOMA to outperform conventional MIMO-OMA. Detailed numerical analysis will be provided in Section VI.

V. COGNITIVE RADIO INSPIRED MIMO-NOMA

In the previous sections, fixed choices of power allocation coefficients have been considered, and in this section, more sophisticated choices will be used. Without loss of generality, we focus on the same case as in Section IV, i.e., the n -th and k -th users from each cluster are selected to perform NOMA and the k -th user has poorer channel conditions, i.e., $n < k$.

An important observation is that there is a dilemma in NOMA systems for choosing $\alpha_{1,k}$. From the perspective of the overall system throughput, an ideal choice of $\alpha_{1,k}$ is $\alpha_{1,k} = 0$, i.e., all power is allocated to the user with better channel conditions. But this choice completely ignores the user fairness, and in this section we focus on two choices of $\alpha_{i,k}$ inspired by the concept of cognitive radio networks.

A. To meet a fixed QoS requirement

Consider that there is a targeted SINR threshold to ensure the QoS requirement at the k -th user, i.e., $SINR_{i,k} \geq \epsilon_{i,k}$. This SINR requirement⁴ imposes the following constraint on the power coefficient $\alpha_{i,k}^2$:

$$1 \geq \alpha_{i,k}^2 \geq \frac{\epsilon_{i,k} \left(|\mathbf{v}_{i,k}^H \mathbf{h}_{i,ik}|^2 + \frac{1}{\rho} \right)}{|\mathbf{v}_{i,k}^H \mathbf{h}_{i,ik}|^2 (1 + \epsilon_{i,k})}. \quad (36)$$

In this paper, we will simply set $\alpha_{i,k}$ as follows:

$$\alpha_{i,k}^2 = \min \left\{ 1, \frac{\epsilon_{i,k} \left(|\mathbf{v}_{i,k}^H \mathbf{h}_{i,ik}|^2 + \frac{1}{\rho} \right)}{|\mathbf{v}_{i,k}^H \mathbf{h}_{i,ik}|^2 (1 + \epsilon_{i,k})} \right\}. \quad (37)$$

This choice of $\alpha_{i,k}^2$ means that the BS will give the k -th user the minimal transmission power needed to meet this user's QoS requirement, and then allocate the remaining power to the n -th user.

The outage probability experienced at the k -th user is equal to $P(\alpha_{i,k} = 1)$ or equivalently $P\left(\frac{\epsilon_{i,k} \left(|\mathbf{v}_{i,k}^H \mathbf{h}_{i,ik}|^2 + \frac{1}{\rho} \right)}{|\mathbf{v}_{i,k}^H \mathbf{h}_{i,ik}|^2 (1 + \epsilon_{i,k})} > 1\right)$, i.e., the k -th user's targeted data rate cannot be supported even if the BS allocates all the power to this user. Following the proof of Theorem 1, it is straightforward to show that a diversity

⁴Note that the power allocation policies considered in this paper are designed to meet the QoS requirements instantaneously, unlike conventional scheduling strategies, such as proportional fairness based ones, which are designed to ensure the QoS requirement in the long term.

order of $(N - k + 1)(N - M + 1)$ is achievable at the k -th user, because

$$P \left(\frac{\epsilon_{i,k} \left(|\mathbf{v}_{i,k}^H \mathbf{h}_{i,ik}|^2 + \frac{1}{\rho} \right)}{|\mathbf{v}_{i,k}^H \mathbf{h}_{i,ik}|^2 (1 + \epsilon_{i,k})} > 1 \right) = P \left(|\mathbf{v}_{i,k}^H \mathbf{h}_{i,ik}|^2 < \frac{\epsilon_{i,k}}{\rho} \right).$$

The following theorem demonstrates the achievable diversity order at the n -th user.

Lemma 3: With the cognitive radio inspired power allocation coefficient $\alpha_{i,k}$ in (37), a diversity gain of $(N - M + 1)(K - k + 1)$ is achievable at the n -th user.

Proof: Please refer to the appendix. \blacksquare

It is important to point out that the diversity gain at the n -th user is constrained by the k -th user's channel condition due to the use of (37), which is consistent with the finding in [10]. Recall that cognitive radio inspired NOMA with single-antenna nodes can achieve a diversity of $(K - k + 1)$ for both users [10]. Therefore one advantage of MIMO-NOMA is that a larger diversity order can be achieved. In addition, the use of MIMO-NOMA can ensure that more users are served simultaneously.

B. To meet a dynamic QoS constraint

Another choice for the QoS requirement at the k -th user is to ensure the following constraint:

$$\log \left(1 + \frac{|\mathbf{v}_{1,k}^H \mathbf{h}_{1,1k}|^2 \alpha_{1,k}^2}{|\mathbf{v}_{1,k}^H \mathbf{h}_{1,1k}|^2 \alpha_{1,n}^2 + \frac{1}{\rho}} \right) > \frac{1}{2} \log \left(1 + |\mathbf{v}_{1,k}^H \mathbf{h}_{1,1k}|^2 \rho \right), \quad (38)$$

which means that the k -th user is willing to perform NOMA with the n -th user only if it can achieve a larger rate compared to the case with conventional MIMO-OMA.

With the same notation as before, the above constraint can be expressed as follows:

$$\frac{(1 + x_k \rho)^2}{(1 + x_k \alpha_{1,n}^2 \rho)^2} > (1 + x_k \rho), \quad (39)$$

from which the constraint for the power coefficient $\alpha_{1,n}^2$ can be obtained as follows:

$$0 \leq \alpha_{1,n} \leq \frac{\sqrt{1 + \rho x_k} - 1}{\rho x_k}. \quad (40)$$

Note that $0 \leq \frac{\sqrt{1 + \rho x_k} - 1}{\rho x_k} \leq 1$ for arbitrary choices of x_k , and therefore $\frac{\sqrt{1 + \rho x_k} - 1}{\rho x_k}$ is a feasible choice. So we will set

$$\alpha_{1,n} = \frac{\sqrt{1 + \rho x_k} - 1}{\rho x_k}, \quad (41)$$

which is the maximal value of the power allocation coefficient given the constraint in (39).

We first focus on the impact of this power coefficient on the outage probability at the k -th user, which is given by

$$\begin{aligned} P_o^k &= P \left(\log \left(1 + \frac{x_k \alpha_{1,k}^2}{x_k \alpha_{1,n}^2 + \frac{1}{\rho}} \right) < R_{1,k} \right) \\ &= P \left(\log \sqrt{1 + \rho x_k} < R_{1,k} \right). \end{aligned} \quad (42)$$

An important conclusion from (42) is that the use of the power coefficient in (41) ensures that the k -th user experiences exactly the same outage probability as the case with conventional MIMO-OMA. This observation is expected since the choice of $\alpha_{1,n}$ is to ensure the constraint in (38), i.e., the k -th user's rate should not be reduced because of the use of NOMA. Following steps similar to those used in the previous section, it is straightforward to show that the diversity gain of this user is $(N - M + 1)(K - k + 1)$.

Because the expression for the power allocation coefficient in (41) is very complicated, an exact expression for the outage probability achieved at the n -th user is difficult to find, but the achievable diversity gain can still be obtained as shown in the following lemma.

Lemma 4: In the proposed CR-MIMO-NOMA system with the dynamic QoS constraint in (38), a diversity order of $(N - M + 1)(K - k + 1)$ is achievable by the n -th user.

Proof: Please refer to the appendix. \blacksquare

It is worth pointing out that the diversity order provided in Lemma 4 is only an achievable one. After carrying out computer simulations, we observe that this diversity lower bound is tight for the case of $R_{1,k} > 1$, and a diversity order larger than $(N - M + 1)(K - k + 1)$ can be achieved for $0 \leq R_{1,k} \leq 1$. A possible reason for this is that a loose bound is used to get the achievable diversity gain for the case of $0 \leq R_{1,k} \leq 1$, as shown in (86).

VI. NUMERICAL RESULTS

In this section computer simulation results will be used to demonstrate the performance of MIMO-NOMA and also verify the accuracy of the developed analytical results. For notational simplicity, we omit the index of the cluster, e.g., $R_{1,k}$ is denoted by R_k .

In Figs. 1 and 2 the performance of MIMO-NOMA with fixed power allocation coefficients is studied first. Particularly, Fig. 1 considers the case in which there are four users grouped into two clusters, with two users in each cluster. Fig. 2 considers the case in which there are three clusters, with three users in each cluster. All users in each cluster will participate in NOMA. Fig. 1 confirms the accuracy of the analytical results developed in Theorem 1 and Corollary 1. Another important observation is that MIMO-NOMA can outperform MIMO-OMA, particularly at high SNR, where different targeted data rates are used for different users in this figure. While MIMO-NOMA achieves better outage performance than MIMO-OMA, both realize the same diversity gain. Fig. 2 demonstrates the accuracy of the high SNR approximation results developed in Theorem 1. In particular, one observation from this figure is that different users experience different diversity orders, which confirms the diversity order results developed in Theorem 1. In Fig. 3 the impact of different power coefficients on the outage performance is investigated. As can be seen from the figure, increasing the value of α_1 causes different performance changes at different users. In particular, by increasing α_1 , the outage probability at the user with stronger CSI is deteriorating, but the outage performance at the other user is improved, since more power is allocated to the user with weaker CSI. This observation is also

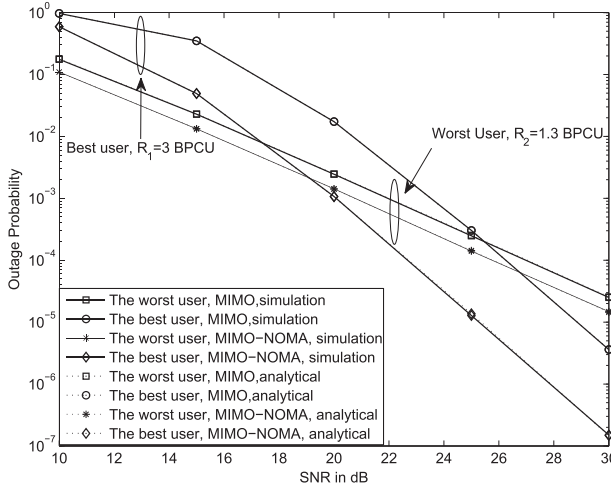


Fig. 1. MIMO-NOMA with a fixed set of power coefficients. $M = 2$, $N = 3$ and $K = 2$. $\alpha_1^2 = \frac{1}{4}$ and $\alpha_2^2 = \frac{3}{4}$. BPCU denotes bits per channel use.

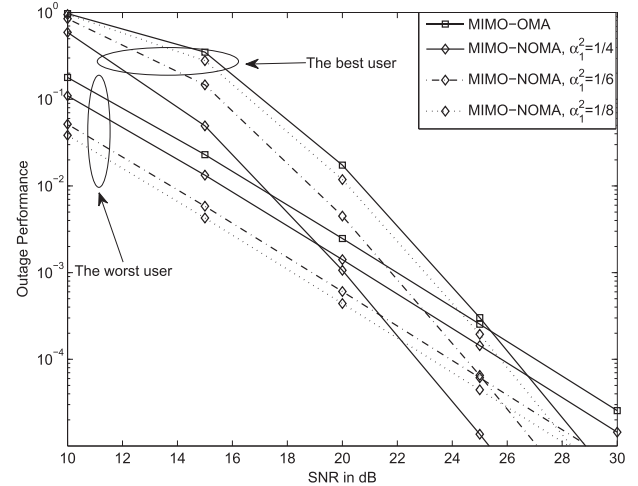


Fig. 3. MIMO-NOMA with different choices of power coefficients. $M = 2$, $N = 3$ and $K = 2$.

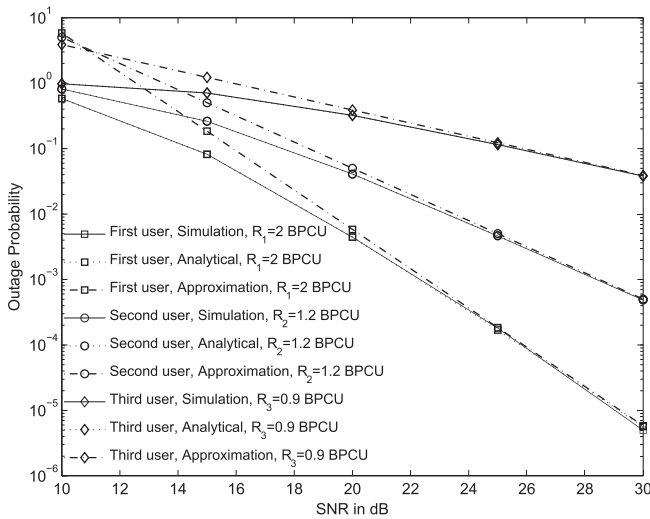
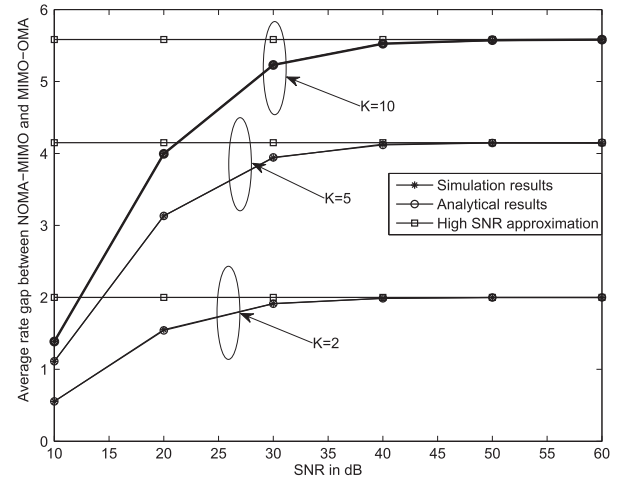


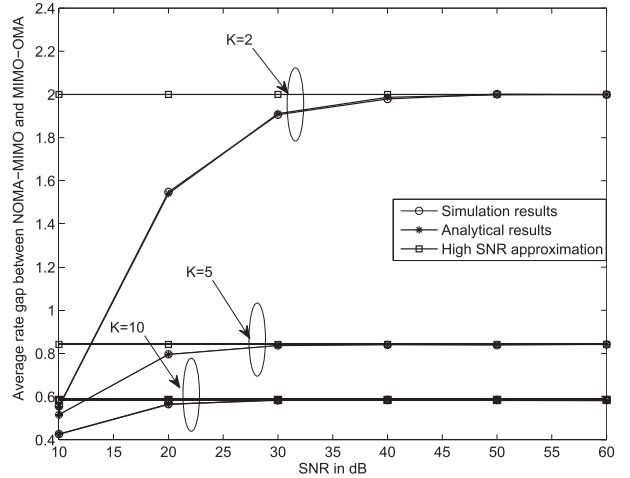
Fig. 2. MIMO-NOMA with a fixed set of power coefficients. $M = 3$, $N = 3$ and $K = 3$. $\alpha_1^2 = \frac{1}{6}$, $\alpha_2^2 = \frac{1}{3}$ and $\alpha_3^2 = \frac{1}{2}$.

consistent with the statement made in [17] that the effect of changing the power allocation coefficients of NOMA is equivalent to choosing different rate pairs along the boundary of the capacity region of broadcast channels.

In Fig. 4 the impact of user pairing is demonstrated by using the average sum-rate gap between MIMO-NOMA and MIMO-OMA. As can be seen from both sub-figures, the exact expression for the average sum-rate gap developed in Lemma 1 matches the simulation results perfectly, and the approximation result developed in the lemma provides a tight bound at high SNR. Comparing Fig. 4(a) to Fig. 4(b), one can observe that the impact of K on the performance gap is much different. In Fig. 4(a), increasing the number of the users in each group, K , can significantly improve the performance gap between MIMO-NOMA and MIMO-OMA. Specifically a gain of two bits per channel use (BPCU) can be obtained when there are two users in each group, and this gap can double when there are five users in each group. The reason for this performance gain is because we schedule the best user and the worst user, i.e., $n = 1$ and $k = K$, and the two selected users' channel information



(a) Case I: $n = 1$ and $k = K$



(b) Case II: $n = 1$ and $k = 2$

Fig. 4. The performance gap offered by MIMO-NOMA, $M = 2$, $N = 2$ and $\alpha_1^2 = \frac{1}{4}$.

becomes very different when increasing K , which is beneficial to the implementation of NOMA. On the other hand, Fig. 4(b) demonstrates that the performance gain of MIMO-NOMA is

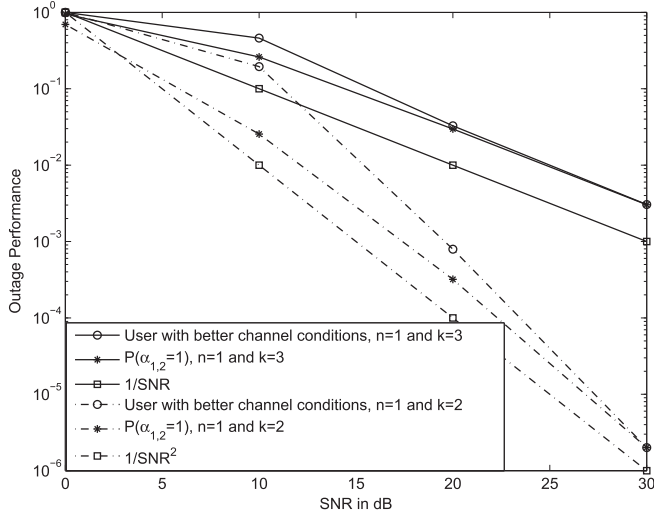
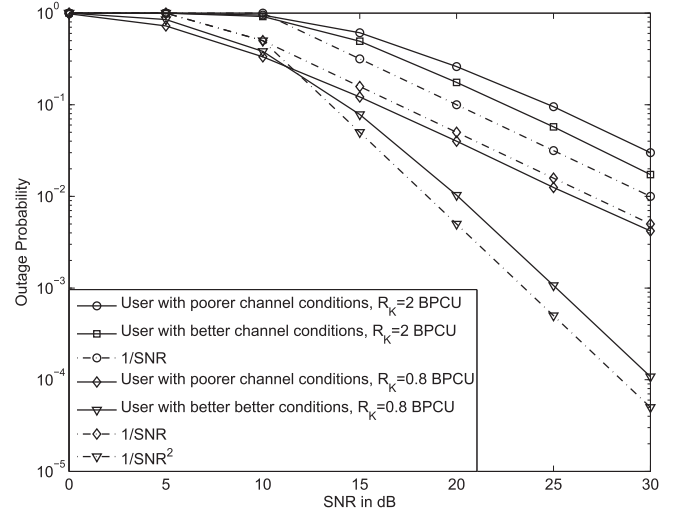


Fig. 5. Cognitive radio inspired MIMO-NOMA with a fixed power constraint, $\epsilon_{1,k} = 1$, $M = 2$, $N = 2$, $K = 3$ and $R_1 = 2$ BPCU.

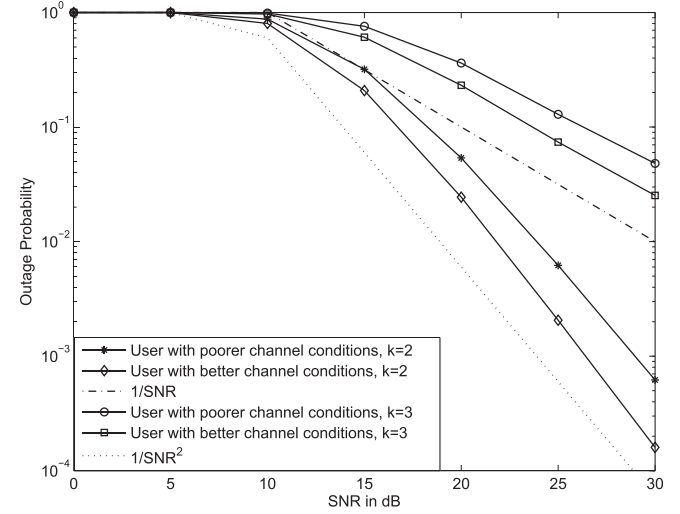
diminishing with increasing K . This is because the user with the best channel conditions and the one with the second best channel conditions are scheduled. When increasing K , the two users' channel conditions become significantly similar, which will reduce the performance gain of NOMA.

In Fig. 5, the performance of cognitive radio (CR) inspired MIMO-NOMA for meeting the fixed QoS requirement in (36) is studied. In this figure three types of curves are provided, one for P_n^o as studied in Lemma 3, one for $P(\alpha_2 = 1)$, and one for $\frac{1}{\rho^{(N-M+1)(K-k+1)}}$. The last is provided to demonstrate the achievable diversity order. As can be seen in the figure, the curves for P_n^o are parallel to the ones for $\frac{1}{\rho^{(N-M+1)(K-k+1)}}$, which demonstrates that the achievable diversity order obtained in Lemma 3 is tight. An interesting observation from the figure is that $P(\alpha_2 = 1)$ is a tight lower bound of P_n^o , particularly at high SNR. This is because CR-MIMO-NOMA tends to satisfy the k -th user's QoS first and therefore the event $\alpha_2^2 = 1$, i.e., the BS allocates all the power to the k -th user, is dominant among the three types of events described in the proof for Lemma 3.

Finally, the performance of CR-MIMO-NOMA in meeting the dynamic QoS requirement in (38) is investigated in Fig. 6. Again the curves for $\frac{1}{\rho^2}$ and $\frac{1}{\rho^2}$ are provided to facilitate the analysis of diversity orders. Both sub-figures demonstrate that a diversity order of $(N - M + 1)(K - k + 1)$ is achievable regardless of the choice of R_k , which confirms the accuracy of Lemma 4. Furthermore, this diversity order of $(N - M + 1)(K - k + 1)$ can be tight depending on the choice of R_k . For example, in Fig. 6(a), when $R_k = 2$ BPCU, the curves for the outage probability for the user with better channel conditions are always parallel to the ones for $\frac{1}{\rho^{(N-M+1)(K-k+1)}}$. In general, our simulation studies reveal that the diversity order of $(N - M + 1)(K - k + 1)$ is exactly what CR-MIMO-NOMA can realize in the case of $R_k > 1$. However, in the case of $0 \leq R_k \leq 1$, a diversity gain larger than $(N - M + 1)(K - k + 1)$ can be achieved, as shown in Fig. 6(b). As discussed in Section V, the reason for this is because the upper bound used in the proof for Lemma 4 is loose in the case of $0 \leq R_k \leq 1$.



(a) $K = 3$, $n = 1$ and $R_n = R_k = 2$ BPCU



(b) $K = 2$, $n = 1$, $k = K$ and $R_n = 2$ BPCU

Fig. 6. CR with a dynamic power constraint, $M = 2$, $N = 2$.

VII. CONCLUSION

In this paper, we have studied the application of MIMO to NOMA systems. A new design of precoding and detection matrices for MIMO-NOMA has been proposed, and its performance has been analyzed. To further improve the performance gap between MIMO-NOMA and conventional OMA, the use of user pairing has been considered in NOMA systems and its impact on the system performance has also been characterized. The cognitive radio inspired choices for power allocation coefficients have also been proposed to meet various QoS requirements. Simulation results have been provided to demonstrate the accuracy of the developed analytical results. In this paper, closed-form analytical results have been developed to demonstrate that NOMA can outperform conventional MIMO-OMA, even with simple choices of power allocation coefficients. A joint design of power allocation and precoding can further improve the performance gain of MIMO-NOMA, as shown in [15]. In addition, in this paper, users have

been randomly divided into multiple groups, and an important future direction is to study the design of low complexity approaches for dynamic clustering/grouping in MIMO-NOMA systems. Note that full-rank channel matrices are assumed in this paper, and therefore it is important to study the design of MIMO-NOMA when channel matrices are rank deficient; the analytical results developed in this paper provide an upper bound for this non-ideal case.

APPENDIX A PROOF FOR THEOREM 1

The proof can be completed in four steps.

A. Density function of effective channel gains

Without loss of generality, we focus only on the users in the first cluster. First recall that these users have been ordered according to the criterion in (6) which can be rewritten as follows:

$$x_1 \geq \dots \geq x_K, \quad (43)$$

where $x_k \triangleq |\mathbf{v}_{1,k}^H \mathbf{h}_{1,1k}|^2$. Define \tilde{x}_k as the unordered counterpart of x_k . Given the choice of $\mathbf{v}_{1,k} = \mathbf{U}_{1,k} \mathbf{z}_{1,k}$ and $\mathbf{z}_{1,k} = \frac{\mathbf{U}_{1,k}^H \mathbf{h}_{1,1k}}{|\mathbf{U}_{1,k}^H \mathbf{h}_{1,1k}|}$, we have

$$|\mathbf{v}_{1,k}^H \mathbf{h}_{1,1k}|^2 = \left(\frac{|\mathbf{U}_{1,k}^H \mathbf{h}_{1,1k}|^2}{|\mathbf{U}_{1,k}^H \mathbf{h}_{1,1k}|} \right)^2 = |\mathbf{U}_{1,k}^H \mathbf{h}_{1,1k}|^2.$$

An important observation is that $\mathbf{U}_{1,k}$ contains the $(N - M + 1)$ orthogonal singular vectors, i.e.,

$$\mathbf{U}_{1,k}^H \mathbf{U}_{1,k} = \mathbf{I}_{N-M+1},$$

and also note that $\mathbf{U}_{1,k}$ is independent of $\mathbf{h}_{1,1k}$. Therefore $\mathbf{v}_{1,k}^H \mathbf{h}_{1,1k}$ represents a unitary transformation of a complex Gaussian vector, which means that $\mathbf{v}_{1,k}^H \mathbf{h}_{1,1k}$ is still an $(N - M + 1) \times 1$ complex Gaussian vector [21]. Therefore this unordered variable, \tilde{x}_k , follows the chi-square distribution, and thus the probability density function (pdf) of \tilde{x}_k is given by

$$f_{\tilde{x}_k}(x) = \frac{e^{-x}}{(N - M)!} x^{N-M}, \quad (44)$$

and its CDF is $F_{\tilde{x}_k}(x) = \int_0^x f_{\tilde{x}_k}(y) dy$. Therefore the ordered variable, x_k , in (43) follows the following pdf [22]:

$$f_{x_k}(x) = \frac{K! f_{\tilde{x}_k}(x) [F_{\tilde{x}_k}(x)]^{K-k} [1 - F_{\tilde{x}_k}(x)]^{k-1}}{(K - k)! (k - 1)!}. \quad (45)$$

B. A unified outage probability expression

Because the users in one cluster carry out different detection strategies, the outage probabilities achieved by different users will be evaluated separately first and then a unified expression for these probabilities will be developed.

1) *Outage probability at the user with the worst channel condition:* The outage probability for the K -th user in the first cluster is given by

$$\mathbb{P}(SINR_{1,K} < \epsilon_{1,K}) = \mathbb{P}\left(\frac{x_K \alpha_{1,K}^2}{x_K \sum_{k=1}^{K-1} \alpha_{1,k}^2 + \frac{1}{\rho}} < \epsilon_{1,K}\right). \quad (46)$$

The above outage probability can be written as the two following forms:

$$\begin{cases} \mathbb{P}\left(x_K < \frac{\epsilon_{1,K}}{\rho(\alpha_{1,K}^2 - \beta_{1,K})}\right), & \text{if } \alpha_{1,K}^2 > \beta_{1,K} \\ 1, & \text{otherwise} \end{cases} \quad (47)$$

where $\beta_{1,K} = \epsilon_{1,K} \sum_{k=1}^{K-1} \alpha_{1,k}^2$.

2) *Outage probability at the k -th user, $1 < k < K$:* The k -th user needs to decode the j -th user's message, $j > k$, before detecting its own message. The overall outage probability for the k -th user to decode its own message can be expressed as follows:

$$\begin{aligned} P_{1,k}^o &= 1 - \mathbb{P}(SINR_{1,k}^j > \epsilon_{1,j}, \forall j \in \{k, \dots, K\}) \\ &= 1 - \mathbb{P}\left(\frac{x_k \alpha_{1,j}^2}{x_k \sum_{l=1}^{j-1} \alpha_{1,l}^2 + \frac{1}{\rho}} > \epsilon_{1,j}, \forall j \in \{k, \dots, K\}\right). \end{aligned} \quad (48)$$

Following steps similar to those used in the previous subsection, the above probability can be rewritten as follows:

$$\begin{aligned} &\mathbb{P}\left(\frac{x_k \alpha_{1,j}^2}{x_k \sum_{l=1}^{j-1} \alpha_{1,l}^2 + \frac{1}{\rho}} > \epsilon_{1,j}, \forall j \in \{k, \dots, K\}\right) \\ &= \begin{cases} \mathbb{P}\left(x_k > \frac{\epsilon_{1,j}}{\rho(\alpha_{1,j}^2 - \beta_{1,j})}, \forall j \in \{k, \dots, K\}\right), & \text{if } \mathbf{C1} \\ 0, & \text{otherwise} \end{cases} \end{aligned} \quad (49)$$

where the condition, $\mathbf{C1}$, denotes $\alpha_{1,j}^2 > \beta_{1,j}$, for all $k \leq j \leq K$, and $\beta_{1,j} = \epsilon_{1,j} \sum_{l=1}^{j-1} \alpha_{1,l}^2$.

Define $\epsilon_{1,k}^* = \max\left\{\frac{\epsilon_{1,j}}{\rho(\alpha_{1,j}^2 - \beta_{1,j})}, k \leq j \leq K\right\}$. The outage probability can be expressed as follows:

$$P_{1,k}^o = \begin{cases} \mathbb{P}(x_k < \epsilon_{1,k}^*), & \text{if } \mathbf{C1} \\ 1, & \text{otherwise} \end{cases} \quad (50)$$

It is interesting to observe that the expression in (47) is a special case of (50). It is straightforward to evaluate that the outage probability expressions in (50) can also be used for the user with the best channel condition by letting $\epsilon_{1,1}^* = \max\left\{\frac{\epsilon_{1,K}}{\rho(\alpha_{1,K}^2 - \beta_{1,K})}, \dots, \frac{\epsilon_{1,2}}{\rho(\alpha_{1,2}^2 - \beta_{1,2})}, \frac{\epsilon_{1,1}}{\rho\alpha_{1,1}^2}\right\}$.

C. Obtaining an exact expression for the outage probability

When the conditions, $\alpha_{1,k}^2 \geq \beta_{1,k}$, are satisfied, the outage probability is given by

$$\begin{aligned} P_{1,k}^o &= \sum_{p=0}^{k-1} \binom{k-1}{p} (-1)^p \frac{K!}{(K-k)!(k-1)!} \\ &\quad \times \int_0^{\epsilon_{1,k}^*} f_{\tilde{x}_k}(x) [F_{\tilde{x}_k}(x)]^{K-k+p} dx \\ &= \sum_{p=0}^{k-1} \binom{k-1}{p} \frac{(-1)^p K! [F_{\tilde{x}_k}(\epsilon_{1,k}^*)]^{K-k+p+1}}{(K-k)!(k-1)!(K-k+p+1)}. \end{aligned} \quad (51)$$

By applying the CDF of the unsorted variable \tilde{x}_k , we obtain

$$P_{1,k}^o = \sum_{p=0}^{k-1} \binom{k-1}{p} \frac{(-1)^p K! \left[\int_0^{\epsilon_{1,k}^*} f_{\tilde{x}_k}(y) dy \right]^{K-k+p+1}}{(K-k)!(k-1)!(K-k+p+1)}.$$

By applying the incomplete gamma function, the exact expression of the outage probability can be obtained as in the theorem.

D. High SNR approximations

By applying the series expansion of the incomplete gamma function [23], the outage probability can be first expanded as shown in (52), shown at the bottom of the page. At high SNR, the outage probability can be approximated as shown in (53), also shown at the bottom of the page. Therefore the theorem is proved.

APPENDIX B

PROOF FOR LEMMA 2

The sum-rate gap will be evaluated separately for two cases in the following subsections.

A. Case I with $n = 1$ and $k = K$

First recall the following integral from Eq. (3.352.4) in [23]:

$$\int_0^\infty \frac{1}{1+x\phi} e^{-lx} dx = -\frac{1}{\phi} e^{\frac{l}{\phi}} \mathbf{E}_i \left(-\frac{l}{\phi} \right)$$

By using the above result and also Lemma 1, $\varphi(1, \phi)$ can be expressed as follows:

$$\begin{aligned} \varphi(1, \phi) &= \frac{\phi}{\ln 2} \int_0^\infty \frac{1}{1+x\phi} \left(1 - \gamma_1 \frac{[\gamma(1, x)]^K}{K} \right) dx \\ &= \frac{1}{\ln 2} \left(\sum_{l=1}^K \binom{K}{l} (-1)^l e^{\frac{l}{\phi}} \mathbf{E}_i \left(-\frac{l}{\phi} \right) \right). \end{aligned} \quad (54)$$

For the worst user, a direct use of Lemma 1 results in a quite complicated expression for $\varphi(K, \phi)$. Instead, we can find a simpler alternative way to calculate this factor, as shown in the following:

$$\begin{aligned} \varphi(K, \phi) &= \int_0^\infty \log(1+x\phi) f_{x_K}(x) dx \\ &= K \log(e) \int_0^\infty \ln(1+x\phi) e^{-Kx} dx \\ &= -\log(e) e^{\frac{K}{\phi}} \mathbf{E}_i \left(-\frac{K}{\phi} \right). \end{aligned} \quad (55)$$

$$\begin{aligned} P_{1,k}^o &= \sum_{p=0}^{k-1} \binom{k-1}{p} (-1)^p K! \frac{\left[(N-M)! \left(1 - e^{-\epsilon_{1,k}^*} \sum_{q=0}^{N-M} \frac{(\epsilon_{1,k}^*)^q}{q!} \right) \right]^{K-k+p+1}}{(K-k)!(k-1)!(K-k+p+1)((N-M)!)^{K-k+p+1}} \\ &= \sum_{p=0}^{k-1} \binom{k-1}{p} (-1)^p K! \frac{\left[(N-M)! \left(1 - e^{-\epsilon_{1,k}^*} \left(e^{\epsilon_{1,k}^*} - \sum_{q=N-M+1}^\infty \frac{(\epsilon_{1,k}^*)^q}{q!} \right) \right) \right]^{K-k+p+1}}{(K-k)!(k-1)!(K-k+p+1)((N-M)!)^{K-k+p+1}}. \end{aligned} \quad (52)$$

$$\begin{aligned} P_{1,k}^o &= \sum_{p=0}^{k-1} \binom{k-1}{p} (-1)^p K! \frac{\left[e^{-\epsilon_{1,k}^*} \sum_{q=N-M+1}^\infty \frac{(N-M)! (\epsilon_{1,k}^*)^q}{q!} \right]^{K-k+p+1}}{(K-k)!(k-1)!(K-k+p+1)((N-M)!)^{K-k+p+1}} \\ &\approx \sum_{p=0}^{k-1} \binom{k-1}{p} (-1)^p K! \frac{\left[\frac{(N-M)! (\epsilon_{1,k}^*)^{N-M+1}}{(N-M+1)!} \right]^{K-k+p+1}}{(K-k)!(k-1)!(K-k+p+1)((N-M)!)^{K-k+p+1}} \\ &\approx \frac{K! \left[\frac{(\epsilon_{1,k}^*)^{N-M+1}}{(N-M+1)!} \right]^{K-k+1}}{(K-k)!(k-1)!(K-k+1)}. \end{aligned} \quad (53)$$

By substituting (54) and (55) into the expression for the rate gap, the expression in (32) can be obtained.

To obtain the high SNR approximation, first recall that the exponential integral function has the following series representation [23]:

$$\mathbf{E}_i(x) = \mathbf{C} + \ln(-x) + \sum_{j=1}^{\infty} \frac{x^j}{j \cdots j!},$$

for $x < 0$. Therefore at high SNR, we have the following approximation:

$$\begin{aligned} \mathcal{E}\{\Delta\} &\approx -\log(e) \left(\mathbf{C} + \ln\left(\frac{K}{\rho}\right) \right) \\ &+ \frac{2}{\ln 2} \left(\sum_{l=1}^K \binom{k}{l} (-1)^l \left(\mathbf{C} + \ln\left(\frac{l}{\rho \alpha_{1,1}^2}\right) \right) \right) \\ &+ 2 \log(e) \left(\mathbf{C} + \ln\left(\frac{K}{\rho \alpha_{1,1}^2}\right) \right) \\ &- \frac{1}{\ln 2} \left(\sum_{l=1}^K \binom{k}{l} (-1)^l \left(\mathbf{C} + \ln\left(\frac{l}{\rho}\right) \right) \right). \end{aligned} \quad (56)$$

After some manipulations the average gap is given by

$$\begin{aligned} \frac{\mathcal{E}\{\Delta\}}{\log e} &\approx \mathbf{C} + \ln\left(\frac{K}{\rho}\right) - 2 \ln \alpha_{1,1}^2 + \sum_{l=1}^K \binom{k}{l} (-1)^l \ln l \\ &+ \sum_{l=1}^K \binom{k}{l} (-1)^l \left(\mathbf{C} + \ln\left(\frac{1}{\rho}\right) \right) \\ &+ 2 \sum_{l=1}^K \binom{k}{l} (-1)^l \left(-\ln \alpha_{1,1}^2 \right) \\ &= \mathbf{C} + \ln\left(\frac{K}{\rho}\right) - 2 \ln \alpha_{1,1}^2 - \left(\mathbf{C} + \ln\left(\frac{1}{\rho}\right) \right) \\ &+ \sum_{l=1}^K \binom{k}{l} (-1)^l \ln l - 2 \left(-\ln \alpha_{1,1}^2 \right) \\ &+ \sum_{l=0}^K \binom{k}{l} (-1)^l \left(\mathbf{C} + \ln\left(\frac{1}{\rho}\right) \right) \\ &+ 2 \sum_{l=0}^K \binom{k}{l} (-1)^l \left(-\ln \alpha_{1,1}^2 \right). \end{aligned}$$

After removing some common factors, the average gap can be simplified as follows:

$$\begin{aligned} \frac{\mathcal{E}\{\Delta\}}{\log e} &\approx \mathbf{C} + \ln\left(\frac{K}{\rho}\right) - 2 \ln \alpha_{1,1}^2 - \left(\mathbf{C} + \ln\left(\frac{1}{\rho}\right) \right) \\ &+ \sum_{l=1}^K \binom{k}{l} (-1)^l \ln l + 2 \ln \alpha_{1,1}^2 \\ &= \ln K + \sum_{l=1}^K \binom{k}{l} (-1)^l \ln l. \end{aligned} \quad (57)$$

And the first part of the lemma is proved.

B. Case II with $n = 1$ and $k = 2$

It is more complicated to evaluate the average gap for Case II due to the complicated expression for x_2 . In particular, the factor $\varphi(2, \phi)$ can be expressed as in (58), shown at the bottom of the page. Since $\int_0^{\infty} \frac{1}{1+x\phi} dx \rightarrow \infty$, therefore it is important to remove the factors in the integral related to $\frac{1}{1+x\phi}$ in order to facilitate the high SNR approximation. Motivated by this, the factor $\varphi(2, \phi)$ can be rewritten as follows:

$$\begin{aligned} \varphi(2, \phi) &= \frac{\phi}{\ln 2} \int_0^{\infty} \frac{1}{1+x\phi} \left(-K \sum_{p=1}^{K-1} \binom{k-1}{p} (-1)^p e^{-px} \right. \\ &\quad \left. + (K-1) \sum_{l=1}^K \binom{k}{l} (-1)^l e^{-lx} \right) dx. \end{aligned}$$

Following the same steps as those used in the previous section, the integral in the above equation can be evaluated and we can have the following:

$$\begin{aligned} \varphi(2, \phi) &= \frac{1}{\ln 2} \left(K \sum_{p=1}^{K-1} \binom{k-1}{p} (-1)^p e^{\frac{p}{\phi}} \mathbf{E}_i\left(-\frac{p}{\phi}\right) \right. \\ &\quad \left. - (K-1) \sum_{l=1}^K \binom{k}{l} (-1)^l e^{\frac{l}{\phi}} \mathbf{E}_i\left(-\frac{l}{\phi}\right) \right). \end{aligned} \quad (59)$$

Substituting (59) and (54) into (30), the exact expression of the average rate gap can be obtained as in the lemma.

At high SNR, the exponential integral function can be simplified as discussed previously, and the average rate gap can be approximated as in (60), shown at the bottom of the next page.

$$\begin{aligned} \varphi(2, \phi) &= \frac{\phi}{\ln 2} \int_0^{\infty} \frac{1}{1+x\phi} \left(1 - \gamma_2 \frac{[1-e^{-x}]^{K-1}}{K-1} + \gamma_2 \frac{[1-e^{-x}]^K}{K} \right) dx \\ &= \frac{\phi}{\ln 2} \int_0^{\infty} \frac{1}{1+x\phi} \left(1 - K \sum_{p=0}^{K-1} \binom{k-1}{p} (-1)^p e^{-px} + (K-1) \sum_{l=0}^K \binom{k}{l} (-1)^l e^{-lx} \right) dx. \end{aligned} \quad (58)$$

With some algebraic manipulations, the high SNR approximation shown in (35) can be obtained, and the lemma is proved.

APPENDIX C PROOF FOR LEMMA 3

Without loss of generality, we will take the users in the first cluster as an example. Recall that the n -th user, $n < k$, in the first cluster tries to decode the k -th user's message with the following SINR:

$$SINR_{1,n}^k = \frac{|\mathbf{v}_{1,n}^H \mathbf{h}_{1,1n}|^2 \alpha_{1,k}^2}{|\mathbf{v}_{1,n}^H \mathbf{h}_{1,1n}|^2 \alpha_{1,n}^2 + \frac{1}{\rho}}.$$

If successful, the n -th user will decode its own message with the following SINR:

$$\begin{aligned} SINR_{1,n}^n &= \rho |\mathbf{v}_{1,n}^H \mathbf{H}_{1,1} \mathbf{p}_1|^2 \alpha_{1,n}^2 \\ &= \rho |\mathbf{v}_{1,n}^H \mathbf{H}_{1,1} \mathbf{p}_1|^2 \left(1 - \frac{\epsilon_{1,k} (|\mathbf{v}_{1,k}^H \mathbf{h}_{1,1k}|^2 + \frac{1}{\rho})}{|\mathbf{v}_{1,k}^H \mathbf{h}_{1,1k}|^2 (1 + \epsilon_{1,k})} \right), \end{aligned} \quad (61)$$

if $\alpha_{1,n}^2 > 0$. Again with the same notation used in the proof of Theorem 1, the SINR at the n -th user can be expressed as follows:

$$SINR_{1,n}^n = \rho x_n \left(1 - \frac{\epsilon_{1,k} (x_k + \frac{1}{\rho})}{x_k (1 + \epsilon_{1,k})} \right), \quad (62)$$

if $\alpha_{1,n}^2 > 0$.

Thus there are two conditions before the SINR expression in (62) can be used. One is $\alpha_{1,n}^2 > 0$ and the other is that the n -th user can decode the k -th user's message, i.e., $\log(1 + SINR_{1,n}^k) > R_{1,k}$. Therefore the outage events at the n -th user can be categorized into the three following types:

- 1) Events with $\alpha_{1,n}^2 = 0$, which means $1 \leq \frac{\epsilon_{1,k} (x_k + \frac{1}{\rho})}{x_k (1 + \epsilon_{1,k})}$ or equivalently

$$x_k \leq \frac{\epsilon_{1,k}}{\rho}. \quad (63)$$

- 2) Events with $\alpha_{1,n}^2 > 0$ and $\log(1 + SINR_{1,n}^k) < R_{1,k}$.

- 3) Events with $\alpha_{1,n}^2 > 0$, $\log(1 + SINR_{1,n}^k) > R_{1,k}$ and $SINR_{1,n}^n < \epsilon_{1,n}$.

Because $|\mathbf{v}_{1,k}^H \mathbf{h}_{1,1k}|^2 < |\mathbf{v}_{1,n}^H \mathbf{h}_{1,1n}|^2$, it is straightforward to show $SINR_{1,n}^k > SINR_{1,k}^k$, which means

$$\begin{aligned} &P\left(\alpha_{1,n}^2 > 0, \log\left(1 + SINR_{1,n}^k\right) < R_{1,k}\right) \\ &= \mathcal{E}_{0 < \alpha_{1,n}^2 \leq 1} \left\{ P\left(SINR_{1,n}^k < \epsilon_{1,k}\right) \right\} \\ &= \mathcal{E}_{0 < \alpha_{1,n}^2 \leq 1} \left\{ P\left(SINR_{1,k}^k = \epsilon_{1,k}, SINR_{1,n}^k < \epsilon_{1,k}\right) \right\} = 0, \end{aligned} \quad (64)$$

i.e., the n -th user can decode the k -th user's information as long as the k -th user can decode its own. But if $\alpha_{1,k}^2 = 1$, i.e., the BS allocates all the power to the k -th user, outage will occur at the n -th user.

Therefore the outage probability experienced by the n -th user is given by

$$\begin{aligned} P_n^o &= P\left(\rho x_n \left(1 - \frac{\epsilon_{1,k} (x_k + \frac{1}{\rho})}{x_k (1 + \epsilon_{1,k})}\right) < \epsilon_{1,n}, x_k > \frac{\epsilon_{1,k}}{\rho}\right) \\ &+ P\left(x_k \leq \frac{\epsilon_{1,k}}{\rho}\right), \end{aligned} \quad (65)$$

which follows from the following simplification:

$$\begin{aligned} &P\left(\alpha_{1,n}^2 > 0, \log(1 + SINR_{1,n}^k) > R_{1,k}, SINR_{1,n}^n < \epsilon_{1,n}\right) \\ &= P\left(\alpha_{1,n}^2 > 0, SINR_{1,n}^n < \epsilon_{1,n}\right). \end{aligned} \quad (66)$$

Define the first factor in the expression for the outage probability in (65) by $Q_2 \triangleq P\left(\rho x_n \left(1 - \frac{\epsilon_{1,k} (x_k + \frac{1}{\rho})}{x_k (1 + \epsilon_{1,k})}\right) < \epsilon_{1,n}, x_k > \frac{\epsilon_{1,k}}{\rho}\right)$. This factor can be evaluated as follows:

$$Q_2 = P\left(x_n \left(\frac{x_k - \frac{\epsilon_{1,k}}{\rho}}{x_k}\right) < \tilde{\epsilon}_{1,n}, x_k > \frac{\epsilon_{1,k}}{\rho}\right) \quad (67)$$

$$= \mathcal{E}_{x_k} \left\{ P\left(x_k < x_n < \frac{\tilde{\epsilon}_{1,n} x_k}{x_k - \frac{\epsilon_{1,k}}{\rho}}\right) \right\}. \quad (68)$$

$$\begin{aligned} \mathcal{E}\{\Delta\} &\approx \frac{1}{\ln 2} \left(K \sum_{p=1}^{K-1} \binom{K-1}{p} (-1)^p \left(C + \ln\left(\frac{p}{\rho}\right) \right) - (K-1) \sum_{l=1}^K \binom{K}{l} (-1)^l \left(C + \ln\left(\frac{l}{\rho}\right) \right) \right) dx \\ &+ \frac{2}{\ln 2} \left(\sum_{l=1}^K \binom{K}{l} (-1)^l \left(C + \ln\left(-\frac{l}{\rho \alpha_{1,1}^2}\right) \right) \right) - \frac{2}{\ln 2} \left(K \sum_{p=1}^{K-1} \binom{K-1}{p} (-1)^p \left(C + \ln\left(-\frac{p}{\rho \alpha_{1,1}^2}\right) \right) \right) \\ &- (K-1) \sum_{l=1}^K \binom{K}{l} (-1)^l \left(C + \ln\left(-\frac{l}{\rho \alpha_{1,1}^2}\right) \right) - \frac{1}{\ln 2} \left(\sum_{l=1}^K \binom{K}{l} (-1)^l \left(C + \ln\left(-\frac{l}{\rho}\right) \right) \right). \end{aligned} \quad (60)$$

where $\tilde{\epsilon}_{1,n} = \frac{(1+\epsilon_{1,k})\epsilon_{1,n}}{\rho}$. It is important to note that the expectation in (68) is taken over the following range:

$$\frac{\epsilon_{1,k}}{\rho} < x_k < \frac{\epsilon_{1,k}}{\rho} + \tilde{\epsilon}_{1,n},$$

where the upper bound is due to the constraint $x_k < \frac{\tilde{\epsilon}_{1,n}x_k}{x_k - \frac{\epsilon_{1,k}}{\rho}}$.

Now the factor Q_2 can be evaluated as follows:

$$\begin{aligned} Q_2 &= \mathcal{E}_{x_k} \left\{ F_{x_n} \left(\frac{\tilde{\epsilon}_{1,n}x_k}{x_k - \frac{\epsilon_{1,k}}{\rho}} \right) - F_{x_n}(x_k) \right\} \\ &= \int_{\frac{\epsilon_{1,k}}{\rho}}^{\frac{\epsilon_{1,k}}{\rho} + \tilde{\epsilon}_{1,n}} \left(F_{x_n} \left(\frac{\tilde{\epsilon}_{1,n}y}{y - \frac{\epsilon_{1,k}}{\rho}} \right) - F_{x_n}(y) \right) f_{x_k}(y) dy. \end{aligned} \quad (69)$$

At high SNR, Q_2 can be upper bounded as follows:

$$Q_2 < \int_{\frac{\epsilon_{1,k}}{\rho}}^{\frac{\epsilon_{1,k}}{\rho} + \tilde{\epsilon}_{1,n}} f_{x_k}(y) dy, \quad (70)$$

since $F_{x_n} \left(\frac{\tilde{\epsilon}_{1,n}y}{y - \frac{\epsilon_{1,k}}{\rho}} \right) - F_{x_n}(y) \leq F_{x_n} \left(\frac{\tilde{\epsilon}_{1,n}y}{y - \frac{\epsilon_{1,k}}{\rho}} \right) \leq 1$.

With this upper bound, the overall outage probability can be upper bounded as follows:

$$\begin{aligned} P_n^o &= Q_2 + \mathbb{P} \left(x_k \leq \frac{\epsilon_{1,k}}{\rho} \right) \\ &\leq \int_{\frac{\epsilon_{1,k}}{\rho}}^{\frac{\epsilon_{1,k}}{\rho} + \tilde{\epsilon}_{1,n}} f_{x_k}(y) dy + \mathbb{P} \left(x_k \leq \frac{\epsilon_{1,k}}{\rho} \right) \\ &= F_{x_n} \left(\frac{\epsilon_{1,k}}{\rho} + \tilde{\epsilon}_{1,n} \right). \end{aligned} \quad (71)$$

To find a high SNR approximation of $F_{x_n}(x)$, first recall that the CDF of x_n is given by

$$F_{x_n}(x) = \gamma_n \sum_{j=0}^{n-1} \binom{n-1}{j} (-1)^j \frac{[F_{\tilde{x}_n}(x)]^{K-n+j+1}}{K-n+j+1}. \quad (72)$$

When $x \rightarrow 0$, we have

$$F_{\tilde{x}_n}(x) = \frac{\gamma(N-M+1, x)}{(N-M)!} \approx \frac{x^{N-M+1}}{(N-M+1)!}. \quad (73)$$

Therefore, the CDF of x_n can be approximated as follows:

$$\begin{aligned} F_{x_n}(x) &\approx \gamma_n \sum_{j=0}^{n-1} \binom{n-1}{j} (-1)^j \frac{\left(\frac{x^{N-M+1}}{(N-M+1)!} \right)^{K-k+j+1}}{K-n+j+1} \\ &\approx \frac{\gamma_n}{K-n+1} \left(\frac{x^{N-M+1}}{(N-M+1)!} \right)^{K-k+1}, \end{aligned} \quad (74)$$

when $x \rightarrow 0$.

Substituting the above approximation into (71), the overall outage probability can be upper bounded as follows:

$$\begin{aligned} P_n^o &\leq \frac{\gamma_n}{K-n+1} \left(\frac{\left(\frac{\epsilon_{1,k}}{\rho} + \tilde{\epsilon}_{1,n} \right)^{N-M+1}}{(N-M+1)!} \right)^{K-k+1} \\ &\rightarrow \frac{1}{\rho^{(N-M+1)(K-k+1)}}. \end{aligned} \quad (75)$$

And the proof is completed.

APPENDIX D PROOF FOR LEMMA 4

With the same notation used in the proof for Theorem 1, the SINR at the n -th user can be expressed as follows:

$$\text{SINR}_{1,n}^n = \rho x_n \frac{\sqrt{1 + \rho x_k} - 1}{\rho x_k}, \quad (76)$$

if it can decode the k -th user's message, i.e.,

$$\log \left(1 + \frac{x_n \alpha_{1,k}^2}{x_n \alpha_{1,n}^2 + \frac{1}{\rho}} \right) > R_{1,k}. \quad (77)$$

Therefore the outage events at the n -th user can be categorized into the two following types:

- Events in which the n -th user cannot decode the k -th user, i.e.,

$$\log \left(1 + \frac{x_n \alpha_{1,k}^2}{x_n \alpha_{1,n}^2 + \frac{1}{\rho}} \right) < R_{1,k}$$

- Events in which the n -th user can decode the k -th user, but cannot decode its own, i.e.,

$$\log \left(1 + \frac{x_n \alpha_{1,k}^2}{x_n \alpha_{1,n}^2 + \frac{1}{\rho}} \right) > R_{1,k},$$

and

$$\log \left(1 + \rho x_n \alpha_{1,n}^2 \right) < R_{1,n}.$$

Therefore the outage probability experienced by the n -th user is given by

$$\begin{aligned} P_n^o &= \mathbb{P} \left(\log \left(\frac{x_n \rho + 1}{x_n \alpha_{1,n}^2 \rho + 1} \right) < R_{1,k} \right) \\ &\quad + \underbrace{\mathbb{P} \left(\log \left(\frac{x_n \rho + 1}{x_n \alpha_{1,n}^2 \rho + 1} \right) > R_{1,k}, \log \left(1 + \rho x_n \alpha_{1,n}^2 \right) < R_{1,n} \right)}_{Q_3}. \end{aligned} \quad (78)$$

The first factor of P_n^o can be calculated as follows:

$$\begin{aligned} Q_4 &\triangleq \mathbb{P} \left(\log \left(\frac{x_n \rho + 1}{x_n \alpha_{1,n}^2 \rho + 1} \right) < R_{1,k} \right) \\ &= \begin{cases} \mathbb{P} \left(x_k < x_n < \frac{2^{R_{1,k}} - 1}{\rho(1 - 2^{R_{1,k}} \alpha_{1,n})} \right), & \text{if } 1 > 2^{R_{1,k}} \alpha_{1,n} \\ 1, & \text{otherwise} \end{cases} \end{aligned} \quad (79)$$

The constraint of $1 > 2^{R_{1,k}} \alpha_{1,n}$ is equivalent to the following one:

$$x_k > \frac{2^{R_{1,k}}(2^{R_{1,k}} - 2)}{\rho}. \quad (80)$$

As a result, $P\left(\log\left(\frac{x_n \rho + 1}{x_n \alpha_{1,n}^2 \rho + 1}\right) < R_{1,k}\right)$ can be expressed as follows:

$$Q_4 = \begin{cases} P\left(x_k < x_n < \frac{2^{R_{1,k}} - 1}{\rho(1 - 2^{R_{1,k}} \alpha_{1,n})}, \text{ if } 1 > 2^{R_{1,k}} \alpha_{1,n} \right. \\ \left. x_k > \frac{2^{R_{1,k}}(2^{R_{1,k}} - 2)}{\rho}\right) & \& R_{1,k} > 1 \\ P\left(x_k < x_n < \frac{2^{R_{1,k}} - 1}{\rho(1 - 2^{R_{1,k}} \alpha_{1,n})}, \text{ if } 1 > 2^{R_{1,k}} \alpha_{1,n} \right. \\ \left. 1, \text{ otherwise} \right) & \& R_{1,k} \leq 1 \end{cases}. \quad (81)$$

Consequently $P\left(\log\left(\frac{x_n \rho + 1}{x_n \alpha_{1,n}^2 \rho + 1}\right) < s R_{1,k}\right)$ can be upper bounded as follows:

$$Q_4 \leq P\left(x_k < x_n < \frac{2^{R_{1,k}} - 1}{\rho(1 - 2^{R_{1,k}} \alpha_{1,n})}\right) + P\left(x_k < \frac{2^{R_{1,k}}(2^{R_{1,k}} - 2)}{\rho}\right), \quad (82)$$

if $R_{1,k} > 1$, otherwise

$$Q_4 = P\left(x_k < x_n < \frac{2^{R_{1,k}} - 1}{\rho(1 - 2^{R_{1,k}} \alpha_{1,n})}\right). \quad (83)$$

Comparing (82) to (83), one can observe that the probability $P\left(x_k < \frac{2^{R_{1,k}}(2^{R_{1,k}} - 2)}{\rho}\right)$ does not need to be taken into consideration for the case of $0 \leq R_{1,k} \leq 1$. In the following, we first focus on the case $R_{1,k} > 1$.

It is important to note that the constraint $x_k < x_n < \frac{2^{R_{1,k}} - 1}{\rho(1 - 2^{R_{1,k}} \alpha_{1,n})}$ yields the following additional constraint for x_k :

$$x_k < \frac{2^{R_{1,k}} - 1}{\rho \left(1 - 2^{R_{1,k}} \frac{\sqrt{1 + \rho x_k} - 1}{\rho x_k}\right)}, \quad (84)$$

which leads to the following inequality:

$$x_k < \frac{2^{2R_{1,k}} - 1}{\rho}. \quad (85)$$

Therefore $P\left(\log\left(\frac{x_n \rho + 1}{x_n \alpha_{1,n}^2 \rho + 1}\right) < R_{1,k}\right)$ can be upper bounded as follows:

$$\begin{aligned} & P\left(\log\left(\frac{x_n \rho + 1}{x_n \alpha_{1,n}^2 \rho + 1}\right) < R_{1,k}\right) \\ & \leq P\left(x_k < x_n < \frac{2^{R_{1,k}} - 1}{\rho(1 - 2^{R_{1,k}} \alpha_{1,n})}, x_k < \frac{2^{2R_{1,k}} - 1}{\rho}\right) \\ & + P\left(x_k < \frac{2^{R_{1,k}}(2^{R_{1,k}} - 2)}{\rho}\right) \\ & \leq P\left(x_k < \frac{2^{2R_{1,k}} - 1}{\rho}\right) + P\left(x_k < \frac{2^{R_{1,k}}(2^{R_{1,k}} - 2)}{\rho}\right). \end{aligned} \quad (86)$$

Following steps similar to those used in the previous section, we have the following asymptotic result:

$$P\left(x_k < \frac{2^{2R_{1,k}} - 1}{\rho}\right) \rightarrow \frac{1}{\rho^{(N-M+1)(K-k+1)}}. \quad (87)$$

The other probabilities have the same asymptotic behavior, and therefore by combining (86) and (87), we have

$$P\left(\log\left(\frac{x_n \rho + 1}{x_n \alpha_{1,n}^2 \rho + 1}\right) < R_{1,k}\right) \leq \frac{1}{\rho^{(N-M+1)(K-k+1)}}, \quad (88)$$

where $a \leq b$ denotes $\lim_{\rho \rightarrow \infty} \frac{\log a}{\log \rho} \geq \lim_{\rho \rightarrow \infty} \frac{\log b}{\log \rho}$ [18]. The above conclusion is also valid for the case $0 \leq R_{1,k} \leq 1$.

The factor Q_3 can be upper bounded as follows:

$$\begin{aligned} Q_3 & \leq P\left(\log\left(1 + \rho x_n \alpha_{1,n}^2\right) < R_{1,n}\right) \\ & = P\left(x_k < x_n < \frac{2^{R_{1,n}} - 1}{\rho \alpha_{1,n}^2}\right) \end{aligned} \quad (89)$$

A hidden constraint on x_k due to $x_k < x_n < \frac{2^{R_{1,n}} - 1}{\rho \alpha_{1,n}^2}$ is

$$x_k < \frac{2^{R_{1,n}} - 1}{\rho \frac{\sqrt{1 + \rho x_k} - 1}{\rho x_k}}, \quad (90)$$

which yields $x_k < \frac{2^{2R_{1,n}} - 1}{\rho}$.

Therefore the factor Q_3 can be further upper bounded as follows:

$$\begin{aligned} Q_3 & \leq P\left(x_k < x_n < \frac{2^{R_{1,n}} - 1}{\rho \alpha_{1,n}^2}, x_k < \frac{2^{2R_{1,n}} - 1}{\rho}\right) \\ & \leq P\left(x_k < \frac{2^{2R_{1,n}} - 1}{\rho}\right) \rightarrow \frac{1}{\rho^{(N-M+1)(K-k+1)}}. \end{aligned} \quad (91)$$

Combing (88), (91) and (92), the overall outage probability can be upper bounded as follows:

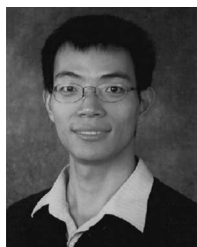
$$P_n^o \rightarrow \frac{1}{\rho^{(N-M+1)(K-k+1)}} \quad (92)$$

And the proof is completed.

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Zhiguo Ding (S'03–M'05) received the B.Eng. degree in electrical engineering from Beijing University of Posts and Telecommunications, Beijing, China, and the Ph.D. degree in electrical engineering from Imperial College London, London, U.K., in 2000 and 2005, respectively. From July 2005 to August 2014, he was working with Queen's University Belfast, Imperial College, and Newcastle University. Since September 2014, he has been with Lancaster University, Lancaster, U.K. as a Chair Professor. His research interests include

5G networks, game theory, cooperative and energy harvesting networks, and statistical signal processing. He is serving as an Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS, the IEEE TRANSACTIONS ON VEHICULAR NETWORKS, the IEEE WIRELESS COMMUNICATION LETTERS, the IEEE COMMUNICATION LETTERS, and the *Journal of Wireless Communications and Mobile Computing*. He was a recipient of the Best Paper Award at the IET Communication Conference on Wireless, Mobile, and Computing 2009, the IEEE Communication Letter Exemplary Reviewer 2012, and the EU Marie Curie Fellowship 2012–2014.



Fumiyuki Adachi (M'79–SM'90–F'00) received the B.S. and Dr. Eng. degrees in electrical engineering from Tohoku University, Sendai, Japan, in 1973 and 1984, respectively. In April 1973, he joined the Electrical Communications Laboratories, Nippon Telegraph and Telephone Corporation (now NTT) and conducted various types of research related to digital cellular mobile communications. From July 1992 to December 1999, he was with NTT Mobile Communications Network, Inc. (now NTT DoCoMo, Inc.), where he led a research group on wide-

band/broadband CDMA wireless access for IMT-2000 and beyond. Since January 2000, he has been with Tohoku University, Sendai, Japan, where he is a Full Professor of communications engineering with the Graduate School of Engineering. His research interests include area of wireless signal processing and networking including broadband wireless access, equalization, transmit/receive antenna diversity, MIMO, adaptive transmission, channel coding, etc. From October 1984 to September 1985, he was a U.K. SERC Visiting Research Fellow with the Department of Electrical Engineering and Electronics, Liverpool University, Liverpool, U.K. He is a VTS Distinguished Lecturer since 2011. He is a Fellow of Institute of Electronics, Information and Communication Engineers of Japan (IEICE). He was a recipient of the IEEE VEHICULAR TECHNOLOGY TRANSACTIONS Best Paper of the Year Award 1980 and 1990, the Avant Garde award 2000, the IEEE VTS Conference Chair Award 2014, the IEICE Achievement Award 2002, the IEICE Transactions Best Paper of the Year Award 1996, 1998, and again 2009, the Thomson Scientific Research Front Award 2004, the Ericsson Telecommunications Award 2008, the Telecom System Technology Award 2009, the Prime Minister Invention Prize 2010, the British Royal Academy of Engineering Distinguished Visiting Fellowship 2011, the KDDI Foundation Research Award 2012, the IEEE VTS Conference Chair Award 2014, and the C&C Prize 2014. He is listed in Highly Cited Researchers 2001.



H. Vincent Poor (S'72–M'77–SM'82–F'87) received the Ph.D. degree in EECS from Princeton University, Princeton, NJ, USA, in 1977. From 1977 to 1990, he was on the faculty of the University of Illinois at Urbana-Champaign, Urbana, IL, USA. Since 1990, he has been on the faculty of Princeton University, where he is the Michael Henry Strater University Professor and Dean of the School of Engineering and Applied Science. He has also held visiting positions at several universities, including most recently at Stanford and Imperial College. His

research interests include wireless networks and related fields. Among his publications in these areas is the recent book *Mechanisms and Games for Dynamic Spectrum Allocation* (Cambridge University Press, 2014). Dr. Poor is a member of the U.S. National Academy of Engineering and the U.S. National Academy of Sciences, and is a foreign member of Academia Europaea and the Royal Society. He is also a fellow of the American Academy of Arts and Sciences, the Royal Academy of Engineering (U.K.), and the Royal Society of Edinburgh. He received the Marconi and Armstrong Awards of the IEEE Communications Society in 2007 and 2009, respectively. Recent recognition of his work includes the 2014 URSI Booker Gold Medal, the 2015 EURASIP Athanasios Papoulis Award, and honorary doctorates from Aalborg University, Aalto University, HKUST, and the University of Edinburgh.