Analysis of maximal-ratio transmit and combining spatial diversity

Fumiyuki Adachi[a)], **Amnart Boonkajay**
Research Organization of Electrical Communication, Tohoku University
2-1-1 Katahira, Aoba-ku, Sendai, Miyagi 980-8577, Japan
a) *adachi@ecei.tohoku.ac.jp*

Abstract: Spatial diversity is still remaining as a powerful means to improve the transmission performance in a multipath fading environment. Various spatial diversity schemes have been proposed such as maximal-ratio transmit and combining (MRTC), selective MRTC, and space-time block coded transmit diversity (STBC-TD). In this paper, we derive a closed form expression for the received signal-to-noise ratio (SNR) achievable with MRTC for the case of base station (BS) having an arbitrary number of antennas and user-equipment (UE) having 2 antennas. Using the derived signal-to-noise power ratio (SNR) expression, the average bit error rate (BER) performance of quaternary phase shift keying (QPSK) transmission in a Rayleigh fading environment is numerically evaluated. It is shown that MRTC provides the BER performance superior to STBC-TD and selective MRTC.
Keywords: Spatial diversity, MRTC, STBC
Classification: Wireless Communication Technologies

References

[1] W. C. Jakes, Jr., Ed., *Microwave mobile communications*, Wiley, New York, 1974.
[2] J. K. Cavers, "Single-user and multiuser adaptive maximal ratio transmission for Rayleigh channels," *IEEE Trans. Vehi. Technol.*, Vol. 49, No.6, pp. 2043-2050, Nov. 2000.
[3] X. Feng and C. Leung, "A new optimal transmit and receive diversity scheme," *Proc. 2001 IEEE Pacific Rim Conference on Communications, Computers and Signal Processing*, Vol. 2, pp. 538-541, 2001.
[4] B. D. Rao and M. Yan, "Performance of maximal ratio transmission with Two receive antennas," *IEEE Trans. Commun.*, Vol. 51, No. 6, pp.894-895, June 2003.
[5] F. Adachi, A. Boonkajay, Y. Seki, T. Saito, S. Kumagai and H. Miyazaki, "Cooperative distributed antenna transmission for 5G mobile communications network," *IEICE Trans. Commun.*, Vol. E100-B, No.8, pp. 1190-1204, Aug. 2017.
[6] F. Adachi and A. Boonkajay, "Selective MIMO diversity with subcarrier-wise UE antenna identication/selection," *IEICE Communications Express*, Vol. 7, Issue 11, pp. 400-408, Nov. 2018.
[7] K. Takeda, T. Itagaki and F. Adachi, "Application of space-time transmit diversity to single-carrier transmission with frequency-domain equaliza-

DOI: 10.1587/comex.2019XBL0015
Received January 20, 2019
Accepted February 7, 2019
Publicized February 20, 2019

tion and receive antenna diversity in a frequency-selective fading channel," *IEE Proc.-Commun.*, vol. 151, No.6, pp. 627-632, Dec. 2004.
[8] A. M. Tulino and S. Verdu, *Random Matrix Theory and Wireless Communications*, Now Publishings, 2004.
[9] R. K. Mallik, "The pseudo-Wishart distribution and its application to MIMO systems," *IEEE Trans. Information Theory*, Vol. 49, No. 10, pp. 2761-2769, Oct. 2003.
[10] J. M. Steele, *The Cauchy-Schwarz Master Class*, Cambridge University Press, 2004.
[11] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, Vol. 16, No.8, pp. 1451-1457, Oct. 1998.
[12] H. Tomeba, K. Takeda and F. Adachi, "Space-time block coded joint transmit/receive diversity in a fequency-nonselective Rayleigh fading channel," *IEICE Trans. Commun.*, Vol.E89-B, No.8, pp. 2189-2195, Aug. 2006.

1 Introduction

Spatial diversity significantly improves the transmission performance in a multipath fading environment. High frequency bands like millimeter wave will be used due to scarcity of available frequency bandwidth. So far, various diversity schemes have been proposed, such as maximal-ratio combining (MRC) [1], selection combining (SC) [1], and maximal-ratio transmission (MRT) [2], and maximal-ratio transmit and combining (MRTC) [3], [4]. The authors proposed space-time block-coded transmit diversity (STBC-TD) [5] which employs MRT and MRC at base station (BS) for downlink and uplink, respectively, and recently proposed selective multi-input multi-output (MIMO) diversity which selects the best UE antenna for MRT and MRC at BS for downlink and uplink, respectively [6] (called selective MRTC in this paper). It should be noted that, for the single-carrier uplink transmission, MRC needs to be replaced by the minimum mean square error combining (MMSEC) [7] since MRC emphasizes the inter-symbol interference (ISI).

BS has a sufficient space to be equipped with a large number of antennas, while user equipment (UE) can be equipped with only a few number of antennas due to its space limitation and hardware complexity limitation. In [4], the probability density function (PDF) of the received signal-to-noise power ratio (SNR) achievable with MRTC is derived for the case of an arbitrary number of transmit antennas and 2 receive antennas. However, to the best of the authors' knowledge, the closed-form expression for the received SNR achievable with MRTC is not available in the literature. This makes it difficult to compare MRTC with other diversity schemes.

In this paper, frequency-nonselective fading is considered. Using M transmit antennas at BS and 2 receive antennas at UE, we theoretically derive the closed-form expression for the conditional SNR achievable with MRTC when $M \times 2$ MIMO channel gains are given. Using the derived expression for the conditional received SNR, the average bit error rate (BER) performance of

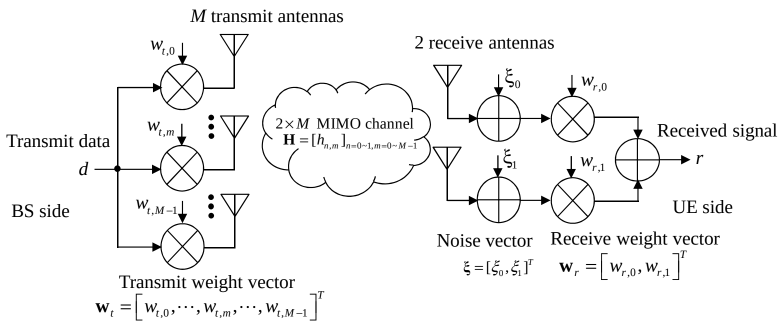

Fig. 1. Downlink transmission system model.

quaternary phase shift keying (QPSK) transmission in a Rayleigh fading environment is numerically evaluated. It is shown that MRTC provides the average BER performance superior to STBC-TD and selective MTRC.

2 Analysis

Throughout the paper, the frequency-nonselective fading is considered. The downlink transmission system model using MRTC is depicted in Fig. 1. BS and UE are assumed to have M antennas and 2 antennas, respectively. The transmit weight vector of size $M \times 1$ and the receive weight vector of size 2×1 are represented as $\mathbf{w}_t = [w_{t,0}, \cdots, w_{t,m}, \cdots, w_{t,M-1}]^T$ and $\mathbf{w}_r = [w_{r,0}, w_{r,1}]^T$, respectively. The downlink channel matrix is of size $2 \times M$ and is represented as $\mathbf{H} = [h_{n,m}]_{n=0\sim1,m=0\sim M-1}$ with $E[|h_{n,m}|^2] = 1$ (where $E[\cdot]$ is the ensemble average operation). The superscripts T, H, and $*$ represent the transpose, Hermitian transpose, and complex conjugate operations, respectively.

2.1 Received SNR

The baseband equivalent received signal r after combining at UE can be expressed as

$$r = \sqrt{2S}(\mathbf{w}_r^T \mathbf{H}\mathbf{w}_t)\, d + \mathbf{w}_r^T \boldsymbol{\xi}, \tag{1}$$

where S is the average signal power, d is the transmit data symbol with $E[|d|^2] = 1$, and $\boldsymbol{\xi} = [\xi_0, \xi_1]^T$ is the complex-valued additive white Gaussian noise (AWGN) vector with the noise power $\frac{1}{2}E[|\xi_n|^2] = \sigma^2$ for $n = 0 \sim 1$. The received SNR γ after coherent detection can be expressed as

$$\gamma = \frac{2S}{\sigma^2} \frac{|\mathbf{w}_r^T \mathbf{H}\mathbf{w}_t|^2}{\mathbf{w}_r^H \mathbf{w}_r}. \tag{2}$$

2.2 Solving the SNR maximization problem

We want to find $(\mathbf{w}_t, \mathbf{w}_r)$ which maximizes the value of γ. This maximization problem can be solved as follows. For the given $\mathbf{w}_r$, γ can be maximized when $\mathbf{w}_t = (\mathbf{w}_r^T \mathbf{H})^H$ subject to $\mathbf{w}_t^H \mathbf{w}_t = 1$ (this is equivalent to the well-known MRT of M transmit antennas) and can be expressed as

$$\gamma = \frac{2S}{\sigma^2} \frac{(\mathbf{w}_r^*)^H (\mathbf{H}\mathbf{H}^H)\mathbf{w}_r^*}{(\mathbf{w}_r^*)^H \mathbf{w}_r^*}, \tag{3}$$

from which the optimal $\mathbf{w}_r$, denoted by $\mathbf{w}_r^+$, can be found. This problem leads to solving the following eigenvalue equation [8]

$$(\mathbf{H}\mathbf{H}^H)\mathbf{w}_r^* = \alpha \mathbf{w}_r^*, \tag{4}$$

where α and $\mathbf{w}_r^*$ are the eigenvalue of $\mathbf{H}\mathbf{H}^H$ and the complex conjugated receive weight vector, respectively. Since $\mathbf{H}\mathbf{H}^H$ is a Hermitian matrix, the eigenvalues become non-negative and real [9].

Since $\mathbf{H}\mathbf{H}^H$ is a matrix of size 2×2, Eq. (4) can be easily solved. The complex conjugated optimal receive weight $(\mathbf{w}_r^+)^*$ is associated with the largest eigenvalue α^+ of $\mathbf{H}\mathbf{H}^H$. The optimal transmit weight is obtained as

$$\mathbf{w}_t^+ = ((\mathbf{w}_r^+)^T \mathbf{H})^H / \sqrt{(\mathbf{w}_r^+)^T (\mathbf{H}\mathbf{H}^H)(\mathbf{w}_r^+)^*}. \tag{5}$$

Another eigenvalue equation can be formulated. For the given $\mathbf{w}_t$, the use of $\mathbf{w}_r = (\mathbf{H}\mathbf{w}_t)^*$ maximizes the value of γ (this is equivalent to the well-known MRC of 2 receive antennas), leading to the following eigenvalue equation

$$(\mathbf{H}^H\mathbf{H})\mathbf{w}_t = \beta \mathbf{w}_t \qquad \text{s.t. } \mathbf{w}_t^H \mathbf{w}_t = 1. \tag{6}$$

Since $\mathbf{H}^H\mathbf{H}$ is a matrix of size $M \times M$, Eq. (6) is very difficult to solve if not impossible for a large M. However, we can show that $\alpha = \beta$ and $\mathbf{w}_t^+$ is equal to the weight given by Eq. (5). Therefore, it is recommended to use Eq. (4) to find the optimal pair of the transmit and receive weights.

2.3 Deriving a closed-form SNR expression

$\mathbf{H}\mathbf{H}^H$ is given as

$$\mathbf{H}\mathbf{H}^H = \begin{bmatrix} a & c^* \\ c & b \end{bmatrix} = \begin{bmatrix} \sum_{m=0}^{M-1} |h_{0,m}|^2 & \left(\sum_{m=0}^{M-1} h_{0,m}^* h_{1,m}\right)^* \\ \sum_{m=0}^{M-1} h_{0,m}^* h_{1,m} & \sum_{m=0}^{M-1} |h_{1,m}|^2 \end{bmatrix}. \tag{7}$$

Using Eqs. (4) and (7), α^+ can be derived (its derivation process is omitted for brevity) as

$$\begin{aligned} \alpha^+ &= \frac{a+b+\sqrt{(a-b)^2+4|c|^2}}{2} \\ &= \frac{1}{2}\sum_{m=0}^{M-1}\left(|h_{0,m}|^2+|h_{1,m}|^2\right) \\ &\quad + \sqrt{\left(\frac{1}{2}\sum_{m=0}^{M-1}(|h_{0,m}|^2-|h_{1,m}|^2)\right)^2 + \left|\sum_{m=0}^{M-1} h_{0,m}^* h_{1,m}\right|^2} \end{aligned}. \tag{8}$$

The achievable maximum SNR γ_{MRTC} and $\mathbf{w}_r^+$ subject to $(\mathbf{w}_r^+)^H \mathbf{w}_r^+ = 1$ are obtained as

$$\begin{cases} \gamma_{\text{MRTC}} = \dfrac{2S}{\sigma^2}\alpha^+ \\ \mathbf{w}_r^+ = \begin{bmatrix} w_{r,0}^+ \\ w_{r,0}^+ \end{bmatrix} = \begin{bmatrix} \mp \dfrac{c}{|c|} \dfrac{(b-\alpha^+)^*}{\sqrt{(b-\alpha^+)^2+|c|^2}} \\ \pm \dfrac{|c|}{\sqrt{(b-\alpha^+)^2+|c|^2}} \end{bmatrix} \end{cases} \tag{9}$$

Finally, the optimal transmit weight $\mathbf{w}_t^+$ can be obtained from Eq. (5).

2.4 Relationship between uplink and downlink optimal weight pairs

So far, we have analyzed MRTC using M transmit and 2 received antennas, which is the downlink transmission case. An interesting problem is to find the optimal transmit and receive weight pair for the uplink transmission case of M BS antennas and 2 UE antennas. Below, the subscript "$\uparrow$" is introduced to indicate the uplink case. According to Sect. 2.3, it is understood that the eigenvalue equation with respect to $\mathbf{w}_{\uparrow t}$ is easier to solve since $\mathbf{H}_{\uparrow}^{H}\mathbf{H}_{\uparrow}$ is of size 2×2. The eigenvalue equation is given by

$$(\mathbf{H}_{\uparrow}^{H}\mathbf{H}_{\uparrow})\mathbf{w}_{\uparrow t} = \alpha_{\uparrow}\mathbf{w}_{\uparrow t}, \tag{10}$$

where $\alpha_{\uparrow}$ is the eigenvalue of $\mathbf{H}_{\uparrow}^{H}\mathbf{H}_{\uparrow}$. Assuming time division duplex (TDD), the uplink and downlink channels are reciprocal and the channel seen for the reception at BS is given by $\mathbf{H}_{\uparrow} = \mathbf{H}^{T}$. Taking the complex conjugate of both sides of Eq. (10) gives

$$(\mathbf{H}\mathbf{H}^{H})(\mathbf{w}_{\uparrow t})^{*} = \alpha_{\uparrow}(\mathbf{w}_{\uparrow t})^{*}, \tag{11}$$

which is the same expression as Eq. (4). Therefore, $\alpha_{\uparrow}^{+} = \alpha^{+}$, $\mathbf{w}_{\uparrow t}^{+} = \mathbf{w}_{r}^{+}$ and $\mathbf{w}_{\uparrow r}^{+} = \mathbf{w}_{t}^{+}$. This reveals that the same SNR can be achieved for the downlink and uplink transmissions and that the optimal transmit (BS) and receive (UE) weights pair for the downlink transmission is the same as the optimal receive (BS) and transmit (UE) weights pair for the uplink transmission.

2.5 Discussion

Below, the performance comparison is provided among MRTC, STBC-TD, and selective MRTC, assuming M BS antennas and 2 UE antennas. STBC-TD employs MRT (downlink)/MRC (uplink) [5]. Selective MRTC selects the best UE antenna while BS uses all antennas for MRT (downlink)/MRC (uplink) [6].

Applying the Cauchy-Schwarz inequality [10] to Eq. (8), the upper-bound of α^{+} can be obtained. The lower-bound of α^{+} can be readily obtained from Eq. (8). Accordingly, we have

$$\begin{aligned} \sum_{m=0}^{M-1}\left(|h_{0,m}|^2 + |h_{1,m}|^2\right) \geq \alpha^{+} &> \max\left\{\sum_{m=0}^{M-1}|h_{0,m}|^2, \sum_{m=0}^{M-1}|h_{1,m}|^2\right\} \\ &\geq \frac{1}{2}\sum_{m=0}^{M-1}\left(|h_{0,m}|^2 + |h_{1,m}|^2\right) \end{aligned}. \tag{12}$$

Using Alamouti's 2×2 STBC having the code rate of 1 [11], STBC-TD achieves the received SNR of $\gamma_{\text{STBC-TD}} = \frac{2S}{\sigma^2}\frac{1}{2}\sum_{m=0}^{M-1}\left(|h_{0,m}|^2 + |h_{1,m}|^2\right)$ (see [12]). We can also show that selective MRTC achieves the received SNR of $\gamma_{\text{selective MRTC}} = \frac{2S}{\sigma^2}\max\left\{\sum_{m=0}^{M-1}|h_{0,m}|^2, \sum_{m=0}^{M-1}|h_{1,m}|^2\right\}$. Therefore, it can be understood that $\gamma_{\text{MRTC}} > \gamma_{\text{selective MRTC}} \geq \gamma_{\text{STBC-TD}}$ and MRTC achieves the highest received SNR.

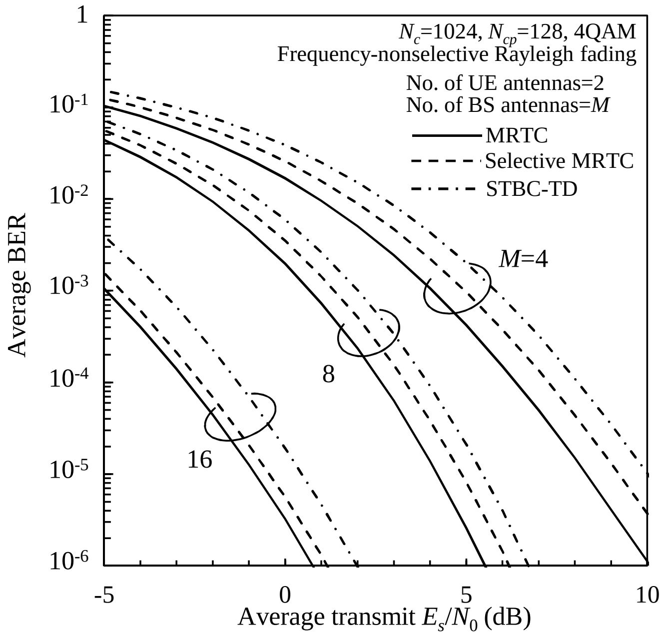

Fig. 2. Average BER performance.

3 Numerical Evaluation

The average BER performances of QPSK transmission achievable with MRTC, STBC-TD, and selective MRTC in a Rayleigh fading environment are numerically evaluated. It is assumed that BS is equipped with M=4, 8, and 16 antennas while UE is equipped with 2 antennas. Assuming a Rayleigh fading environment, $\{h_{n,m}\}, n = 0 \sim 1, m = 0 \sim M-1$, are modeled as independent and identically distributed (i.i.d.) complex Gaussian random variables with unit variance. The average BER of QPSK transmission is obtained by averaging the conditional BER $p_e(\gamma) = \frac{1}{2}erfc(\sqrt{\gamma/4})$ over all possible $\{h_{n,m}\}$.

The numerically obtained average BER performances achievable with MRTC, STBC-TD, and selective MRTC are plotted as a function of the average transmit symbol energy-to-noise power spectrum density ratio $E_s/N_0(= S/\sigma^2)$ in Fig. 2. It can be seen from the figure that MRTC provides the best BER performance, followed by selective MRTC. The performance gap between MRTC and selective MRTC becomes narrower as M increases; when M=8 and 16, it is respectively about 1.0 dB and 0.5 dB in terms of the required average transmit E_s/N_0 for achieving BER=10^{-3}.

4 Conclusion

The closed-form expression for the received SNR achievable with MRTC was derived for the case of an arbitrary number of BS antennas and 2 UE antennas. Using the derived SNR expression, the average BER performance of QPSK transmission achievable with MRTC in a frequency-nonselective Rayleigh fading environment was numerically evaluated and was compared with those achievable with STBC-TD and selective MRTC. It was confirmed that MRTC provides the best BER performance, followed by selective MRTC.

The received SNR expression derived in this paper assuming the frequency-nonselective fading environment can be applied with slight modification to numerically evaluate the average BER performances of orthogonal frequency division multiplexing (OFDM) and discrete Fourier transform (DFT)-spread OFDM (a family of single-carrier transmission) in a frequency-selective fading environment.

Acknowledgments

The results presented in this paper have been achieved by "The research and development project for realization of the fifth-generation mobile communications system, commissioned to Tohoku University by The Ministry of Internal Affairs and Communications (MIC), Japan.