社団法人 電子情報通信学会
THE INSTITUTE OF ELECTRONICS,
INFORMATION AND COMMUNICATION ENGINEERS

信学技報
TECHNICAL REPORT OF IEICE

周波数領域等化を用いる一般化 OFDM

ガチャニンハリス[†] 高岡辰輔[‡] 安達文幸[‡]

東北大学大学部工学研究科　電気・通信工学 〒980-8579 宮城県仙台市青葉区荒巻字青葉 05
E-mail: [†]haris@mobile.ecei.tohoku.ac.jp, [‡] (takaoka, adachi)@ mobile.ecei.tohoku.ac.jp

あらまし OFDM などのマルチキャリア伝送では送信信号のピーク対平均信号電力比（PAPR）が大きくなるので，線形電力増幅器の負担が増加してしまうという問題がある．最近では，PAPR の問題の少ない周波数領域等化を用いるシングルキャリア（SC）伝送が注目されている．本論文では，PAPR の問題を避けるため周波数領域等化を用いる一般化 OFDM(GOFDM)を提案している．次いで，OFDM の平均誤り率（BER）特性を計算機シミュレーションにより求め，従来の OFDM および周波数領域等化を用いる SC 伝送のそれと比較している．

キーワード ピーク対平均信号電力比(PAPR)，周波数領域等化，OFDM.

Generalized OFDM with Frequency-domain Equalization

Haris GACANIN[†] Shinsuke TAKAOKA[‡] and Fumiyuki ADACHI[‡]

Electrical and Communication Engineering, Graduate School of Engineering, Tohoku University
05 Aza-Aoba, Aramaki, Aoba-ku, Sendai, 980-8579 Japan
E-mail: [†]haris@ mobile.ecei.tohoku.ac.jp, [‡](takaoka, adachi)@ mobile.ecei.tohoku.ac.jp

Abstract A possible problem of orthogonal frequency division multiplexing (OFDM) is its high peak-to-average power ratio (PAPR). Recently, the single carrier (SC) transmission system with frequency-domain equalization is attracting much attention. In this paper, a novel approach to alleviate the PAPR problem of OFDM is proposed. A generalized OFDM (GOFDM) with frequency-domain equalization is presented and its performance in a frequency-selective fading channel is evaluated and compared with those of conventional OFDM and single carrier (SC) systems.

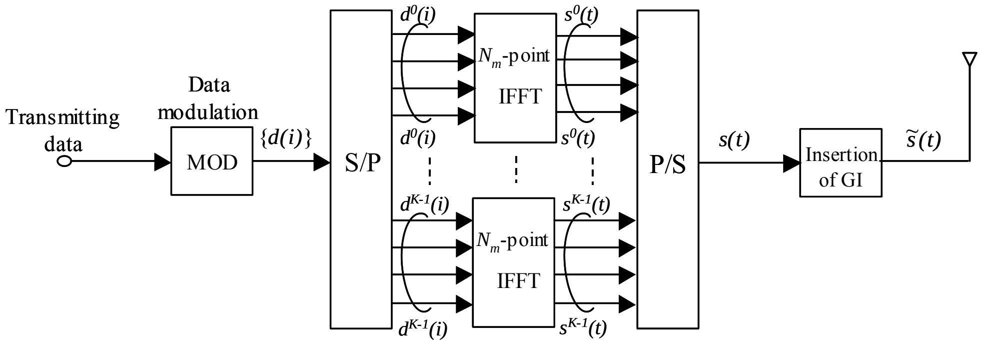
Keyword: OFDM**,** peak-to-average-power ratio (PAPR), frequency-domain equalization

1. INTRODUCTION

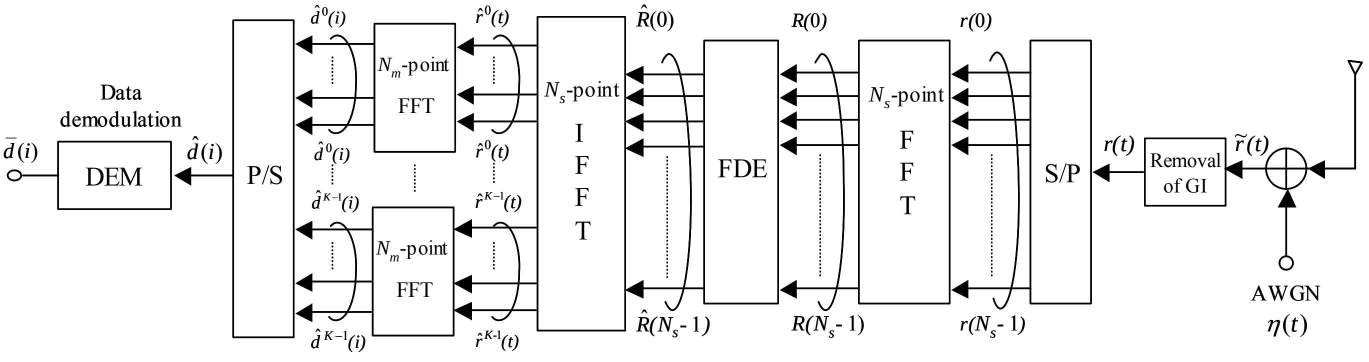
In wireless channel, the presence of many propagation paths with different time delays produces frequency-selective multipath fading, which produces inter-symbol interference (ISI) and severely degrades the transmission performance [1]. To improve the transmission performance, some adaptive equalization techniques (e.g., maximum likelihood sequence estimation (MLSE) [1,2]) must be employed. Recently, orthogonal frequency division multiplexing (OFDM) has kept attracting much attention [3]. In OFDM, high-speed data is transmitted in parallel using a number of orthogonal subcarriers, where each modulated subcarrier bandwidth is narrow enough to experience frequency-nonselective fading and therefore, sophisticated adaptive equalization can be avoided. A possible problem of OFDM is its high peak-to-average-power ratio (PAPR). Power amplifiers, analog-to-digital (A/D) and digital-to-analog (D/A) converters with a large dynamic range are required. To alleviate this PAPR problem, some techniques have been proposed [4, 5]. Remembering that the PAPR is proportionate to the number of subcarriers, a simple approach is to reduce the number of subcarriers but keeping the data rate the same. However, this approach decreases the frequency efficiency since the guard interval (GI) length needs to be the same.

The single carrier (SC) transmission system does not have the problem of high PAPR; however, the transmission performance severely degrades due to the ISI in a frequency-selective fading channel. Recently, it has been shown that application of frequency-domain equalization (FDE) to a SC system can take an advantage of frequency-selectivity of the channel and achieve a much improved transmission performance, compared to the frequency-nonseletive channel case [6].

In this paper, for overcoming the PAPR problem of OFDM system, we propose a novel OFDM, called generalized OFDM (GOFDM). The GOFDM is designed to time-multiplex multiple OFDM symbols with reduced number of subcarriers during the time window of fast Fourier transform (FFT), while keeping the transmission data rate the same as the conventional OFDM system. Between OFDM symbols within one FFT time window, the GI is not inserted. Therefore, ISI can distort the GOFDM signal. Viewing the GOFDM signal as a SC signal, we apply various FDE techniques using zero forcing (ZF), maximal ratio (MR), and minimum mean square error (MMSE)



(a) Transmitter



(b) Receiver

Figure 1. Transmission system model.

weights, used in multicarrier CDMA (MC-CDMA) [7, 8] and quite recently in DS-CDMA [9]. The GOFDM system becomes the single carrier (SC) system when the number of subcarriers is reduced to one. So, the proposed GOFDM system can bridge between the SC system and the conventional OFDM system.

Remainder of this paper is organized as follows. In Sect. 2, the GOFDM is presented. Sect. 3 evaluates by the computer simulation the BER performance of GOFDM system in a frequency-selective Rayleigh fading channel and than, compares it with the conventional OFDM and SC systems. Sect. 4 provides some conclusions and future work.

2. GOFDM

2.1. TRANSMIT SIGNAL

Figure 1 illustrates the transmitter/receiver block diagram of the GOFDM system. The sequence of N_s modulated data symbols $\{d(i);\ i=0\sim N_s-1\}$ is transmitted during an FFT time window. $\{d(i)\}$ is divided into K blocks of N_m ($\leq N_s$) data symbols each. The k-th block data sequence is denoted by $\{d^k(i);\ i=0\sim N_m-1\}$, where $\{d^k(i)=d(kN_m+i);\ k=0\sim K-1\}$ and $N_m=N_s/K$. OFDM signal $\{s^k(t)\}$ having N_m subcarriers is generated using $\{d^k(i)\}$ data sequence as shown in Fig. 2.

Throughout the paper, the discrete time signal representation is used. The GOFDM signal can be expressed using the equivalent lowpass representation as

$$s(t)=\sum_{k=0}^{K-1} s^k(t-kN_m)u(t-kN_m)\,,\quad (1)$$

where $s^k(t)$ is the k-th OFDM symbol with N_m subcarriers, given by

$$s^k(t)=\sqrt{\frac{2E_c}{T_c}}\frac{1}{N_m}\sum_{i=0}^{N_m-1} d^k(i)\exp\left[j2\pi t\frac{i}{N_m}\right]\quad (2)$$

with E_c and T_c representing the signal energy per FFT sample and sample period, respectively and $u(t)$=1(0) for $t=0\sim N_m-1$ (elsewhere). The sequence of K OFDM signals makes one frame of GOFDM signal with N_s samples. Figure 3 shows the spectrum of generated GOFDM signal with N_s=8 for K=1, 2, 4 and 8 (N_m=8, 4, 2 and 1, respectively). The bandwidth of the GOFDM signal is the same as that of the conventional OFDM signal (K=1). Before transmission, the last N_g samples of the frame is copied as a cyclic prefix and inserted at the beginning of the frame as the GI. The GI-inserted sample sequence $\tilde{s}(t)$ with N_s+N_g samples is transmitted over the frequency-selective fading channel. Note that in the conventional OFDM system, one frame contains only one OFDM symbol with N_s subcarriers, i.e., K=1. The PAPR of the GOFDM signal is reduced by a factor of $K=N_s/N_m$, compared to the conventional OFDM system.

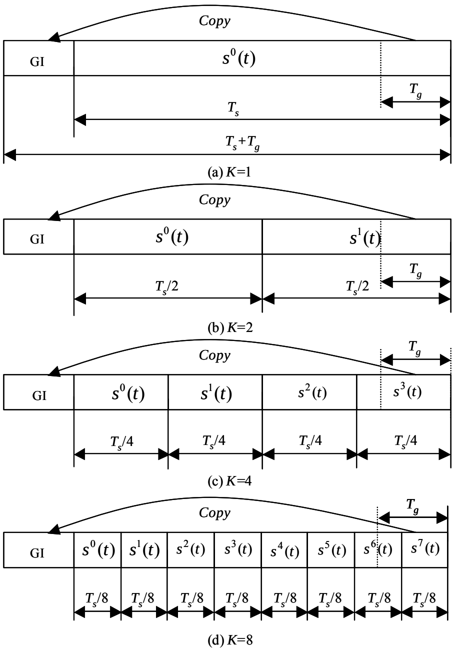


Figure 2. GOFDM signal structure.

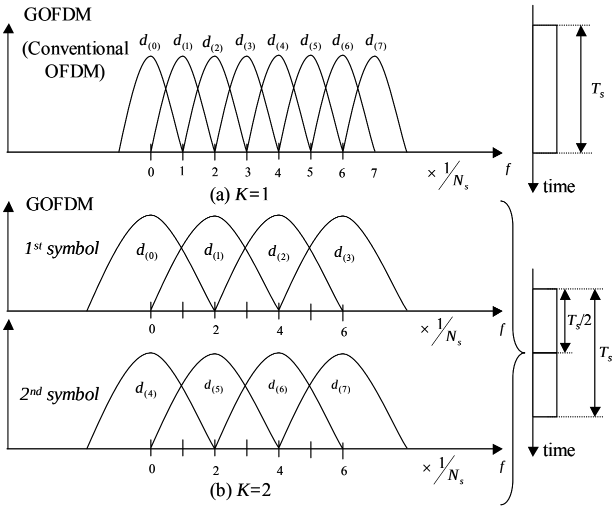


Figure 3. Frequency spectrum of GOFDM signal.

2.2. RECEIVED SIGNAL

In this paper, it is assumed that the fading channel is composed of L discrete propagation paths; the l-th path gain and time delay being denoted by h_l and lT_c, respectively. Discrete-time impulse response $h(t)$ is given as

$$h(t)=\sum_{l=0}^{L-1} h_l\delta(t-lT_c)\ . \quad (3)$$

The received signal can be represented as

$$r(t)=\sum_{l=0}^{L-1}\tilde{s}(t-l)h_l+\eta(t) \quad (4)$$

for t=-N_g~N_s-1, where $\eta(t)$ is the zero-mean noise sample with a variance of $2N_0/T_c$ due to additive white Gaussian noise (AWGN) with single-sided power spectrum density N_0.

2.3. FREQUENCY-DOMAIN EQUALIZATION

The receiver procedure for recovery of N_s decision variables corresponding to the transmitted N_s data modulated symbols is shown in Fig. 4. After removing the GI from the received signal $\{r(t);\ t=0\sim N_s-1\}$, the received signal is decomposed into N_s subcarrier components $\{R(n);\ n=0\sim N_s-1\}$ by applying N_s-point FFT:

$$\begin{aligned}R(n)&=\sum_{t=0}^{N_s-1} r(t)\exp\left[-j2\pi n\frac{t}{N_s}\right]\\&=S(n)H(n)+\Omega(n)\end{aligned}\ , (5)$$

where $S(n)$, $H(n)$ and $\Omega(n)$ are respectively given by

$$\begin{cases}S(n)=\sum_{t=0}^{N_s-1} s(t)\exp\left[-j2\pi n\frac{t}{N_s}\right]\\ H(n)=\sum_{l=0}^{L-1} h_l\exp\left[-j2\pi n\frac{l}{N_s}\right]\\ \Omega(n)=\sum_{t=0}^{N_s-1} \eta(t)\exp\left[-j2\pi n\frac{t}{N_s}\right]\end{cases}\ . (6)$$

Subcarrier-by-subcarrier one-tap FDE is carried out as follows:

$$\hat{R}(n)=w(n)R(n)\ , (7)$$

where $w(n)$ is the equalization weight for the nth subcarrier, given by

$$w(n)=\begin{cases}\dfrac{H^*(n)}{|H(n)|^2} & \text{for ZF}\\ H^*(n) & \text{for MRC}\\ \dfrac{H^*(n)}{|H(n)|^2+\left(\dfrac{E_c}{N_0}\right)^{-1}} & \text{for MMSE}\end{cases}\ . (8)$$

The equalized nth subcarrier component $\hat{R}(n)$ is transformed back to the time-domain signal $\hat{r}(t)$ by applying N_s-point IFFT to obtain:

$$\begin{aligned}\hat{r}(t)&=\frac{1}{N_s}\sum_{n=0}^{N_s-1}\hat{R}(n)\exp\left[j2\pi t\frac{n}{N_s}\right]\\&=\sum_{k=0}^{K-1}\hat{r}^k(t-kN_m)u(t-kN_m)\end{aligned} \quad (9)$$

for t=0~N_s-1, where $\hat{r}^k(t)$ is the k-th OFDM signal with N_m subcarriers. Then, N_m-point FFT is applied to $\hat{r}^k(t)$ to obtain:

$$\hat{d}^k(i)=\sum_{t=0}^{N_m-1}\hat{r}^k(t)\exp\left[-j2\pi i\frac{t}{N_m}\right] \quad (10)$$

for i=0~N_m-1 and k=0~K-1, which is the decision variable for data demodulation.

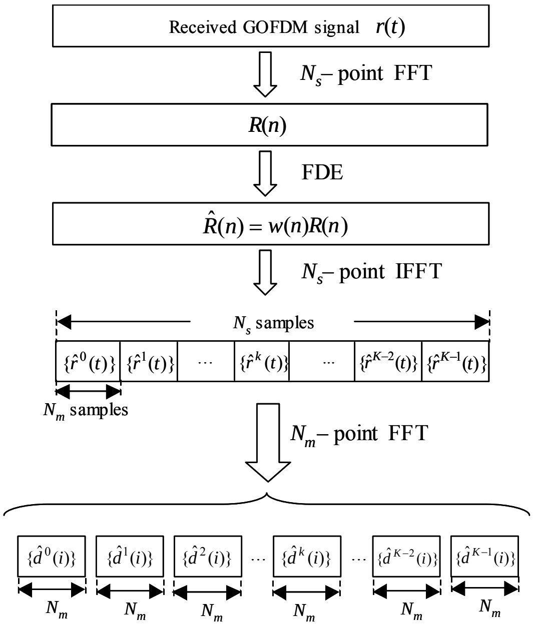


Figure 4. Recovery of N_s decision variables from received GOFDM signal.

3. COMPUTER SIMULATION

The average BER performance of GOFDM system is evaluated by computer simulation. Table 1 summarizes the simulation condition. In the simulation, quadrature phase shift keying (QPSK) data modulation is assumed. The number of OFDM subcarriers is given by $N_m=N_s/K$ with N_s=256 and the GI length is assumed to be N_g=32. The fading channel is assumed to be L=16-path frequency-selective Rayleigh fading channel having exponential power delay profile with decay factor β. Ideal channel estimation is assumed.

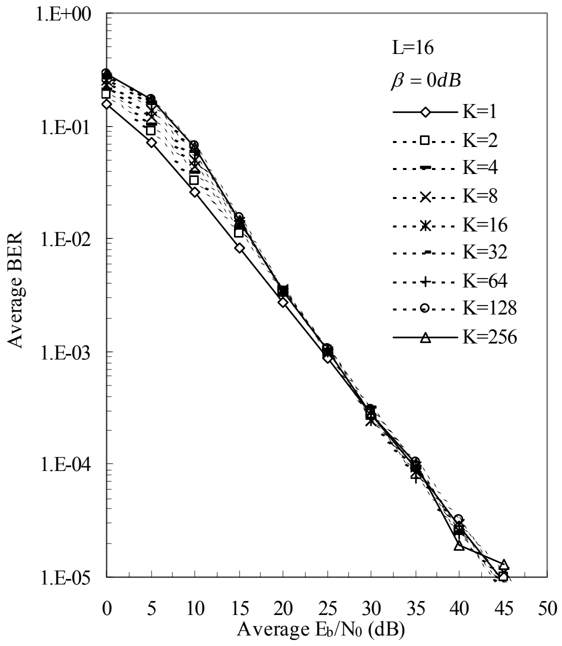
3.1. BER PERFORMANCE

The simulated BER performance of the GOFDM system is plotted in Fig. 5 as a function of the average E_b/N_0 for various values of K (K=1~256), where E_b/N_0=0.5E_c/N_0. It can be clearly understood that the MMSE equalization provides the best BER performance because the noise enhancement is avoided by giving up perfect restoration of frequency-nonselectivity. It can be seen that the BER performance with ZF equalization is almost insensitive to K. Because the ZF perfectly restores the frequency-nonselective channel but produces the noise enhancement, the BER performance is worse than with MMSE. On the other hand, the BER performance with MRC degrades rapidly as K increases due to the enhanced frequency-selectivity, producing the enhanced ISI. In the following, only MMSE is considered.

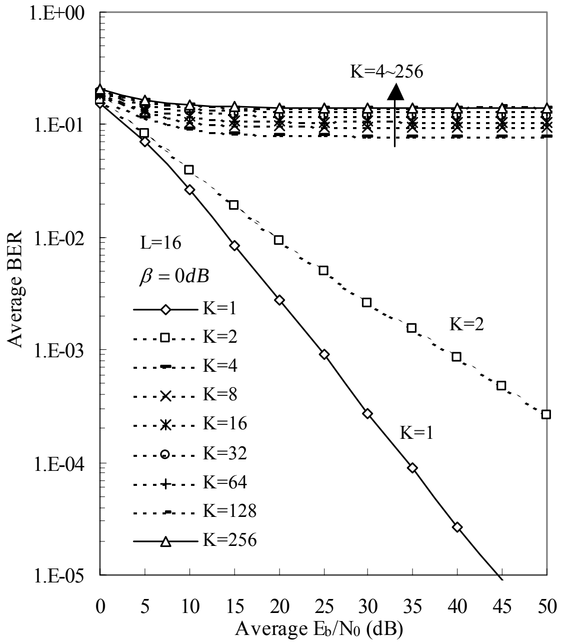
Table 1. Simulation conditions

Transmitter	Data modulation	QPSK
	Number of FFT points	N_m=256/K
	Number of symbols	K=1~256
	Frame length	N_s = 256
	GI	N_g = 32
Channel	Fading	Frequency - selective Rayleigh fading
	Number of paths	L = 16
Receiver	Number of FFT points	N_s = 256 N_m = 256/K
	Frequency-domain equalization	ZF, MRC, MMSE
	Channel estimation	Ideal

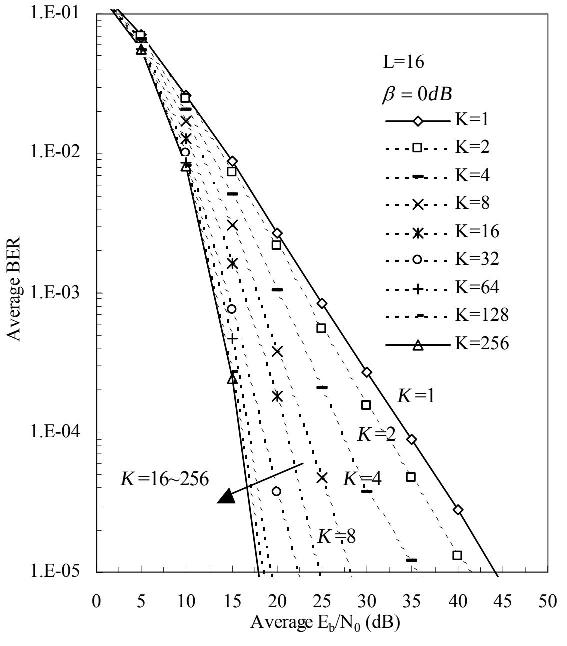
The GOFDM system becomes the conventional OFDM system when K=1 and the SC system when K=256. It is interesting to note that the BER performance with MMSE equalization improves as going from conventional OFDM (K=1) system to the SC (K=256) system. The worst BER performance is provided when K=1, while the best BER performance is achieved when K=256. Figure 6 shows that the BER performance is better when the channel is more frequency-selective (i.e., better BER performance is obtained with β=0 dB than β=2, 4 or 10 dB). If channel is more frequency-selective, the larger frequency diversity effect is obtained, resulting in the improved BER performance.



(a) ZF



(b) MRC



(c) MMSE

Figure 5. Simulated BER performance with *K* as a parameter.

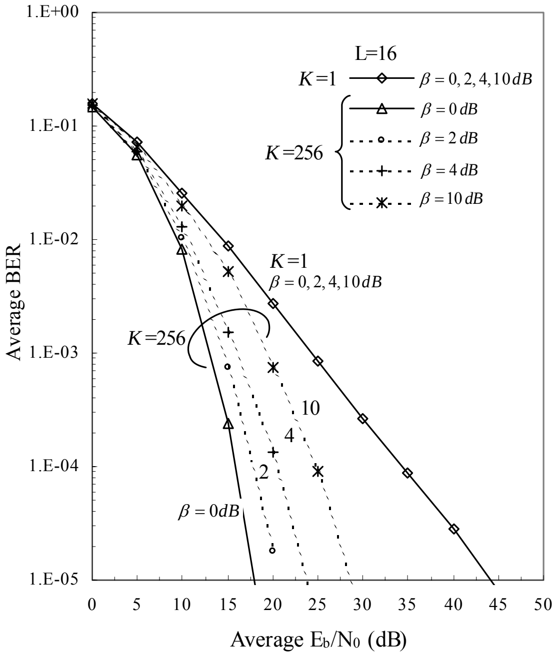


Figure 6. Dependency of the BER performance on β.

4. CONCLUSIONS

In this paper a novel GOFDM system was proposed for alleviating the problem of high PAPR in the OFDM system. With GOFDM, consecutive OFDM symbols with reduced number of subcarriers are time-multiplex during the FFT time window to alleviate the high PAPR problem. The GOFDM fills the gap between SC and OFDM system. GOFDM system achieves a better BER performance in comparison to the conventional OFDM. The BER performance of GOFDM is bounded between conventional OFDM (as upper bound) and SC (as lower bound). Very important to note is that GOFDM does not degrade BER performance at all, compared to the conventional OFDM; on the contrary, the BER performance improves. It is easy to control PAPR of the transmitted GOFDM signal.

In this paper, we have assumed the ideal channel estimation. Channel estimation error may degrade the GOFDM performance. Performance can be improved by antenna diversity reception, transmit diversity, higher-level modulations, coding, etc. These are interesting future work.

REFERENCES

[1] J.G.Proakis and Masoud Salehi, *Communication system engineering*, 2nd Ed., Prentice Hall.

[2] J.G.Proakis, *Digital communications*, McGraw-Hill, 3rd Ed., McGraw Hill.

[3] Richard D.J. van Nee, Ramjee Prasad and Richard Van Nee, *OFDM for wireless multimedia communications,* Artech House, January 2000.

[4] Xiaodong Li and Leonard J. Cimini, “Effects of clipping and filtering on the performance of OFDM”, IEEE Commun. Letters, Vol.2, No.5, May 1998.

[5] Vahid Tarokh and Hamid Jafarkhani, “On the computation and reduction of the peak-to-average power

ration in multicarrier communications", IEEE Trans. Commun., Vol.48, No.1, pp.37-44, Jan. 2000.

[6] D. Falconer, S.L. Ariyavisitakul, A. Benyamin-Seeyar, B. Eidson, "Frequency-domain equalization for single-carrier broadband wireless systems", IEEE Commun. Mag., Vol. 40, pp.58-66, April 2002.

[7] S. Hara and R. Prasad, "Overview of multicarrier CDMA", IEEE Commun. Mag., pp.126-144, Dec 1997.

[8] M. Helard, R. Le Gonable, J.-F. Helard, and J.-Y. Banaais, "Multicarrier CDMA techniques for future wideband wireless access", Ann. Telecommun., Vol. 56, pp. 260-274, 2001.

[9] T. Sao and F. Adachi, "Comparative study of various frequency equalization techniques for downlink of a wireless OFDM-CDMA system", IEICE Trans. Commun., Vol. E86B, No.1, pp.352-364, Jan. 2003.