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A distributed learning–based user association for heterogeneous networks

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Abstract
The coexistence of various base stations (BSs) in heterogeneous networks (HetNets) has emerged as a promising approach to meet the ever increasing network capacity. In these networks, one of the important issues is the problem of associating user equipments (UEs) to BSs. In this paper, we investigate the UE association (UEA) problem in heterogeneous networks and propose a load-aware UEA mechanism based on the BSs' estimated load and signal-to-interference-and-noise ratio. The proposed mechanism can capture the trade-off between UE's quality-of-service requirement and delay. We model this strategic UE-BS association as a noncooperative game. To solve the game, we develop a fully distributed algorithm inspired by machine learning techniques, whereby the proposed UEA scheme corresponds to a Markov chain. In the proposed scheme, each UE senses its environment and decides which BS to select based on the satisfaction technique. Therefore, it achieves a high level of satisfaction for UEs. Furthermore, use of historical information helps UEs select BSs with better long-term performance. Simulation results show that the proposed mechanism reduces fractional transfer time and the number of unsatisfied UEs, respectively, up to about 46% and 52.3% and improves BS throughput up to about 15.4% compared to a benchmark algorithm that is based on the received signal strength and the BS's estimated load.

1 | INTRODUCTION

The tremendous growth in mobile data traffic and the unprecedented increase in smartphone penetration pose significant challenges to existing cellular networks. Accordingly, the deployment of heterogeneous networks (HetNets) has been recognized as an essential technology to deal with the aforementioned issues for next-generation wireless networks.[1-4] Therefore, cellular networks are gravitating toward increasing heterogeneity, especially through the deployment of small-cell base stations (SBSs).[5] The examples of SBSs are microcells, picocells, and femtocells, which differ primarily in maximum transmit power, coverage range, backhaul, ease of deployment, and cost of installation and maintenance.[6-8]

In HetNets, macrocell base stations (MBSs) and SBSs coexist to meet the ever increasing network capacity demand and improve the energy efficiency by transmitting at low power.[9-11] Moreover, SBSs allow operators to offload traffic load from MBSs and enhance UE performance,[12,13] thereby boosting the spectral efficiency (SE). Unlike MBSs, SBSs can be deployed in an ad hoc and unplanned manner by either the operator or the user, consequently enabling flexible and low-cost deployment.[14]

In HetNets, one of the main problems is user equipment association (UEA), in which a user equipment (UE) is associated with a particular base station (BS). In these networks, the cochannel interference and different transmission power levels of BSs may lead to an unbalanced distribution of traffic load.[15] User equipment association plays an important role in balancing load among BSs and suppressing unnecessary interference. Thus, it significantly impacts the network performance by offloading the UEs from the overloaded BSs to the lightly loaded BSs.[16,17] In this regard, substantial research has been devoted to investigating the UEA problem in HetNets. In the UEA problem, 1 decision is made according to the quality of the radio link and the availability of resource. In current networks, the UEA policies based on the received signal strength is a common approach, where a UE typically associates to the serving BS that offers the best received signal strength indication (RSSI).[18] However, the RSSI-based UEA schemes may lead to major drawbacks such as load imbalance and result in decreasing the SE. Thus, the UE's throughput is dependent on the BS's load.[19] Accordingly, it is desirable to consider intelligent mechanisms that explicitly account for the traffic load conditions of BSs. In the work of Bejerano et al,[20] to provide fair service to UEs, which is obtained when load balancing is achieved, an association scheme is proposed. In the work of Zhou et al,[21] a load-aware UEA scheme for HetNets, which is formulated as a network-wide weighted utility maximization problem, is developed. To solve the problem, a gradient descent method is adopted. A UEA scheme based on the BS's estimated load is considered in the work of Samarakoon et al,[22] in which each BS broadcasts its estimated load through a beacon signal in downlink transmission. In the work of Ye et al,[23] to implement UE-BS association, a logarithmic utility maximization problem is developed, in which a fractional association assumption is used to convert the problem into a convex optimization problem. In the work of Ali et al,[24] a network-centric approach using cell range expansion (CRE) for UEA is proposed, in which BSs take decisions on selecting CRE, and then, UEs follow the association rule.

Based on the signaling overhead, UEA schemes can be classified into centralized and distributed approaches. In centralized approaches, there is a central entity that decides which UE to associate with which BS, by using the collected information.[16] These approaches can provide optimal UE-BS association for networks at the cost of signaling exchange, and thus, it may not be feasible for dense UE and/or BS deployment. In contrast with centralized approaches, in distributed UEA schemes, UEs and BSs make UEA decisions independently and in a distributed manner without the need for a central entity. Therefore, distributed UEA approaches are more attractive due to their lower implementation complexity and lower signaling overhead.[25]

1.1 | Contributions and organization

The main contribution of this paper is in proposing a load-aware UEA mechanism for downlink in a HetNet scenario. Game theory is used as a fundamental tool for decision-making, in the process of UE interactions. Since our focus is on distributed UE-centric approaches, we model the UEA problem as a noncooperative game, in which UEs are considered as players. We assume that UEs do not have knowledge of other UEs' actions and payoffs. Therefore, the proposed UEA scheme is performed in a fully distributed manner without the need to exchange information among UEs. To solve the game, we propose a satisfaction-based learning procedure, in which UEs try to maximize their own satisfaction and which results in minimizing the number of UEs in outages. The UEs' outages depend on several factors such as resource utilization in BS and UE distribution. In the proposed mechanism, a player that has been satisfied by its payoff would not change its strategy, whereas an unsatisfied player may decide to change its strategy. Then, the unsatisfied UEs should decide to associate with other BSs and perform new associations according to the proposed scheme. The proposed mechanism achieves a desirable trade-off between load balancing and throughput by selecting the serving BS based on the BS's estimated load and received signal-to-interference-and-noise ratio (SINR) at the location of UEs. In this respect, each BS periodically broadcasts its estimated load through a control message.[22,26]

Our adaptive proposed scheme is based on the learning algorithm. In this regard, we use historical information on the UE's performance instead of completely relying on the instantaneous measurement. The UEs assign a set of probability distributions with the serving BSs. Then, the probability distributions are updated using a learning rule, in which the UEs perform the updates by assessing their payoffs and satisfying their quality-of-service (QoS) requirements. Furthermore, it allows a trade-off between exploitation and exploration with selecting the greedy and nongreedy strategies, respectively. Exploitation is used to maximize the expected payoff on the 1-shot play, whereas exploration may lead to greater reward in the long run and make better strategy selection in the succeeding runs.[27] The challenge of the proposed approach lies in effectively balancing load among BSs while simultaneously improving the QoS of UEs. By combining these goals, the proposed scheme is seen as a promising approach to overcoming this challenge. The proposed solution has several advantages. First, it is a distributed algorithm, in which each UE can independently select its serving BS by interacting with the environment. Thus, it can reduce the signaling overhead in the network. Second, a decision taken by a UE is independent of the number of UEs in the network. Therefore, it is a suitable approach for when the number of UEs varies over time in the network. Since there is no centralized controller, it can enhance the robustness of the network against failures and attacks. Furthermore, historical information can be used to select strategies with better long-term performance.

TABLE 1 List of notations

Symbol	Semantics
$\mathcal{B}$	BSs
$\mathcal{B}_M$	MBSs
$\mathcal{B}_S$	SBSs
$\mathcal{K}$	UEs
$\mathcal{O}$	Unsatisfied UEs
ρ_b	Load of BS b
$\hat{\rho}_b$	Estimated load of BS b
$\gamma_{b,k}$	Received SINR from BS b at receiver of UE k
$R_{b,k}$	Achievable transmission rate of UE k from BS b
P_b	Transmission power of BS b
$\boldsymbol{x}_b$	Location coordinates of BS b
$\boldsymbol{x}_k$	Location coordinates of UE k
$g(\boldsymbol{x}_b, \boldsymbol{x}_k)$	Channel gain between BS b and UE k
σ^2	Noise power
I_M	Interference caused by MBSs
I_S	Interference caused by SBSs
$\mathcal{L}_b$	Coverage area of BS b
$\boldsymbol{A}$	Association matrix
$a_{b,k}$	Association relation between UE k and BS b
s_k	Action of player k
$\boldsymbol{s}_{-k}$	Actions of all players other than player k
$\boldsymbol{s}$	Action profile
$\mathcal{S}_k$	Action set of player k
$\mathcal{U}_k$	Payoff of player k
π_k	Probability distribution which player k assigns to its action set
λ_k	Learning rate of UE k

The rest of this paper is organized as follows. Section 2 presents our system model including the network layout and the load of the BS. In Section 3, we present the proposed UEA mechanism. The simulation results are presented in Section 4. Finally, conclusions are drawn in Section 5.

Notations. The regular and boldface symbols refer to scalars and matrices, respectively. $||\boldsymbol{x}||$ and $||A||_0$ denote the Euclidean norm of vector $\boldsymbol{x}$ and the l_0-norm, ie, number of nonzero elements in matrix A, respectively. A^{T} represents the transpose of matrix A. The cardinality of set $\mathcal{A}$ is denoted by $|\mathcal{A}|$. The function $\mathbb{1}_{(event)}$ denotes the indicator function, which is equal to 1 if *event* is true, and 0 otherwise. The set of real numbers is denoted by $\mathbb{R}$. For any finite set $\mathcal{S}$, the set of all probability distributions over it is denoted by $\triangle(\mathcal{S})$.

2 | SYSTEM MODEL

We consider the downlink transmission of a 2-tier HetNet consisting of a set of MBSs, $\mathcal{B}_{\mathrm{M}}$, and a set of SBSs, $\mathcal{B}_{\mathrm{S}}$, in which MBSs and SBSs constitute tier 1 and tier 2, respectively. The set of UEs and the set of the total BSs are denoted by $\mathcal{K} = \{1, \dots, |\mathcal{K}|\}$ and $\mathcal{B} = \{1, \dots, |\mathcal{B}|\}$, ie, $\mathcal{B} = \mathcal{B}_{\mathrm{M}} \cup \mathcal{B}_{\mathrm{S}}$, respectively. We assume that BSs transmit on the same frequency spectrum, ie, cochannel deployment. Moreover, we assume an open-access scheme for BSs in the network, ie, UEs are allowed to associate with any tier's BSs. For the convenience of reading, we summarize the terminologies and notations frequently used throughout this paper in Table 1.

Without loss of generality, we assume that for each macrocell coverage area, an MBS is located at the center of the area, and SBSs and UEs are uniformly distributed within the coverage of the MBS, as illustrated in Figure 1. Each BS $b \in \mathcal{B}$ is characterized by its coverage area $\mathcal{L}_b$, ie, the area covered by the transmission range of BS b. Let P_b and $g(\boldsymbol{x}_b, \boldsymbol{x}_k)$ denote the transmit power of BS b, and the channel gain between BS b located at $\boldsymbol{x}_b \in \mathbb{R}^2$ and UE k located at $\boldsymbol{x}_k \in \mathbb{R}^2$, respectively. Here, $g(\boldsymbol{x}_b, \boldsymbol{x}_k) = 10^{-\frac{L(\boldsymbol{x}_b, \boldsymbol{x}_k)}{10}}$, in which $L(\boldsymbol{x}_b, \boldsymbol{x}_k)$ is the path loss between BS b and UE k in decibels. In a HetNet, CRE bias can help

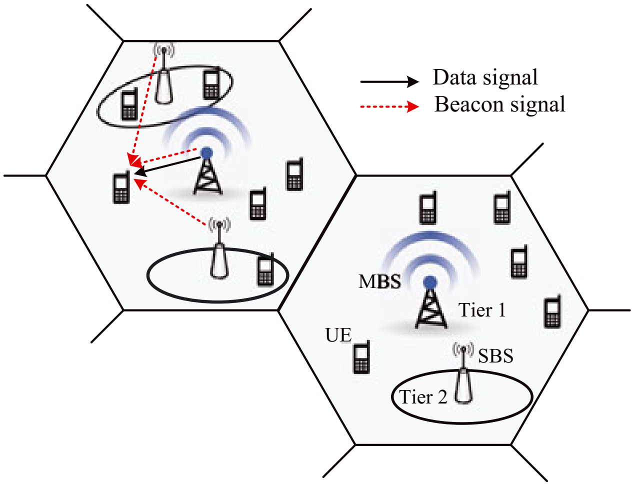

FIGURE 1 A typical example of a 2-tier HetNet

offload UEs from one network tier to another. Therefore, BSs can adaptively optimize CRE based on the network's conditions.[24] Since, in this paper, our focus is on the UE-centric approach, we assume that the BSs have bias 1. Therefore, the received SINR from MBS m at the receiver of UE k is given by

$$\gamma_{m,k}(\boldsymbol{x}_m, \boldsymbol{x}_k) = \frac{P_m g(\boldsymbol{x}_m, \boldsymbol{x}_k)}{\underbrace{\sum_{m' \in \mathcal{B}_{\mathrm{M}} \setminus m} P_{m'} g(\boldsymbol{x}_{m'}, \boldsymbol{x}_k)}_{\mathrm{I_M}} + \underbrace{\sum_{s' \in \mathcal{B}_{\mathrm{S}}} P_{s'} g(\boldsymbol{x}_{s'}, \boldsymbol{x}_k)}_{\mathrm{I_S}} + \sigma^2}, \tag{1}$$

where σ^2 is the additive white Gaussian noise power at the receiver of UEs and assumed to be constant for all UEs. The interference caused by MBSs and SBSs is denoted by $\mathrm{I_M}$ and $\mathrm{I_S}$, respectively. The received SINR from SBS s at the receiver of UE k is given by

$$\gamma_{s,k}(\boldsymbol{x}_s, \boldsymbol{x}_k) = \frac{P_s g(\boldsymbol{x}_s, \boldsymbol{x}_k)}{\underbrace{\sum_{m' \in \mathcal{B}_{\mathrm{M}}} P_{m'} g(\boldsymbol{x}_{m'}, \boldsymbol{x}_k)}_{\mathrm{I_M}} + \underbrace{\sum_{s' \in \mathcal{B}_{\mathrm{S}} \setminus s} P_{s'} g(\boldsymbol{x}_{s'}, \boldsymbol{x}_k)}_{\mathrm{I_S}} + \sigma^2}. \tag{2}$$

From Shannon's capacity formula, the achievable transmission rate of UE k from MBS m is given by

$$R_{m,k}(\boldsymbol{x}_m, \boldsymbol{x}_k) = \omega \log_2 \left(1 + \gamma_{m,k}(\boldsymbol{x}_m, \boldsymbol{x}_k)\right), \tag{3}$$

where ω is the total bandwidth. The achievable transmission rate of UE k from SBS s is given by

$$R_{s,k}(\boldsymbol{x}_s, \boldsymbol{x}_k) = \omega \log_2 \left(1 + \gamma_{s,k}(\boldsymbol{x}_s, \boldsymbol{x}_k)\right). \tag{4}$$

For each BS, a set of orthogonal resource blocks in the time domain is considered to serve the associated UEs. Let $\lambda(\boldsymbol{x}_k)$ and $\frac{1}{\mu(\boldsymbol{x}_k)}$ be the mean arrival rate and the mean packet size of UE k, respectively. Thus, the fraction of time required to serve the traffic load from BS b to the location of UE $k \in \mathcal{L}_b$ is defined as follows: $\varrho_b(\boldsymbol{x}_k) = \frac{\lambda(\boldsymbol{x}_k)}{\mu(\boldsymbol{x}_k) R_{b,k}(\boldsymbol{x}_b, \boldsymbol{x}_k)}$. Consequently, the load of BS b is given by

$$\rho_b = \min \left\{ \sum_{\boldsymbol{x}_k \in \mathcal{L}_b} \varrho_b(\boldsymbol{x}_k), 1 \right\}. \tag{5}$$

When the load of a BS exceeds 1, some UEs associated with it may experience a sudden drop in their received throughput and cannot get the radio service, which is referred to as UE outages.[28] In this regard, to cope with overloading the BS, the following procedure can be applied:

1. **Sort UEs**: The UEs associated with an overloaded BS are sorted in descending order based on the system load density, ie, $\varrho_b(\boldsymbol{x}_k) = \frac{\lambda(\boldsymbol{x}_k)}{\mu(\boldsymbol{x}_k) R_{b,k}(\boldsymbol{x}_b, \boldsymbol{x}_k)}$.

2. **Drop the UE**: The UE with the highest system load density is dropped.
3. **Stop**: If the load of the BS is less than the maximum load, then stop; otherwise, go to step 2.

Therefore, the load of BSs is an important parameter from the UE's perspective. Using Equation (5), the total number of flows, which is proportional to the expected delay, can be calculated as follows: $\sum_{b\in\mathcal{B}}\frac{\rho_b}{1-\rho_b}$.[29]

3 | TOWARD PROPOSED SATISFACTION-BASED LEARNING USER EQUIPMENT ASSOCIATION

In this section, we present our proposed UEA approach. At each time t, the HetNet can be configured dynamically depending on the BSs' load vector, $\boldsymbol{\rho}(t) = [\rho_b(t)]_{|\mathcal{B}|\times 1}$, and the association matrix between the UEs and BSs, $\boldsymbol{A}(t)$.

3.1 | Preliminaries

In order to study the behavior of individuals in a strategic scenario, game theory is considered as a useful tool. Based on the ability to communicate between players, games can be classified into 2 categories: noncooperative games and coalitional games or cooperative games.[30] Here, we introduce some game-theoretic notions to model the problem as a noncooperative game. A Game consists of a set of players, a set of strategies, and payoffs for players, which can be defined as follows:

- *Player*: A player is the decision maker in the game.
- *Strategy (or action)*: A strategy is the decision taken by the player.
- *Payoff*: The utility that each player realizes for a particular strategy.

3.2 | The proposed user equipment association

In HetNets, associating UEs to BSs is identified as a major issue. In a conventional UEA scheme, each UE is associated with the BS that offers the highest RSSI. Since SBSs transmit with low power levels and are generally less loaded compared to MBSs, associating UEs to BSs just according to the RSSI metric is not suitable. Moreover, these schemes may lead to major drawbacks such as load imbalance and result in decreasing the network performance. In such context, one challenge for UEs is to achieve maximum throughput by choosing appropriate BSs. Thus, it is desirable to consider intelligent mechanisms that explicitly account for the traffic load conditions of BSs. In the following, we propose our UEA mechanism.

In the network, the UEs periodically assess their actual performance. Thus, if they are not satisfied with their current associations, they should decide to seek new associations. Furthermore, we assume that each UE is associated with, at most, 1 BS at each time t. We define an association matrix $\boldsymbol{A} \in \{0,1\}^{|\mathcal{B}|\times|\mathcal{K}|}$ where each single binary element $a_{b,k}$ in matrix $\boldsymbol{A}$ represents the association relation between UE k and BS b such that

$$a_{b,k} = \begin{cases} 1, & \text{if UE } k \text{ is associated with BS } b, \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

We propose a distributed learning approach for the UEA problem. In this approach, the UEs assign probability distributions to the serving BSs in order to associate to the appropriate BSs such that load balancing is achieved among BSs. As a result, it improves the SE of the network. We note that the proposed UEA is executed in a fully distributed manner without any need for information exchange among UEs.

Now, our goal is to develop a fully distributed algorithm that minimizes the number of unsatisfied UEs. Our UEA problem is thus given by

$$\min_{\boldsymbol{A}\in\{0,1\}^{|\mathcal{B}|\times|\mathcal{K}|}} \quad |\mathcal{K}| - ||\boldsymbol{A}^{\mathrm{T}}.\vec{\boldsymbol{I}}||_0, \tag{7a}$$

$$\text{subject to} \quad \sum_{b\in\mathcal{B}} a_{b,k} \le 1 \qquad \forall k \in \mathcal{K}, \tag{7b}$$

$$a_{b,k} \in \{0,1\} \qquad \forall b \in \mathcal{B}, \forall k \in \mathcal{K}, \tag{7c}$$

where $\vec{\mathbf{1}} \in \mathbb{R}^{|\mathcal{B}|}$ denotes the all-1's vector. The constraints in Equation (7b) force every UE to be connected to, at most, 1 BS. Note that the association problem in Equations (7a) and (7b) is a combinatorial problem and, thus, intractable if $|\mathcal{B}| \times |\mathcal{K}|$ is large. In this regard, we model the UEA problem as a noncooperative game where the players are the UEs and the actions (or strategies) available for each player are the set of BSs. The objective of each player is to maximize its payoff. The satisfaction form of the game is expressed as $\mathcal{G} = \langle \mathcal{K}, \{\mathcal{S}_k\}_{k\in\mathcal{K}}, \{f_k\}_{k\in\mathcal{K}} \rangle$, with the following variable definitions:

- $\mathcal{K}$ is the set of players.
- $\mathcal{S}_k$ is the set of available actions for player k , which comprises the set of BSs $\mathcal{B}$. The set of actions of all players is denoted by $\mathcal{S} = \mathcal{S}_1 \times \cdots \times \mathcal{S}_{|\mathcal{K}|}$. We also denote by $\boldsymbol{s} = (s_1, \dots, s_{|\mathcal{K}|}) \in \mathcal{S}$ an action profile composed of the action of all players where $s_k \in \mathcal{S}_k$ is an action of player k.
- f_k denotes the correspondence of player k which determines the set of actions of player k that allows its satisfaction. Here, $f_k = \{s_b \in \mathcal{S}_b : \mathcal{U}_k(s_k, \boldsymbol{s}_{-k}) = 1\}$, where $\mathcal{U}_k(s_k, \boldsymbol{s}_{-k}) : \mathcal{S} \to \{-1, 1\}$ is the payoff function of player k. This payoff is viewed as the gain observed by the player, in which $\mathcal{U}_k(s_k, \boldsymbol{s}_{-k}) = -1$ if player k is not satisfied by choosing action s_k, and $\mathcal{U}_k(s_k, \boldsymbol{s}_{-k}) = 1$ otherwise. Here, $\boldsymbol{s}_{-k} = (s_1, \dots, s_{k-1}, s_{k+1}, \dots, s_{|\mathcal{K}|})$ denotes the actions of all players other than player k.

Please note that the player's payoff depends not only on his own action but also on the actions played by the other players. This is mainly due to the fact that the player's satisfaction and/or outages depend on the load of the BS selected by the player and the other associated UEs.

At each time t, the UEs assess their actual throughput, and if they are not satisfied, they must decide on the UEA process. Moreover, they are learning to play an efficient action profile that yields significant network performance. Our proposed approach is a satisfaction-based learning process that allows the UEs to achieve satisfaction equilibria in finite time. In this respect, as long as a UE is able to satisfy its QoS condition and is not dropped by its current serving BS, it has no incentive to change its current action. The learning process is described in the following section. To reduce the number of UE outages, we consider the load of BSs in the learning procedure.

3.2.1 | The first iteration

Each player $k \in \mathcal{K}$ assigns a probability distribution $\pi_k = (\pi_{k,1}, \dots, \pi_{k,|\mathcal{S}_k|})$ to the set of actions. Here, each element π_{k,i_k} corresponds to the probability with which UE k chooses action $s_{k,i_k} \in \mathcal{S}_k$. In addition, $i_k \in \{1, \dots, |\mathcal{S}_k|\}$ represents the element's index of each set $\mathcal{S}_k, \forall k \in \mathcal{K}$.

For player k, let $\boldsymbol{e}_i^{(k)} = (e_{i,1}^{(k)}, \dots, e_{i,|\mathcal{S}_k|}^{(k)})$ be a unit probability vector with dimension $1 \times |\mathcal{S}_k|$ where a single element is 1 and the rest of the elements are 0. That is, $\forall j \in \{1, \dots, |\mathcal{S}_k|\} \setminus \{i\}, e_{i,j}^{(k)} = 0$, and $e_{i,i}^{(k)} = 1$. Here, k, i, and j denote the kth player, i the index of element that is equal to 1, and the index of each element in the vector, respectively. Thus, the probability distribution vector for $t = 1$ is considered as $\pi_k(t = 1) = \boldsymbol{e}_{i_k^*}^{(k)}$, where $i_k^* = \operatorname{argmax}_{b\in\mathcal{B}} \quad 10\log_{10}\Upsilon_{k,b}(t)$. $\Upsilon_{k,b}(t)$ is the metric defined as follows:

$$\Upsilon_{k,b}(t) = \gamma_{b,k}(\boldsymbol{x}_b, \boldsymbol{x}_k)(1 - \hat{\rho}_b(t))^{\eta_k}, \tag{8}$$

where $\hat{\rho}_b(t)$ is the estimated load of BS b at time t, which is broadcasted by the BS through a beacon signal and is given by

$$\hat{\rho}_b(t) = \left(1 - \left(\frac{1}{t}\right)^{\alpha}\right) \hat{\rho}_b\,(t-1) + \left(\frac{1}{t}\right)^{\alpha} \rho_b\,(t-1)\,, \tag{9}$$

where $\alpha > 0$ denotes the learning rate exponent for the load estimation. To estimate the load, the learning tool used in Equation (9) considers the long-term history and the instantaneous load, and it balances between them.[22] Let $\eta_k \geq 0$ be a private UE-dependent parameter that characterizes the personal preference of UE k for the effect of the BS's estimated load to the UEA metric. It balances the trade-off between the probability of UE outages and received SINR. With increasing η_k, UE k is inclined to associate to lightly loaded BSs even if the highly loaded BSs have a better SINR at the receiver of UE k.

Later, the players select their actions using a mapping function $T : \pi_k \to s_{k,i} \in \mathcal{S}_k$, which maps the probability distributions to an element in the set of actions. Each UE computes its payoff based on the actual throughput according to

$$\mathcal{U}_k(s_k, \boldsymbol{s}_{-k}) = \begin{cases} -1, & \text{if } k \in \mathcal{O} \\ 1, & \text{otherwise,} \end{cases} \tag{10}$$

where $\mathcal{O}$ denotes the set of UEs in outages, referred to as unsatisfied UEs, due to the overloading of their serving BSs.

3.2.2 | The learning process

Once the UEs compute their payoffs, each UE k in the set of UEs that experienced an outage proceeds by updating its probability distribution. In other words, the probability distribution is updated if the highest payoff is not achieved. The updates of the probability distributions are the key elements of the learning process. When the payoff is equal to 1, ie, the player is satisfied, the UE keeps playing the current action during the next iteration as follows:

$$s_k(t+1) = s_k(t), \tag{11}$$

where $s_k(t)$ is the action of player k at time t. Otherwise, the player must update its probability distribution with the hope of increasing its payoff. Moreover, the probability assigned to the current action must be decreased to augment exploring the other actions. Hence, the probability corresponding to each action $s_{k,i_k} \in \mathcal{S}_k \setminus \{s_k(t)\}$ is updated as follows:

$$d_k\left(\pi_{k,i_k}(t)\right) = \pi_{k,i_k}(t) + \lambda_k(t)\frac{\Upsilon_{k,i_k}(t)}{\Upsilon_{\max,k}(t)}\left(1-\pi_{k,i_k}(t)\right), \tag{12}$$

where

$$\Upsilon_{\max,k}(t) = \max_{i_k} \Upsilon_{k,i_k}(t), \tag{13}$$

where $\lambda_k(t)$ denotes the learning rate. The probability corresponding to the selected action at iteration t, ie, $s_{k,i_k} = s_k(t)$, is updated as follows:

$$d'_k\left(\pi_{k,i_k}(t)\right) = \pi_{k,i_k}(t) + \lambda_k(t)\frac{\Upsilon_{\max,k(t)} - \Upsilon_{k,i_k}(t)}{2\Upsilon_{\max,k}(t)}\left(-\pi_{k,i_k}(t)\right). \tag{14}$$

Therefore, the UE selects the action according to

$$s_k(t+1) = \begin{cases} s_k(t), & \text{if } \mathcal{U}_k(t) = 1 \\ \sim \pi_k(t+1), & \text{otherwise,} \end{cases} \tag{15}$$

where $\sim \pi_k(t+1)$ means according to the probability distribution $\pi_k(t+1)$. Therefore, the probability distribution is updated as follows:

$$\pi_{k,i_k}(t+1) = \begin{cases} \pi_{k,i_k}(t), & \text{if } \ \mathcal{U}_k(t) = 1 \\ \beta_k(t+1)d_k\left(\pi_{k,i_k}(t)\right), & \text{if } \ \mathcal{U}_k(t) = -1, s_{k,i_k} \neq s_k(t) \\ \beta_k(t+1)d'_k\left(\pi_{k,i_k}(t)\right), & \text{if } \ \mathcal{U}_k(t) = -1, s_{k,i_k} = s_k(t). \end{cases} \tag{16}$$

Let $\beta_k(t+1) = \frac{1}{\sum_{i_k \in \mathcal{A}_k} d_k\left(\pi_{k,i_k}(t)\right) + d'_k\left(\pi_{k,i_k}(t)\right)}$ be a normalization factor for UE k.

The convergence and obtaining the long-term behavior of the proposed UEA scheme relies on an ordinary differential equation.[31] Moreover, the learning rate is chosen according to the following conditions[27,32]:

$$\lim_{t\to\infty} \sum_{n=1}^{t} \lambda_k(n) = +\infty \tag{17}$$

and

$$\lim_{t\to\infty} \sum_{n=1}^{t} \lambda_k^2(n) < +\infty. \tag{18}$$

Equation A.1 ensures that the learning rates are large enough to overcome any random fluctuations or initial conditions. To assure convergence, Equation (18) guarantees that the learning rates diminish with iterations and eventually become small enough. The pseudo code of the proposed UEA scheme is given in Algorithm 1. The operation of the proposed mechanism is illustrated in Figure 2.

Algorithm 1 Proposed learning-based UEA algorithm

Input: $\mathcal{B}, \mathcal{K}$
Output: $\boldsymbol{A}, \mathcal{U}_k \;\; \forall k \in \mathcal{K}$
Initialization: $\mathcal{B}, \mathcal{K}, t = 1, \hat{\rho}_b(t)$ for $\forall b \in \mathcal{B}$
The first iteration:
for $\forall k \in \mathcal{K}$ **do**
  Advertise estimated load $\hat{\rho}_b(t)$ for $\forall b \in \mathcal{B}$
  for $\forall b \in \mathcal{B}$ **do**
    Calculate $Y_{k,b}(t)$, using (8)
  end for
  $i_k^* = \text{argmax}_{b \in \mathcal{B}} \, 10\log_{10} Y_{k,b}(t)$
  $\boldsymbol{\pi}_k(t = 1) = \boldsymbol{e}_{b^{(k)}_{i_k^*}}$
  Action selection using a mapping function $f : \boldsymbol{\pi}_k \rightarrow s_{k,i} \in \mathcal{S}_k$
  Associate the UEs to the corresponding BSs
  Update matrix $\boldsymbol{A}$
  for $\forall s_{k,i} \in \mathcal{S}_k$ **do**
    Update the probability distribution π_{k,i_k}, (16)
  end for
end for
The learning process:
while true **do**
  $t \leftarrow t + 1$
  Advertise estimated load $\hat{\rho}_b(t)$ for $\forall b \in \mathcal{B}$
  for $\forall k \in \mathcal{O}$ **do**
    Action selection using a mapping function $f : \boldsymbol{\pi}_k \rightarrow s_{k,i} \in \mathcal{S}_k$
    Update $\boldsymbol{A}$
    for $\forall s_{k,i} \in \mathcal{S}_k$ **do**
      Update the probability distribution π_{k,i_k}, (16)
    end for
  end for
end while

The proposed learning-based UEA corresponds to a Markov chain. Let $\boldsymbol{\pi}(t) = (\boldsymbol{\pi}_1(t), \ldots, \boldsymbol{\pi}_{|\mathcal{K}|}(t)) \in \triangle(\mathcal{S}_1) \times \cdots \times \triangle(\mathcal{S}_{|\mathcal{K}|})$ be the collection of the probability distributions updated at each iteration. In Algorithm 1, each player selects an action and updates the probability distribution associated to its set of actions. In other words, a transition from the current probability distributions vector $\pi(t)$ to another $\pi(t+1)$ takes place when some UEs update their serving BSs according to their observations on their payoffs. Thus, the transition only depends on the current distributions vector $\pi(t)$. Therefore, in the learning-based UEA approach, only $|\mathcal{S}|$ information is stored. Here, we define the satisfaction equilibrium as follows.

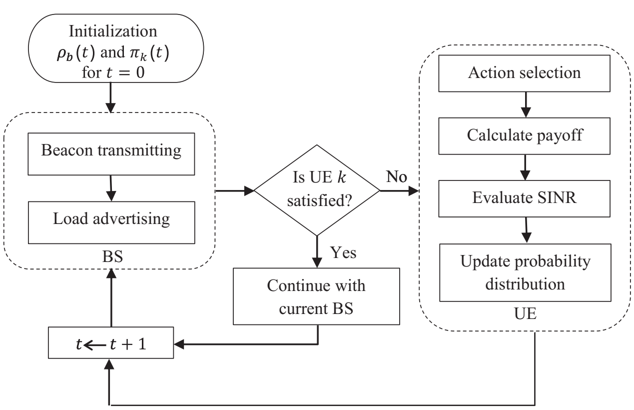

FIGURE 2 Flow diagram of the proposed UEA mechanism

TABLE 2 System-level simulation parameters

System Parameters		
Parameter		**Value**
Physical link type		Downlink
Carrier frequency		2 GHz
Total bandwidth		10 MHz
Noise power		−174 dBm/Hz
Mean packet arrival rate		1800 Kbps
Learning rate exponent α		0.9
	BS Parameters	
Parameter	**MBS**	**SBS**
Maximum power	46 dBm	30 dBm
Cell radius	250 m	40 m
Distance-dependent path loss model	128.1+37.6$\log_{10}(d)$$d$ in Km	140.7+36.7$\log_{10}(d)$$d$ in Km
Minimum distance	MBS-SBS: 75 m	SBS-SBS: 40 m
	MBS-UE: 35 m	SBS-UE: 10 m

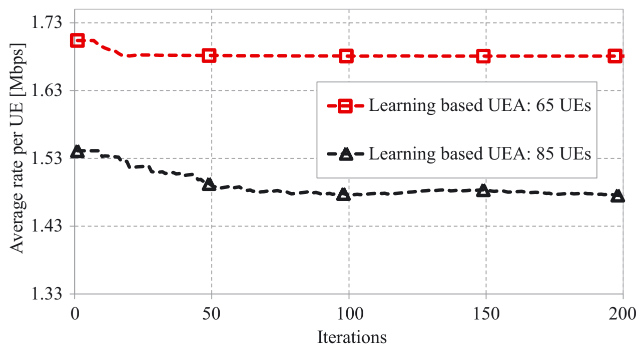

FIGURE 3 Convergence of the average rate per UE of the proposed UEA ($|\mathcal{B}_S| = 5$ and $|\mathcal{K}| = 65$ and 85)

Definition 1. Satisfaction Equilibrium: A strategy profile $\boldsymbol{s}^*$ is an equilibrium for game $\widehat{\mathcal{G}}$ if

$$\forall k \in \mathcal{K}, \qquad s_k^* \in f_k(\boldsymbol{s}_{-k}^*). \tag{19}$$

The convergence of the proposed learning procedure is proven based on Theorem 1.

Theorem 1. *Let $\pi(t) = (\pi_1(t), \ldots, \pi_{|\mathcal{K}|}(t)) \in \triangle(\mathcal{S}_1) \times \cdots \times \triangle(\mathcal{S}_{|\mathcal{K}|})$ be the collection of the probability distributions updated at each iteration. Under Algorithm 1, as $t \to +\infty$, the probability distributions $\pi(t)$ converges to an equilibrium of the game $\mathcal{G} = \langle \mathcal{K}, \{\mathcal{S}_k\}_{k\in\mathcal{K}}, \{f_k\}_{k\in\mathcal{K}} \rangle$ in finite time for a network in which the total required rate does not exceed the total capacity of the network.*

The proof of Theorem 1 *is presented in Appendix* A. *In Appendix* B, *a discussion about the computational requirements of the proposed mechanism is presented.*

4 | SIMULATION RESULTS

In this section, we conduct simulations to evaluate the performance of the proposed UEA scheme. For our simulations, we consider a single hexagonal cell served by 1 MBS and a set of SBSs. The system parameters used for the simulations are summarized in Table 2. For comparison purposes, we consider an additional UEA scheme in which the UEs select their serving BSs based on the RSSI and the BS's estimated load.[22] It is referred to hereinafter as "baseline UEA".

To investigate the convergence behavior of our proposed UEA, we consider a network with 5 SBSs. Figure 3 shows the average rate per UE of the proposed UEA for the number of UEs= 65 and 85. We can observe that by increasing the number of UEs, the iterations for convergence increase. The convergence is achieved in about 25 and 77 iterations for the number of UEs = 65 and 85, respectively. Furthermore, with increasing the number of UEs in the network, the average rate per UE decreases.

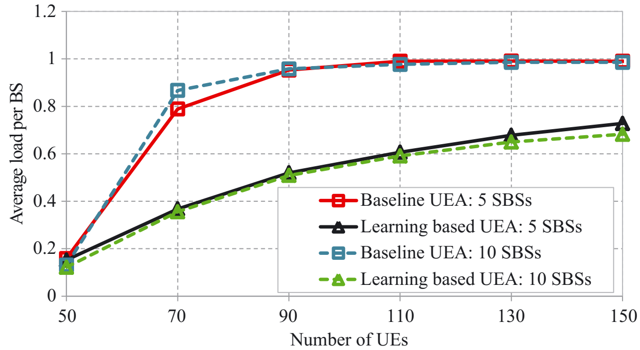

FIGURE 4 Average load per BS versus the number of UEs ($|\mathcal{B}_S| = 5$ and 10)

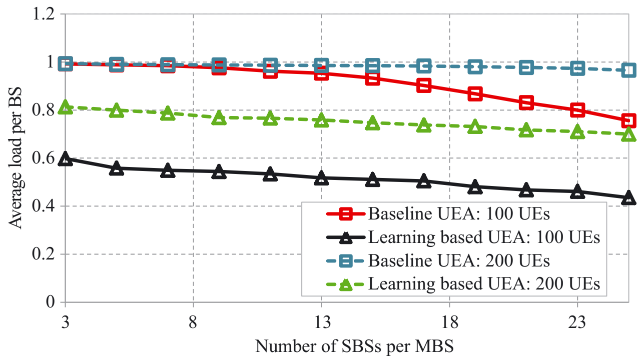

FIGURE 5 Average load per BS versus the number of SBSs ($|\mathcal{K}| = 100$ and 200)

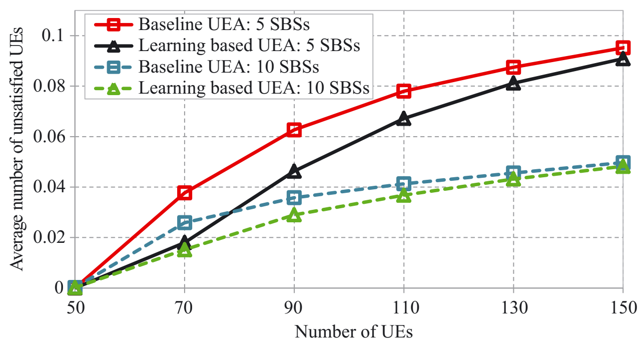

FIGURE 6 Average number of unsatisfied UEs versus the number of UEs ($|\mathcal{B}_S| = 5$ and 10)

The main reason is that with increasing the number of UEs, the BS's load increases. Therefore, it may lead to overloading some BSs and, thus, decreasing the UE's rate.

In order to evaluate the effect of our proposed UEA scheme on the delay, we depict the average load per BS versus the number of UEs under the number of SBSs = 5 and 10. In Figure 4, we can observe that, with increasing the number of UEs, average load increases. Moreover, the learning-based UEA scheme balances load among BSs by offloading UEs associated with highly loaded BSs to lightly loaded BSs and results in decreasing delay. In Figure 5, we depict the average load per BS for 100 and 200 UEs. As the number of SBSs increases, the average load per BS decreases. From Figure 5, it can be seen that the proposed UEA scheme balances the load among BSs and yields up about 46% compared to the baseline UEA approach for a network with 100 UEs and 13 SBSs.

Figure 6 shows the average number of unsatisfied UEs versus the number of UEs for 5 and 10 SBSs. As the number of UEs increases, the average number of unsatisfied UEs increases. Since our proposed UEA scheme has a good performance in load balancing, it decreases the number of unsatisfied UEs. Furthermore, with increasing the SBSs in the network, the average load per BS decreases. Thus, it yields significant satisfaction of UEs in the network. We also observe about 52.3% reduction in the average number of unsatisfied UEs in the learning-based UEA scheme compared to the baseline approach for a network with 5 SBSs and 70 UEs.

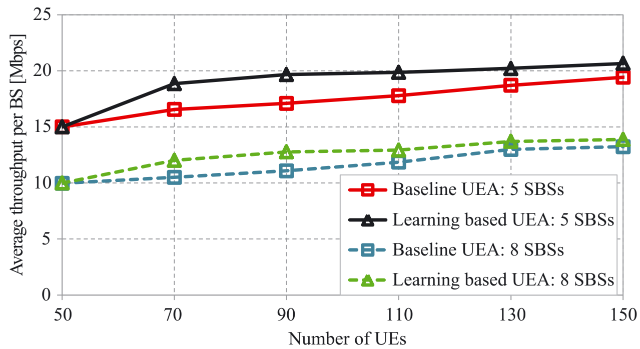

FIGURE 7 Average throughput per BS versus the number of UEs ($|\mathcal{B}_\mathrm{S}| = 5$ and 8)

In Figure 7, we show the average throughput per BS when increasing the number of UEs for 5 and 8 SBSs. For a given number of SBSs in the network, as the number of UEs increases, the average throughput per BS increases. It is observed that the proposed UEA scheme achieves a higher throughput compared to the baseline approach because of the load balancing behavior of our proposed UEA scheme. For instance, the learning-based UEA scheme yields up to about 15.4% improvement over the baseline approach for a network with 8 SBSs and 90 UEs.

5 | CONCLUSION

In this paper, we have proposed a load balancing UEA approach and studied its performance on the downlink of a 2-tier HetNet. We have modeled the UEA problem as a noncooperative game, in which each UE tries to associate to an appropriate BS. In order to solve the game, we have proposed a satisfaction-based learning process that is implemented in a fully distributed manner. Simulation results have shown that our proposed UEA scheme balances load among BSs and improves the UE's QoS over the baseline approach. Improvements of 46% in the delay and 15.4% in the average BS throughput are obtained in the learning-based approach. Furthermore, our proposed scheme outperforms the baseline for the average number of unsatisfied UEs in the network. Our proposed UEA approach can also be extended to K-tier HetNets to further improve the energy efficiency and SE.

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APPENDIX A : PROOF OF THEOREM 1

Here, we assume that the total required rate does not exceed the total capacity of the network. This is because for a UE, if there are not enough BSs with enough resources within the coverage of the UE, it may not be accommodated. Therefore, a sufficient condition for the existence of a satisfaction equilibrium is that the total required rate does not exceed the total capacity of the network or equivalently:

$$\begin{aligned} &\textit{if} \quad \forall k \in \mathcal{K} \qquad \exists b \in \mathcal{B} : \boldsymbol{x}_k \in \mathcal{L}_b, \\ &\sum_{b\in\mathcal{B}} \sum_{\boldsymbol{x}_k\in\mathcal{L}_b} \varrho_b(\boldsymbol{x}_k) \leq \frac{\lambda}{\mu} \sum_{b\in\mathcal{B}} \sum_{\boldsymbol{x}_k\in\mathcal{L}_b} \frac{1}{R_{b,k}(\boldsymbol{x}_b, \boldsymbol{x}_k)} < |\mathcal{B}|, \end{aligned} \tag{A.1}$$

where

$$\frac{\lambda}{\mu} = \max_k \frac{\lambda(\boldsymbol{x}_k)}{\mu(\boldsymbol{x}_k)}. \tag{A.2}$$

To drive an upper bound on the maximum required rate, we use an equivalent network with symmetric setting with the same number of BSs and UEs, in which

$$\forall k \in \mathcal{K}, \forall b \in \mathcal{B} \qquad \gamma_{b,k}(\boldsymbol{x}_b, \boldsymbol{x}_k) = \gamma. \tag{A.3}$$

Moreover, we consider the worst case where all UEs have the same QoS requirement with the maximum required rate. Therefore, we have

$$\frac{\lambda}{\mu} \sum_{b\in\mathcal{B}} \sum_{\boldsymbol{x}_k\in\mathcal{L}_b} \frac{1}{\omega \log_2(1+\gamma_{b,k}(\boldsymbol{x}_b, \boldsymbol{x}_k))} = \frac{\lambda}{\mu} \frac{|\mathcal{K}|}{\omega \log_2(1+\gamma)} < |\mathcal{B}|. \tag{A.4}$$

Thus

$$\frac{\lambda}{\mu} < \frac{|\mathcal{B}|}{|\mathcal{K}|} \omega \log_2(1+\gamma). \tag{A.5}$$

Since the game $\mathcal{G} = \langle \mathcal{K}, \{\mathcal{S}_k\}_{k\in\mathcal{K}}, \{f_k\}_{k\in\mathcal{K}} \rangle$ is a finite game, it has at least 1 equilibrium.[33] Moreover, for $t > 1$, we have $\pi_{k,i_k}(t) > 0$, $\forall k \in \mathcal{O}$ and $i_k \in \{1, \dots, |\mathcal{S}_k|\}$. Using a similar argument as in the work of Perlaza et al,[34] this condition implies that each action profile can be played at least once during sufficiently large iterations. Since at least 1 satisfaction equilibrium exists, the UEs will select a satisfaction equilibrium at least once during playing game. According to Equation (15), when 1 equilibrium is played, the UEs would not change their actions. Therefore, this concludes the proof of converging the learning procedure.

APPENDIX B : COMPUTATIONAL REQUIREMENTS

Here, with considering digital signal processors (DSPs), we present the computational requirements of the proposed UEA mechanism. We assume that addition, multiplication, and comparison operations require 1 DSP cycle.[35] For division operation, 42 operations are considered. Since the computational analysis does not take into account the compiler optimizations and the ability of DSPs to execute various instructions, the analysis provides an upper bound on computations. Table A1 summarizes the computational instructions for the proposed UEA approach.

TABLE A1 Computational requirements for the proposed UEA approach

Operations	Required Instructions
Sum	4
Multiplication	6
Division	2
Comparison	$\vert\mathcal{B}\vert - 1$
Total number of operations	$93 + \vert\mathcal{B}\vert$