# Sojourn Time-Based Velocity Estimation in Small Cell Poisson Networks

Arvind Merwaday, *Student Member, IEEE*, İsmail Güvenç, *Senior Member, IEEE*, Walid Saad, *Senior Member, IEEE*, Abolfazl Mehbodniya, *Member, IEEE*, and Fumiyuki Adachi, *Fellow, IEEE*

***Abstract*—Due to the increasing density of small cells, mobility management in heterogeneous networks has become a challenging task. One key challenge facing the development of advanced mobility management techniques is the accurate estimation of the users' velocity. One simple way to estimate a user's velocity is via the use of sojourn time samples. In this letter, the Cramer-Rao lower bound (CRLB) for the sojourn time-based velocity estimation is analyzed. Stochastic geometry is used for the spatial modeling of small cells, and the CRLB is derived using the tools from estimation theory. An asymptotically unbiased velocity estimator is also derived. Our analysis shows that the sojourn time-based velocity estimation exhibit a lower CRLB compared to the CRLB of classical velocity estimation using handover count.**

***Index Terms*—5G, Cramer-Rao lower bound (CRLB), heterogeneous networks (HetNets), long term evolution (LTE), mobility state estimation, phantom cell, small cells, sojourn time.**

## I. Introduction

THE DEMAND for wireless data traffic has increased significantly in the past decade and is expected to continue to do so [1], [2]. Small cells are being rapidly deployed over existing macro-cellular networks in order to keep up with the increasing data traffic demands. Due to the random and dense deployment of small cells, mobility management has emerged as one of the most critical challenges, particularly when dealing with high-speed user equipment (UE). Consequently, an accurate estimate of a UE's velocity can help in efficient handling of the handovers and thereby providing a good quality of service to the UE. One simple way to estimate a UE's velocity is by using its *sojourn time* samples. The sojourn time is defined as the amount of time a mobile UE spends in a cell before it is handed over to another cell.

There are only a few studies available in the literature that investigate the use of sojourn time information for mobility management. In [3], considering a random waypoint model for UE mobility, expressions for the probability density function (PDF) and the expected value of the sojourn time are derived for homogeneous networks, assuming that the base stations (BSs) are distributed according to a Poisson point process (PPP). The authors in [4] estimated the velocity of a UE in two-tier cellular networks using the sojourn times of the UE, while assuming a classical Manhattan cell model [5]. To the best of authors' knowledge, there are no studies in the literature that assess the performance limits of sojourn time based velocity estimation.

Handover-count based method is another closely related approach, in which the UE's mobility is estimated using the number of handovers made by the UE in a predefined time window [6], [7]. Existing long term evolution (LTE) and LTE-Advanced technologies use the handover-count method for detecting the mobility state of a UE into three broad classes: low, medium, and high-mobility [6]. In [7], handover-count based velocity estimation is analyzed and the estimation accuracy is characterized through Cramer-Rao lower bound (CRLB). Despite the earlier work in [3], [4], [6], and [7], fundamental bounds for the accuracy of sojourn-time based UE velocity estimation have not been studied in the literature.

The main contribution of this letter is to introduce a novel technique for UE velocity estimation based on sojourn-time samples, and analyze its accuracy through CRLBs, when the density of small cell BSs (SBSs) is known. Our contributions will also include: 1) The derivation of a closed form expression for the joint PDF (JPDF) of $N$ sojourn-time samples; 2) the derivation, using the JPDF, of a closed form expression for the CRLB of velocity estimation; and 3) the derivation of an asymptotically unbiased velocity estimator whose accuracy is investigated for various UE speeds and SBS densities.

## II. System Model

Consider a two-tier heterogeneous network (HetNet) consisting of macro cell BSs (MBSs) and SBSs that use different frequency bands, as in the phantom cell architecture [8], [9]. The SBSs are randomly distributed according to a homogeneous PPP with intensity $\lambda$. Assuming that the SBSs are densely deployed, as expected in future wireless networks [10], the coverage of small cells can be modeled effectively using a Poisson-Voronoi tessellation as shown in Fig. 1. For simplicity, we assume that the UE travels with a constant velocity $v$ along a linear trajectory as shown in Fig. 1. We let $T$ be the sojourn time and $L$ the distance traveled within a cell. During the travel, we assume that the UE can detect its boundary crossings, and therefore can measure the sojourn times. Since the SBSs are randomly distributed, $T$ is a continuous random variable.

### A. Sojourn Time PDF

The PDF of the distance $L$ traveled by a UE in an arbitrary Poisson-Voronoi cell is derived in [11] as:

$$f_L(l) = \int_0^{\pi} \int_0^{\pi-\alpha} \frac{\pi^2 \lambda l^2 \rho_\alpha \rho_\beta \left(2\pi\lambda l^2 b_0^2 \rho_\alpha^2 - c_0\right)}{\sin(\alpha+\beta)} \times e^{-\pi\lambda l^2 V_2(\alpha,\beta)}\, \mathrm{d}\beta \mathrm{d}\alpha, \quad (1)$$

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A. Merwaday and İ. Güvenc are with the Department of Electrical and Computer Engineering, Florida International University, FL 33174 USA (e-mail: amerw001@fiu.edu; iguvenc@fiu.edu).

W. Saad with Wireless@VT, Bradley Department of Electrical and Computer Engineering, Virginia Tech, Blacksburg, VA 24061 USA, and also with the Department of Computer Engineering, Kyung Hee University, Kyung, South Korea (e-mail: walids@vt.edu).

A. Mehbodniya and F. Adachi are with Wireless Signal Processing and Networking Lab, Department of Communication Engineering, Tohoku University, Sendai 980-8579, Japan (e-mail: mehbod@mobile.ecei.tohoku.ac.jp; adachi@ecei.tohoku.ac.jp).

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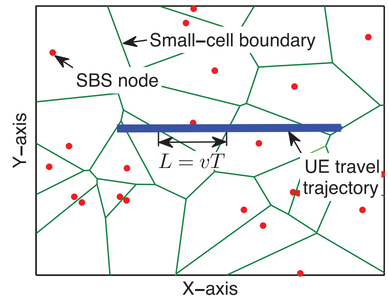


Fig. 1. Poisson-Voronoi tessellation of small cells and the UE travel trajectory.

$$\text{where } V_2(\alpha,\beta)=\left(1+\rho_\beta^2-2\rho_\beta\cos\alpha\right)\left(1-\frac{\beta}{\pi}+\frac{\sin 2\beta}{2\pi}\right)+\rho_\beta^2\left(1-\frac{\alpha}{\pi}+\frac{\sin 2\alpha}{2\pi}\right),$$

$$\rho_\alpha=\frac{\sin\alpha}{\sin(\alpha+\beta)},\quad \rho_\beta=\frac{\sin\beta}{\sin(\alpha+\beta)},$$

$$b_0=\frac{(\pi-\beta)\cos\beta+\sin\beta}{\pi},\text{ and } c_0=\frac{(\pi-\beta)+\sin\beta\cos\beta}{\pi}.$$

The details of (1) can be found in [11]. Therein, $L$ is denoted as the chord length. Next, the PDF of sojourn time will be expressed in terms of the chord length PDF $f_L(\cdot)$.

*Theorem 1:* Consider a Poisson-Voronoi tessellation formed by the SBSs distributed according to a PPP of intensity $\lambda$. A UE traveling along a straight line trajectory with constant velocity $v$ has a sojourn time PDF which can be expressed as

$$f_T(t;v)=vf_L(vt), \tag{2}$$

where the function $f_L(\cdot)$ is the chord length PDF given in (1).

*Proof:* The sojourn time is the ratio of distance traveled by the UE in a cell to the velocity of the UE, $T = L/v$. Using this relationship, we can express sojourn time distribution as,

$$F_T(t;v)=P(T\le t;v)=P\left(\frac{L}{v}\le t\right)=F_L(vt).$$

By differentiating the sojourn time distribution with respect to $t$, we can obtain the sojourn time PDF as,

$$f_T(t;v)=\frac{\partial F_T(t;v)}{\partial t}=\frac{\partial F_L(vt)}{\partial t}=vf_L(vt).$$ ■

The plots of sojourn time PDF for different SBS densities are shown in Fig. 2 for a UE velocity $v = 60$ km/h.

## III. CRLB for Velocity Estimation

In this section, we derive the CRLB for velocity estimation using the sojourn time samples obtained during a fixed time interval $T_w$. The number of sojourn time samples $N$ is a random variable which is equal to the number of handovers during the time interval $T_w$. We first derive JPDF of the $N$ sojourn time samples, and then use this JPDF to derive the CRLB.

Let $\boldsymbol{T} = [T_n : n = 0, 1, ..., N-1]$ be a vector of $N$ sojourn time samples. Using extensive simulations, we have found that the correlation between the sojourn time samples of adjacent cells is less than 5.9%, which is a reasonably small value. Hence, for mathematical tractability, we assume that the sojourn time samples are independent. Therefore, the JPDF of $N$ sojourn time samples will be:

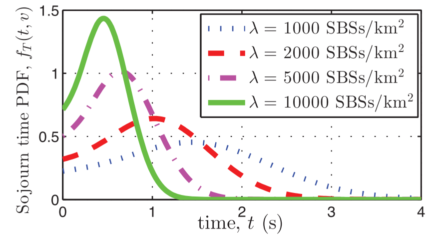


Fig. 2. Sojourn time PDF for different SBS densities, with $v = 60$ km/h.

$$f_{\boldsymbol{T}}(\boldsymbol{t};v)=\prod_{n=0}^{N-1}f_T(t_n;v)=v^N\prod_{n=0}^{N-1}f_L(l_n), \tag{3}$$

where $\boldsymbol{t} = [t_n \ge 0 : n = 0, 1, ..., N-1]$ is a vector parameter, and $l_n = vt_n$ is the chord length corresponding to the $n$th sojourn time sample. A pre-requisite for the CRLB derivation is that the JPDF must satisfy the regularity condition as will be shown next.

*Lemma 1:* The JPDF of $N$ sojourn time samples which is shown in (3) satisfies the following regularity condition:

$$E\left[\frac{\partial \log f_{\boldsymbol{T}}(\boldsymbol{t};v)}{\partial v}\right]=0 \qquad \text{for all } v,$$

with $E[\cdot]$ being the expectation operator.

*Proof:* See Appendix A. ■

Since the JPDF satisfies the regularity condition, the CRLB for velocity estimation can be obtained using

$$\mathrm{var}(\hat{v})\ge\frac{1}{E\left[\left(\frac{\partial \log f_{\boldsymbol{T}}(\boldsymbol{t};v)}{\partial v}\right)^2\right]}, \tag{4}$$

where var(·) is the variance operator. Next, we show that the CRLB in (4) can be expressed in closed form.

*Theorem 2:* The CRLB for velocity estimation of a UE using the sojourn time samples measured within a time interval $T_w$ is given by:

$$\mathrm{var}(\hat{v})\ge\frac{\pi v}{17.52T_w\sqrt{\lambda}}. \tag{5}$$

*Proof:* See Appendix B. ■

## IV. Sojourn Time Based Velocity Estimator

In this section, we derive an estimator which takes $N$ sojourn time samples as the input and estimates the UE velocity, provided that the SBS density $\lambda$ is known. We start deriving the estimator by using mean sojourn time given by:

$$E[T]=E\left[\frac{L}{v}\right]=\frac{E[L]}{v}=\frac{\pi}{4v\sqrt{\lambda}}. \tag{6}$$

The last step in (6) was obtained through the expression for mean cross-sectional length which is derived in [12]. By rearranging the terms in (6) and using $E[T]=1/N\sum_{n=0}^{N-1}T_n$, we can obtain a velocity estimator as

$$\hat{v}=\frac{\pi N}{4\sqrt{\lambda}\sum_{n=0}^{N-1}T_n}. \tag{7}$$

An estimator is said to be unbiased if its expected value is same as the true value of the parameter being estimated.

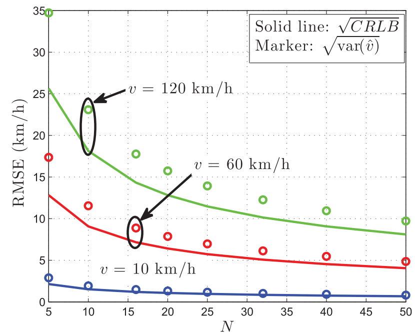


Fig. 3. CRLB and estimator variance versus $N$, for different UE velocities.

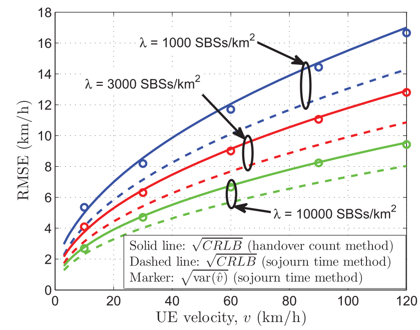


Fig. 4. Comparison between the RMSEs of handover count and sojourn time methods for the velocity estimation of a UE.

Unbiasedness ensures that, on the average, the estimator yields the true value of the unknown parameter. To check if (7) is an unbiased estimator, we derive the expectation as

$$E[\hat{v}] = \frac{\pi N}{4\sqrt{\lambda}} E\left[\frac{1}{\sum_{n=0}^{N-1} T_n}\right] \neq v. \tag{8}$$

Since $E[\hat{v}] \neq v$, $\hat{v}$ in (7) is not an unbiased estimator. However, as $N$ tends to infinity, we get,

$$\lim_{N\to\infty} E[\hat{v}] = \frac{\pi}{4\sqrt{\lambda}} E\left[\frac{1}{\lim_{N\to\infty} \frac{1}{N}\sum_{n=0}^{N-1} T_n}\right], \\ = \frac{\pi}{4\sqrt{\lambda}} E\left[\frac{1}{E[T_n]}\right] = \frac{\pi}{4\sqrt{\lambda}} E\left[\frac{4v\sqrt{\lambda}}{\pi}\right] = v. \tag{9}$$

Therefore, the estimator in (7) is asymptotically unbiased.

The variance of the velocity estimator can be expressed as

$$\text{var}(\hat{v}) = E\left[\hat{v}^2\right] - (E[\hat{v}])^2, \\ = \left(\frac{\pi N}{4\sqrt{\lambda}}\right)^2 \left(E\left[\left(\frac{1}{\sum_{n=0}^{N-1} T_n}\right)^2\right] - \left(E\left[\frac{1}{\sum_{n=0}^{N-1} T_n}\right]\right)^2\right). \tag{10}$$

## V. Numerical Results

The plots of CRLB and estimator variance are shown in Fig. 3 for different $N$ and $v$ values, with $\lambda = 3000$ SBSs/km$^2$. The CRLB plots were obtained using (5), while the plots of estimator variance were obtained through numerical computation of (10). It can be observed that the root mean squared error (RMSE) increases with the increasing UE velocity. On the other hand, the RMSE decreases as the number of sojourn time samples increases. Also, as $N$ increases, the difference between the CRLB and the estimator variance is reduced.

Next, we compare the CRLBs of the handover-count based and the sojourn-time based velocity estimation methods. The handover-count based velocity estimation method is analyzed in [7], where a UE's velocity is estimated based on the number of small cells traversed by the UE during a predefined time interval. In [7], CRLB for the handover-count based velocity estimation is derived to be

$$\text{var}(\hat{v}_{\text{ho}}) \geq \left[\frac{1}{2}\left(\frac{0.41 T_{\text{w}}\sqrt{\lambda}}{\sigma^2}\right)^2 + \left(\frac{\mu}{v\sigma}\right)^2\right]^{-1}, \tag{11}$$

where $\mu = 4vT_{\text{w}}\sqrt{\lambda}/\pi$, and $\sigma^2 = 0.07 + 0.41vT_{\text{w}}\sqrt{\lambda}$. The CRLBs of sojourn-time method and handover-count method are compared in Fig. 4. The measurement time interval is fixed to $T_{\text{w}} = 12$ seconds for both the methods. It can be observed in Fig. 4 that the CRLB of sojourn-time method is smaller than the CRLB of handover-count method, for all the UE velocities and the densities of SBSs. This is because the handover count method estimates the UE velocity by making use of only the handover count information. In contrast, the sojourn time method uses both handover count information and sojourn time information for the velocity estimation. Hence, theoretically the sojourn time method provides better accuracy.

The variance of sojourn-time based velocity estimator is also plotted in Fig. 4 in comparison with the CRLBs. It can be seen that the estimator variance does not attain the CRLB of sojourn-time method. Deriving an *efficient estimator* that can attain the CRLB of sojourn-time method is a challenging task due to the complexity of the sojourn-time PDF expression, and it is left as a future work.

## VI. Conclusion

In this letter, we have derived a closed-form expression for the CRLB of sojourn-time based velocity estimation, which is a function of small cell density, UE velocity, and measurement time interval. We have also derived an asymptotically unbiased velocity estimator. Theoretically, the sojourn-time based velocity estimation has been shown to be more accurate than the handover-count based velocity estimation. Future work includes incorporating more realistic mobility model, developing efficient UE velocity estimator that uses sojourn time samples, and modeling uncertainties in the boundary detection of small cells.

## Appendix A
## Proof for Lemma 1: Regularity Condition

By taking natural logarithm of the JPDF in (3) and then differentiating with respect to $v$, we get

$$\frac{\partial \log f_{\boldsymbol{T}}(\boldsymbol{t}; v)}{\partial v} = \frac{N}{v} + \sum_{n=0}^{N-1} \frac{t_n}{f_L(l_n)} \frac{\partial f_L(l_n)}{\partial l_n}. \tag{12}$$

The differential term in (12) can be derived to be

$$\frac{\partial f_L(l_n)}{\partial l_n} = \frac{2}{l_n}\left(f_L(l_n) + f'_L(l_n)\right), \tag{13}$$

where,

$$f_L'(l) = \int_0^{\pi}\int_0^{\pi-\alpha}\left(\frac{2\pi^3\lambda^2 l^4\rho_\alpha^3\rho_\beta b_0^2\left(1-\pi\lambda l^2 V_2(\alpha,\beta)\right)}{\sin(\alpha+\beta)} + \frac{\pi^3\lambda^2 l^4\rho_\alpha\rho_\beta c_0 V_2(\alpha,\beta)}{\sin(\alpha+\beta)}\right)e^{-\pi\lambda l^2 V_2(\alpha,\beta)}\,\mathrm{d}\beta\mathrm{d}\alpha.$$

By substituting (13) into (12), we get,

$$\frac{\partial \log f_{\boldsymbol{T}}(\boldsymbol{t};v)}{\partial v} = \frac{1}{v}\left(3N + 2\sum_{n=0}^{N-1}\frac{f_L'(l_n)}{f_L(l_n)}\right). \tag{14}$$

Given $N$, to check whether the JPDF satisfies the regularity condition, we take the expectation of (14) w.r.t. $L$ as,

$$E\left[\frac{\partial \log f_{\boldsymbol{T}}(\boldsymbol{t};v)}{\partial v}\right] = \frac{1}{v}E_L\left(3N + 2\sum_{n=0}^{N-1}\frac{f_L'(l_n)}{f_L(l_n)}\right), = \frac{1}{v}\left(3N + 2\sum_{n=0}^{N-1}E_L\left[\frac{f_L'(l_n)}{f_L(l_n)}\right]\right). \tag{15}$$

The expectation function w.r.t. $L$ in (15) can be derived as

$$E_L\left[\frac{f_L'(l_n)}{f_L(l_n)}\right] = \int_0^\infty f_L'(l)\,\mathrm{d}l = -1.5. \tag{16}$$

The last step in (16) was obtained by numerically evaluating the integral $\int_0^\infty f_L'(l)\,\mathrm{d}l$ in MATLAB with an error tolerance of $10^{-6}$. By substituting (16) into (15), we get,

$$E\left[\frac{\partial \log f_{\boldsymbol{T}}(\boldsymbol{t};v)}{\partial v}\right] = \frac{3N + 2(-1.5)N}{v} = 0. \tag{17}$$

Since the expectation function in (17) equates to zero irrespective of the values of $N$ and $v$, the JPDF in (3) satisfies the regularity condition for all $N$ and $v$.

## Appendix B
## Proof for Theorem 2: CRLB Derivation

By squaring (14) and taking the expectation, we get,

$$E\left[\left(\frac{\partial \log f_{\boldsymbol{T}}(\boldsymbol{t};v)}{\partial v}\right)^2\right] = \frac{1}{v^2}E\left[9N^2 + 4\left(\sum_{n=0}^{N-1}\frac{f_L'(l_n)}{f_L(l_n)}\right)^2 + 12N\sum_{n=0}^{N-1}\frac{f_L'(l_n)}{f_L(l_n)}\right], = \frac{1}{v^2}E_N\left(9N^2 + 4\sum_{n=0}^{N-1}E_L\left[\left(\frac{f_L'(l_n)}{f_L(l_n)}\right)^2\right] + 8\sum_{m<n}E_L\left[\frac{f_L'(l_n)f_L'(l_m)}{f_L(l_n)f_L(l_m)}\right] + 12N\sum_{n=0}^{N-1}E_L\left[\frac{f_L'(l_n)}{f_L(l_n)}\right]\right). \tag{18}$$

The first two expectations w.r.t. $L$ in (18) can be derived as follows:

$$E_L\left[\left(\frac{f_L'(l_n)}{f_L(l_n)}\right)^2\right] = \int_0^\infty\left(\frac{f_L'(l)}{f_L(l)}\right)^2 f_L(l)\,\mathrm{d}l = 3.345. \tag{19}$$

The last step in (19) was obtained by numerically evaluating the integral in MATLAB with an error tolerance of $10^{-6}$. Next,

$$E_L\left[\frac{f_L'(l_n)f_L'(l_m)}{f_L(l_n)f_L(l_m)}\right] = \iint_0^\infty\frac{f_L'(l_n)f_L'(l_m)}{f_L(l_n)f_L(l_m)}f_L(l_n)f_L(l_m)\mathrm{d}l_n\mathrm{d}l_m, = \int_0^\infty f_L'(l_m)\left(\int_0^\infty f_L'(l_n)\,\mathrm{d}l_n\right)\mathrm{d}l_m.$$

Substituting (16) into the above equation, we get,

$$E\left[\frac{f_L'(l_n)f_L'(l_m)}{f_L(l_n)f_L(l_m)}\right] = -1.5\int_0^\infty f_L'(l_m)\,\mathrm{d}l_m = 2.25. \tag{20}$$

By substituting (16), (19) and (20) into (18), we get,

$$E\left[\left(\frac{\partial \log f_{\boldsymbol{T}}(\boldsymbol{t};v)}{\partial v}\right)^2\right] = \frac{4.38}{v^2}E[N]. \tag{21}$$

The expectation of $N$ in (21) is basically the mean number of handovers, the expression for which is derived in [3] as

$$E[N] = 4vT_{\mathrm{w}}\sqrt{\lambda}/\pi. \tag{22}$$

By substituting (22) into (21), we get,

$$E\left[\left(\frac{\partial \log f_{\boldsymbol{T}}(\boldsymbol{t};v)}{\partial v}\right)^2\right] = \frac{4.38}{v^2}\frac{4vT_{\mathrm{w}}\sqrt{\lambda}}{\pi} = \frac{17.52T_{\mathrm{w}}\sqrt{\lambda}}{\pi v}.$$

By substituting the above equation into (4), we obtain the CRLB as expressed in (5). This completes the proof.

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