社団法人 電子情報通信学会
THE INSTITUTE OF ELECTRONICS,
INFORMATION AND COMMUNICATION ENGINEERS

信学技報
TECHNICAL REPORT OF IEICE

周波数選択性フェージングチャンネルにおける周波数領域等化 OFDM/TDM ターボ符号化効果

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あらまし 直交周波数分割多重(OFDM)と時分割多重(TDM)を組み合わせた OFDM/TDM は，シングルキャリア(SC)と OFDM を橋渡しする伝送方式である．平均2乗誤差最小周波数領域等化(MMSE-FDE)を用いる符号化 OFDM/TDM には，符号化利得と周波数ダイバーシチ効果との間にトレードオフが存在する．本論文は，ターボ符号化 OFDM/TDM の平均ビット誤り率(BER)特性を計算機シミュレーションにより明らかにしている．OFDM/TDM では，１フレーム当りのスロット数 (K) が重要な設計パラメータとなる(K=1 (256)は OFDM(SC)に相当する)．Kを大きくすれば周波数ダイバーシチ効果は大きくなるが，周波数インタリーブ効果が小さくなるため符号化利得は小さくなる．本論文は、K がターボ符号化 OFDM/TDM の BER 特性に与える影響は，無符号化時のそれとは異なることを示している．シミュレーション結果より，K=1 及び 256 を用いる OFDM/TDM はほぼ同様の平均 BER を示し，他の K を用いる場合より優れた特性を示すことが分かった．

Turbo coded OFDM/TDM with Frequency-domain Equalization

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Abstract: Orthogonal frequency division multiplexing (OFDM) combined with time division multiplexing (TDM) bridges the conventional OFDM and single carrier (SC) transmission. For a coded OFDM/TDM system with frequency-domain equalization (FDE) based on minimum mean square error (MMSE) criterion in a frequency-selective fading channel, there is a trade-off between coding gain and frequency diversity effect. In this paper, we evaluate by computer simulation the bit error rate (BER) performance of turbo coded OFDM/TDM. The number K of slots in an OFDM/TDM frame is an important design parameter. It was shown that K impacts the BER performance differently in the coded case from uncoded case. As K increases from 1, the better frequency diversity effect is obtained but coding gain tends to be lost due to less frequency interleaving effect; extreme cases are K=1 (conventional OFDM) and K=256 (SC). We have found that the BER performance is almost the same for K=1 and 256 while for other value of K, the BER performance degrades. We also consider the required peak transmit power because it is an important design parameter of transmit power amplifiers. It was found that OFDM/TDM reduces peak transmit power and improves the BER performance in comparison to conventional OFDM; for both uncoded and coded cases, the worst BER performance is achieved with K=1 (conventional OFDM) while the best performance is achieved with K=256 (SC).

Keyword: *OFDM/TDM, frequency-domain equalization, channel coding, frequency-selective fading*

1. Introduction

The next generation mobile communications systems are required to provide very high data rate services. However, the wireless channel for high data transmission becomes severe frequency-selective giving rise to inter-symbol interference (ISI) [1]. The main challenge is to achieve a reliable high data rate transmission under severe frequency-selective fading environment. Orthogonal frequency division multiplexing (OFDM) has been considered as a promising transmission technique for next generation mobile communication systems [1], [2]. However, OFDM has high peak-to-average-power ratio (PAPR) problem [2]. Recently [3], [4], we have shown that, OFDM combined with time division multiplexing (TDM) [5], called OFDM/TDM, alleviates the high PAPR and bridge conventional OFDM and single carrier (SC) transmission when frequency-domain equalization (FDE) based on minimum mean square error (MMSE) criterion is used. The N_c-point inverse fast Fourier transform (IFFT) time window for the conventional OFDM is divided into K slots (which constitutes the OFDM/TDM frame). Within each slot, an OFDM signal with reduced number of subcarriers ($N_m=N_c/K$) is transmitted.

Until now, only the BER performance for uncoded OFDM/TDM was presented. Turbo coding [6] has been under the intensive research for various scenarios using both

SC (e.g., DS-CDMA) and multi-carrier (e.g., OFDM) systems [7], [8]. For uncoded OFDM/TDM, it was found that as K increases the BER performance consistently improves due to larger frequency diversity effect. However, in the coded case, as K increases, each data symbol tends to be spread over a wider bandwidth and hence each data symbol tends to suffer from the similar frequency-selective fading. An extreme case is when $K=N_c$ and all data symbols suffer from the same fading. This reduces the coding gain. Therefore there is a trade off relationship between frequency diversity effect and coding gain. Because of this trade-off the BER performance of the coded OFDM/TDM with MMSE-FDE shows different behavior as K changes compared to uncoded case. In this paper we evaluate, by computer simulation, the turbo coded BER performance of OFDM/TDM with MMSE-FDE and discuss about trade-off (due to change of K) between frequency diversity effect and channel coding gain.

The rest of the paper is organized as follows. Section 2 presents the turbo-coded OFDM/TDM transmission model. Sect. 3 evaluates, by computer simulations, the BER performance of the turbo-coded OFDM/TDM in a frequency-selective Rayleigh fading channel and then, discusses about the frequency diversity-coding gain trade-off. Sect. 4 provides some conclusions.

2. Coded OFDM/TDM Transmission Model

OFDM/TDM transmitter and receiver structures are illustrated in Fig. 1. The OFDM/TDM frame structure is illustrated in Fig. 2 in comparison to conventional OFDM with N_c subcarriers. In OFDM/TDM, we concatenate K slots, during each slot an OFDM signal with reduced number N_m ($=N_c/K$) of subcarriers is transmitted. The structure of turbo encoder and iterative decoder is shown in Fig. 3. In this paper, we use turbo encoder that is consisted of two parallel-concatenated recursive systematic convolutional (RSC) encoders C_1 and C_2, connected by a internal interleaver of size N. Overall coding rate is $R=1/3$, but in this paper, puncturing is used to achieve a coding rate of $R=1/2$.

2.1. OFDM/TDM Transmit Signal

The information bit sequence of length N is turbo coded with coding rate R, bit interleaved and transformed into the data-modulated symbol sequence $\{d_c(i);\ i=0\sim N/R-1\}$. This sequence is divided into m ($=N/RN_c$) frames of N_c symbols each. The m-th frame symbol sequence is denoted as $\{d^m(i);\ i=0\sim N_c-1\}$, where $d^m(i)=d(mN_c+i)$ with $E[|d(i)|^2]=1$ ($E[\cdot]$ denotes the ensemble average operation). For OFDM/TDM modulation [3], [4], an each frame is divided into K slots with N_m symbols and an N_m-symbol sequence in each slot is fed into N_m-point IFFT to generate OFDM signal with N_m subcarriers. The resulting K OFDM signals are concatenated to form an OFDM/TDM frame with a length of $N_c=KN_m$ samples, where each OFDM signal is transmitted during one slot, as shown in Fig. 2 (b). The resulting OFDM/TDM signal in one frame can be expressed using the equivalent lowpass representation as

$$s(t)=\sqrt{\frac{2E_s}{T_cN_m}}\sum_{i=0}^{N_m-1}d^{\left\lfloor t/N_m\right\rfloor}(i)\exp\left[j2\pi t\frac{i}{N_m}\right] \qquad (1)$$

for $t=0\sim N_c-1$, where E_s and T_c represent the symbol energy and the sampling period, respectively and $\lfloor x\rfloor$ represents the largest integer smaller than or equal to x. Finally, N_g sample GI is inserted at the beginning of OFDM/TDM frame and transmitted over the a frequency-selective fading channel.

2.2. Received Signal and FDE

An L-path channel is assumed. The received time-domain signal can be expressed as

$$r(t)=\sum_{l=0}^{L-1}h_l s(t \bmod N_c-\tau_l)+\eta(t) \qquad (2)$$

for $t=-N_g\sim N_c-1$, where h_l and τ_l are the l-th path gain and time delay, and $\eta(t)$ is the zero-mean complex Gaussian noise process with a variance of $2N_0/T_c$ due to the additive white Gaussian noise (AWGN). After removal of the GI, the received signal is decomposed into N_c frequency components $\{R(k);\ k=0\sim N_c-1\}$ by applying N_c-point FFT over the entire OFDM/TDM frame as

$$\begin{aligned}R(n)&=\frac{1}{N_c}\sum_{t=0}^{N_c-1}r(t)\exp\left[-j2\pi n\frac{t}{N_c}\right],\\&=S(n)H(n)+\Omega(n)\end{aligned} \qquad (3)$$

where $S(n)$, $H(n)$ and $\Omega(n)$ are the signal component, the propagation channel gain and the noise component at the nth frequency, respectively, and are given by

$$\begin{cases}S(n)=\dfrac{1}{N_c}\displaystyle\sum_{t=0}^{N_c-1}s(t)\exp\left[-j2\pi n\frac{t}{N_c}\right]\\H(n)=\displaystyle\sum_{l=0}^{L-1}h_l\exp\left[-j2\pi n\frac{\tau_l}{N_c}\right]\\\Omega(n)=\dfrac{1}{N_c}\displaystyle\sum_{t=0}^{N_c-1}\eta(t)\exp\left[-j2\pi n\frac{t}{N_c}\right]\end{cases}. \qquad (4)$$

Since, the GI is not inserted between consecutive slots but only at the beginning of each OFDM/TDM frame, the ISI may arise due to frequency-selective fading and degrade the BER performance. To overcome this problem, we apply one-tap FDE as

$$\hat{R}(n)=w(n)R(n) \qquad (5)$$

where $w(k)$ is the MMSE equalization weight given by [9]

$$w(n)=\frac{H^*(n)}{|H(n)|^2+\left(\dfrac{E_s}{N_0}\right)^{-1}}. \qquad (6)$$

By applying N_c-point IFFT to $\{\hat{R}(n);\ n=0\sim N_c-1\}$, we obtain the time-domain OFDM/TDM signal $\hat{r}(t)$ as

$$\hat{r}(t)=\frac{1}{N_c}\sum_{n=0}^{N_c-1}\hat{R}(n)\exp\left(j2\pi t\frac{n}{N_c}\right) \qquad (7)$$

for $t=0\sim N_c-1$.

2.3. OFDM/TDM Demodulation

The decision variable for the ith data symbol of the kth OFDM signal can be obtained by applying N_m-point FFT as [3], [4]

$$\hat{d}^k(i)=\frac{1}{N_m}\sum_{t=kN_m}^{(k+1)N_m-1}\hat{r}(t)\exp\left(-j2\pi i\frac{t-kN_m}{N_m}\right),(8)$$

2.4. Turbo Decoding

Turbo decoding principle is based on iterative algorithm that requires soft decision values as input. Log-likelihood ratio (LLR) approximation is used for the generation of the soft values. The decision variable expressed by Eq. (8) includes the ISI and noise due to AWGN. Assuming that the ISI can be approximated as a zero-mean complex-valued Gaussian variable, the sum of ISI and noise due to the AWGN can be treated as a new zero-mean complex-valued Gaussian noise with variance: $2\sigma^2$.

The LLR approximation for the bth bit in the nth symbol (b=0 and 1) is given by [10], [8]

$$L(b)\approx\frac{\left|\hat{d}_c(n)-\hat{H}(n)\hat{s}_0\right|^2}{2\sigma^2}-\frac{\left|\hat{d}_c(n)-\hat{H}(n)\hat{s}_1\right|^2}{2\sigma^2}. \quad (9)$$

Here, $\hat{s}_0$ or ($\hat{s}_1$) is the candidate symbol, with 0 (or 1) in the bth bit position, for which the Euclidian distance from $\hat{d}_c(n)$ is minimum. The LLR values are computed for all the bits in the symbol and turbo decoding is performed using these LLR values as soft input. The iterative decoding process is shown in Fig. 3 (b).

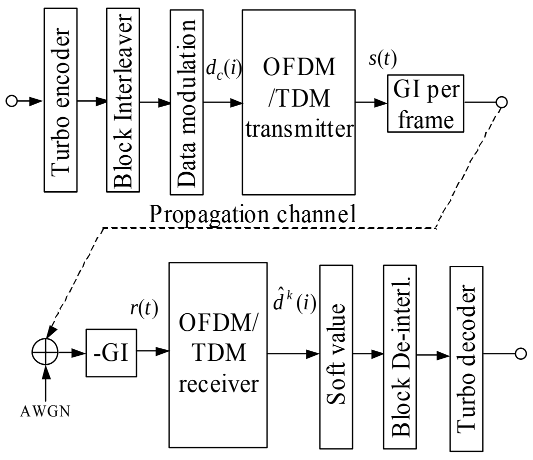
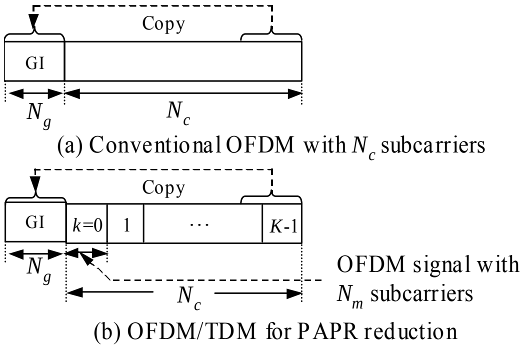


Figure 1. OFDM/TDM transmission system model.

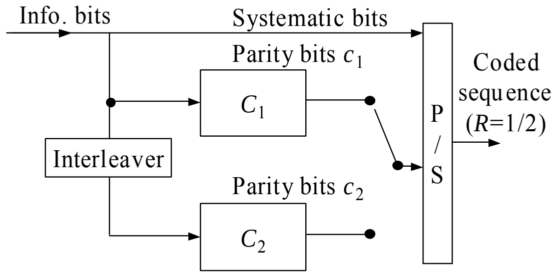
(UBACITI d^c poslije soft value)



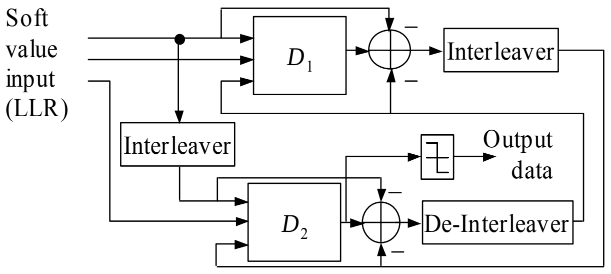
(a) Conventional OFDM with N_c subcarriers

(b) OFDM/TDM for PAPR reduction

Figure 2. OFDM/TDM frame structure.



(a) Encoder



(b) Iterative decoder

Figure 3. Turbo coder and decoder.

3. Performance Evaluation

3.1. Simulation conditions

The simulation parameters are shown in Table 1. We assume QPSK modulation with $|d(i)|$=1, N_c=256 and N_g=32. The propagation channel is an L=16-path frequency-selective block Rayleigh fading channel having exponential power delay profile with decay factor β as shown in Fig. 4. It is assumed that the l-th path time delay is l samples (i.e., $\tau_l=l$) where path gains stay constant over the one OFDM/TDM frame and all the path delays are within the GI. The normalized Doppler frequency f_DT_s=0.0014, which corresponds to a mobile velocity of about 100 km/h when the carrier frequency is 5 GHz and the transmission data rate is 100 Msymbols/s; where $1/T_s=1/(T_c(1+N_g/N_c))$ is the transmit symbol rate. We assume ideal channel estimation.

A rate 1/3 turbo encoder with constraint length 4 and (13, 15) RSC component encoders is assumed. The parity sequences are punctured to obtain coding rate R=1/2. The internal interleaver for turbo coding is S-random ($S=N^{1/2}$) interleaver. The data sequence length is taken to be N=1024 bits. Before data-modulation the turbo coded and punctured sequence is interleaved by channel interleaver. A block interleaver used as channel interleaver in the simulation is of size $2^a \times 2^b$ block interleaver, where a and b are the maximum allowable integers for a given sequence size so that we can obtain an interleaver as close as possible to a square one. Log-MAP decoding with 8 iterations is carried out at the receiver.

Table 1. Simulation parameters.

Transmitter	Data modulation	QPSK
	No. of IFFT points	N_m=256/K
	No. of slots per frame	K=1~256
	Frame length	N_c=256
	GI	N_g=32
Turbo coding	Coding rate R=1/2 (13, 15) RSC encoder, Log-MAP decoding with 8 iterations	
Channel interleaver	Block interleaver	
Channel	L=16-path frequency-selective block Rayleigh fading with decay factor β	
Receiver	No. of FFT points	N_c=256
	FDE	MMSE
	Channel estimation	Ideal

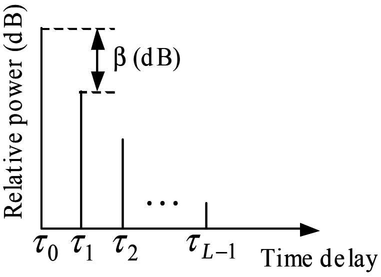


Figure 4. Channel power delay profile.

3.2. Average Power Efficiency

3.2.1. Impact of *K*

Figure 5 plots the average BER performance of the uncoded and coded OFDM/TDM with MMSE-FDE as a function of the average bit energy-to-AWGN power spectrum density ratio E_b/N_0, where $E_b/N_0=0.5\times R\times(E_s/N_0)\times(1+N_g/N_c)$, with K as a parameter. For uncoded OFDM/TDM system it was shown that [3], [4] MMSE-FDE takes advantage of frequency diversity effect and the BER performance improves due to enhanced frequency diversity effect as K increases.

However, for coded OFDM/TDM there is a trade-off between coding gain and frequency diversity gain. For K=1, there is no frequency diversity gain at all and the BER performance improves only because of coding gain due to frequency interleaving effect. For K=2 and 4, the frequency diversity gain due to MMSE-FDE slightly increases (as K increases [3]) but the main reason for BER improvement in comparison to uncoded case is due to coding gain. However, for higher K, coding gain decreases due to less frequency interleaving effect but the frequency diversity gain increases due to MMSE-FDE and the BER performance improves in comparison to uncoded case.

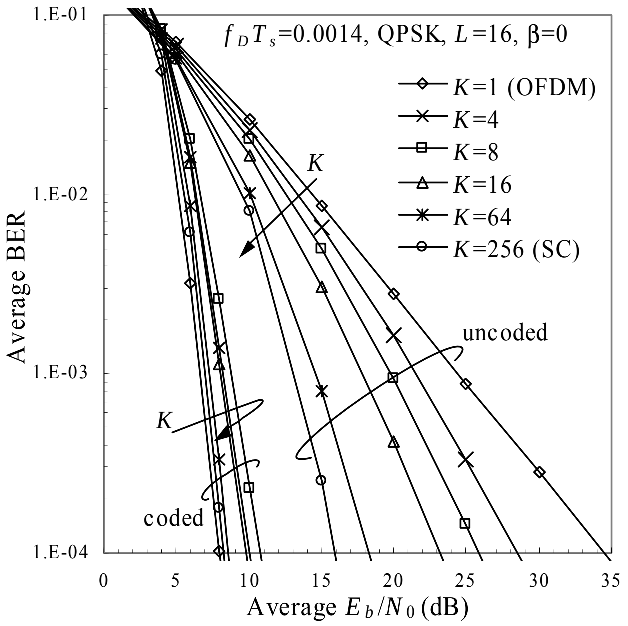


Figure 5. Average BER performance versus average E_b/N_0.

3.2.2. Impact of β

The channel frequency-selectivity is determined by the decay factor β. As β increases, the channel is becoming less frequency-selective and when β→∝ dB becomes a frequency-nonselective channel (single-path channel). Fig. 6 plots the uncoded and coded average BER performance as a function of the decay factor β. For uncoded case, the best BER performance is obtained always with K=256 and gradually degrades to K=1. However, for coded case, K=1 (OFDM) always provide the best performance irrespective of β. K=256 (SC) is worse but very close to K=1 (OFDM). The worst case is K=4~16 irrespective of β; however the performance gap between K=1 and 4 (or 16) becomes narrower as β increases.

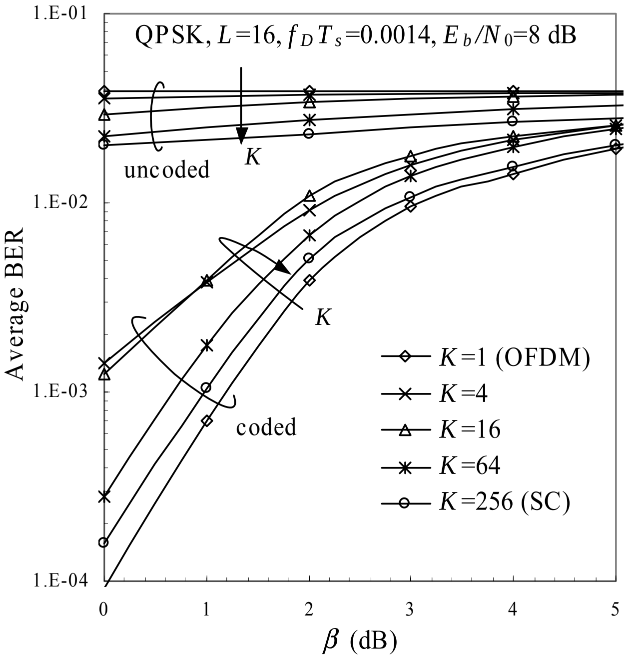


Figure 6. Impact of β on coded average BER.

3.3. Peak Power Efficiency

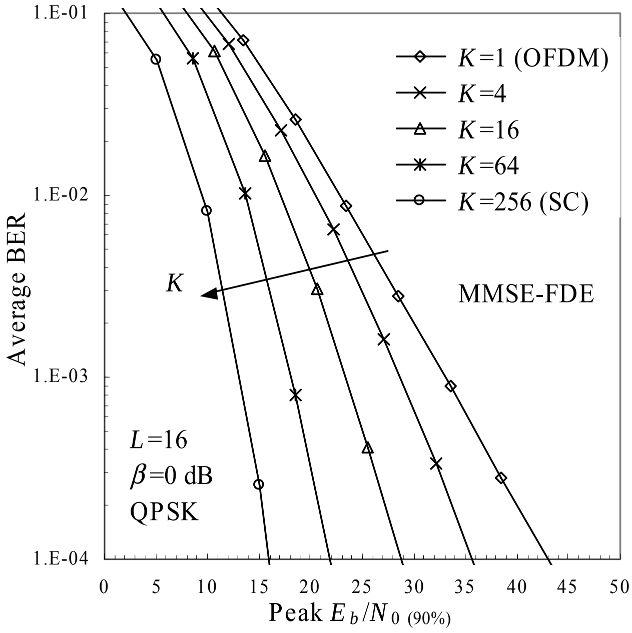
We also need to consider the required peak transmit power because it is an important design parameter of transmit power amplifiers. For conventional OFDM transmission, high PAPR causes signal degradation due to non-linear power amplification and the BER performance degrades. In this section, we evaluate effect of peak transmit power on the BER performance of OFDM/TDM with MMSE-FDE.

3.3.1. Impact of *K*

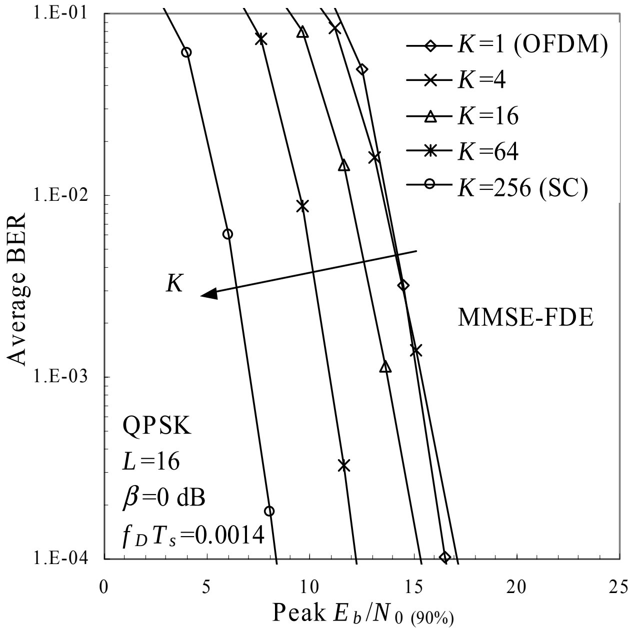
Figs. 7. (a) and (b), respectively, plot the achievable BER performance of the uncoded and coded OFDM/TDM with MMSE-FDE as a function of the peak transmit power for various values of *K* (*K*=1~256). We consider the PAPR$_{10\%}$ level, which the PAPR of OFDM/TDM exceeds with a probability of 10%. PAPR$_{10\%}$ is about 8.5, 7.2, 5.7 and 3.7 dB for *K*=1, 16, 64 and 256, respectively. We assume uniform power delay profile (β=0 dB), which produces the strongest frequency-selectivity for the given number of paths.

For uncoded case the required peak power for the average BER=10^{-4} can be reduced by about 7.3, 14.2, 21.1 and 27 dB, compared to the conventional OFDM, when *K*=4, 16, 64 and 256, respectively (see Fig. 7(a)). For coded case *K*=1 and 4 gives almost the same BER performance while, for other *K* values, the required peak power for the average BER=10^{-4} can be reduced by about 1.7, 5 and 8.6 dB, compared to the conventional OFDM, when *K*=16, 64 and 256, respectively (see Fig. 7(b)).

For both uncoded and coded cases, the worst BER performance is provided when *K*=1 (conventional OFDM), while the best BER performance is achieved when *K*=256 (SC). Since peak transmit power is a very important parameter in system design, we can see that the benefit of the coded OFDM vanishes. However, for OFDM/TDM, as *K* increases the required transmit peak power reduces and the BER performance improves in comparison to conventional OFDM.



(a) Uncoded

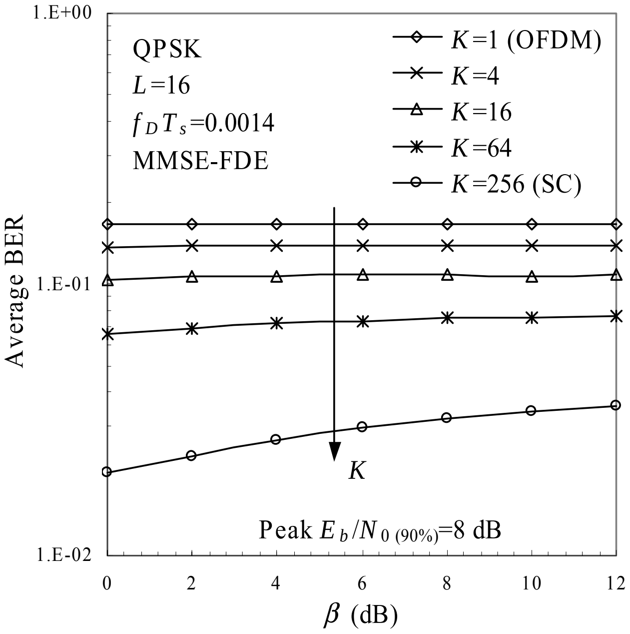


(b) Coded

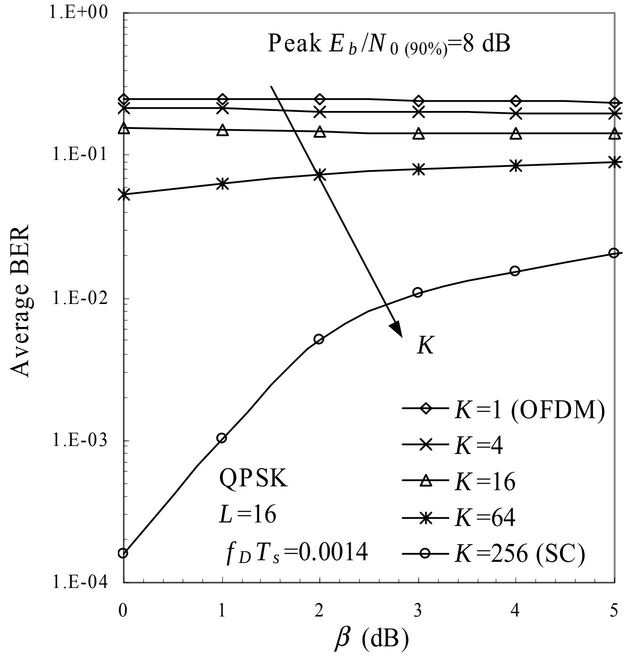
Figure 7. Average BER performance versus peak E_b/N_0.

3.3.2. Impact of β

Fig. 8 (a) and (b), respectively, plot the uncoded and coded average BER performance as a function of the decay factor β. For both uncoded and coded cases, the best BER performance is obtained with *K*=256 (SC) and the worst with *K*=1 (conventional OFDM) irrespective of β.



(a) Uncoded



(b) Coded

Figure 8. Impact of β

4. Conclusions

In this paper, we evaluated the BER performance of the turbo coded OFDM/TDM with MMSE-FDE by computer simulations in a frequency-selective Rayleigh fading channel.

When channel coding is used, a trade-off among frequency diversity gain and coding gain due to better frequency interleaving effect is observed; as K increases, the coding gain decreases while frequency diversity gain increases. The BER performance is almost the same for K=1 (conventional OFDM) and 256 (SC) while for other K, the BER performance degrades.

OFDM/TDM reduces peak transmit power and improves the BER performance in comparison to conventional OFDM; for both uncoded and coded cases, the worst BER performance is achieved with K=1 (conventional OFDM) while the best performance is achieved with K=256 (SC).

Acknowledgment

The authors would like to thank Ms. Deepshikha Garg for her helpful support and encouragement.

References

[1] S. Hara and R. Prasad, *Multicarrier Techniques for 4G Mobile Communications*, Artech House, June 2003.

[2] R. D.J. van Nee, R. Prasad and R. van Nee, *OFDM for wireless multimedia communications,* Artech House, Jan. 2000.

[3] H. Gacanin, et. al., “Generalized OFDM with frequency-domain equalization,” IEICE Technical Report, RCS2003-306, pp.19-24, March 2004.

[4] H. Gacanin, et al., “Generalized OFDM for Bridging between OFDM and Single-carrier transmission,” Proc. 9th Intern. Conf. on Comm. Syst., Singapore, Sept. 2004.

[5] C. V. Sinn, et. al.,“Common architectures for TD-CDMA and OFDM based mobile radio systems without the necessity of a cyclic prefix,” MS-SS Workshop, Oberpfaffenhofen, Germany, Sept. 2001.

[6] C. Berrou, A. Glavieux, and P. Thitimajshima, “Near optimum error correcting coding and decoding: Turbo codes,” IEEE Trans. Comm., vol. 44, pp. 1261-1271, 1996.

[7] D. Garg and F. Adachi, “Performance comparison of turbo-coded DS-CDMA, MC-CDMA and OFDM with frequency-domain equalization and higher-level modulation,” Proc. of IEEE VTC05-Fall, USA, Sept. 2005.

[8] D. Garg and F. Adachi, “Diversity-coding-orthogonality trade-off for coded MC-CDMA with high level modulation,” IEICE Trans. Comm., Vol.E88-B, No.1, pp.76-83, Jan. 2005.

[9] S. Hara and R. Prasad, “Overview of multicarrier CDMA,” IEEE Commun. Mag., pp.126-144, Dec. 1997.

[10] A. Stefanov and T. Duman, “Turbo coded modulation for wireless communications with antenna diversity,” Proc. IEEE VTC99-Fall, pp. 1565-1569, Netherlands, Sept. 1999.