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# STTD とアンテナダイバーシチを用いる MMSE 周波数領域等化 OFDM/TDM の理論検討

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あらまし 本論文は, 周波数選択性フェージングチャネルにおける STTD とアンテナ受信ダイバーシチを用いるときの MMSE 周波数領域等化 OFDM/TDM の平均ビット誤り率(BER)を理論的に検討している．FDE 後の残留符号間干渉成分をガウス近似することにより，チャネル利得が与えられた時の条件付 BER を導出している．モンテカルロ数値計算により，MMSE 規範に基づく FDE を用いる OFDM/TDM の平均 BER 特性を理論的に明らかにしている．

# Performance Analysis of Joint STTD and Receive Antenna Diversity with MMSE-FDE for OFDM/TDM

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**Abstract**: In this paper, a theoretical BER analysis of orthogonal frequency division multiplexing combined with time division multiplexing (OFDM/TDM) is presented for joint use of space-time transmit diversity (STTD) and receive antenna diversity with minimum mean square error (MMSE) frequency-domain equalization (FDE). The conditional BER expression is derived, based on a Gaussian approximation of the inter-symbol interference (ISI) arising from channel frequency-selectivity, for the given set of channel gains. The average BER performance is evaluated by Monte-Carlo numerical computation method using the derived conditional BER expression.

**Keywords**: *OFDM, time division multiplexing, frequency-domain equalization, frequency-selective fading.*

## 1. Introduction

Orthogonal frequency division multiplexing (OFDM) has been adopted as a signaling technique for wireless local area network (WLAN) in both HyperLan II and IEEE 802.11 standards [1]. However, OFDM signals have a problem with high peak-to-average power ratio (PAPR). Recently [2], OFDM combined with time division multiplexing (OFDM/TDM) [3] was proposed to alleviate this problem and it has been shown that OFDM/TDM with minimum mean square error (MMSE) frequency-domain equalization (FDE) can bridge conventional OFDM and single carrier (SC) transmissions owing to frequency diversity effect while reducing the PAPR. For achieving further transmission performance improvement, the use of multiple transmit and receive antennas is very effective. Recently, space-time transmit diversity (STTD) [4] has been gaining much attention since its use at a base station alleviates the complexity problem of mobile receivers. When STTD encoding is applied to OFDM signal transmission each subcarrier is encoded independently to achieve spatial diversity gain.

Recently [5], we proposed joint frequency-domain STTD and antenna diversity reception based on MMSE criterion for OFDM/TDM transmissions. At a transmitter, an OFDM/TDM signal frame with $K$ OFDM signals, where each OFDM signal has $N_c/K$ subcarriers ($N_c$ is the fast Fourier transform (FFT) window size for FDE at a receiver) is generated. Then, unlike conventional STTD encoding, we perform STTD encoding for OFDM/TDM on a frame-by-frame basis (i.e., not on each OFDM signal separately). The encoding is performed over each OFDM/TDM frequency component unlike the conventional STTD encoding which encodes each OFDM subcarrier independently. At a receiver, frame-by-frame based joint frequency-domain STTD decoding and antenna diversity reception based on MMSE criterion is carried out to achieve both spatial and frequency diversity gains.

So far, we have presented only the computer simulation results to show the average BER performance of OFDM/TDM with joint STTD and receive antenna diversity based on MMSE criterion. This paper is intended to give a theoretical foundation to this topic. The paper is organized as follows. Sect. 2 briefly presents the STTD encoded OFDM/TDM transmission system model. An expression for the conditional BER in a frequency-selective Rayleigh fading channel is derived for the given set of channel gains in Sect. 3. In Sect. 4, the theoretical average BER performance is evaluated by Monte-Carlo numerical computation method using the derived BER expression and compared with the computer simulation results to confirm the theoretical analysis. Section 5 concludes the paper.

## 2. Joint STTD and Receive Antenna Diversity Reception Based on MMSE Criterion [5]

Throughout the paper, discrete-time representation of OFDM/TDM signal is used. The OFDM/TDM transmission system model with joint STTD, antenna diversity reception, and MMSE-FDE is shown in Fig. 1. We use STTD with $N_t$=2 transmit antennas and $N_r$-receive antennas.

### 2.1. OFDM/TDM signal

For OFDM/TDM transmission, a data-modulated symbol sequence is mapped into the sequence of $N_c$-symbol data frames. The $q$th frame of the data-modulated symbol sequence is denoted as $\{d_q(i);\ i=0\sim N_c-1\}$. Then, the $q$th frame sequence is divided into $K$ blocks of $N_m$ symbols. The $k$-th block symbol sequence is denoted as $\{d_q^k(i);\ i=0\sim N_m-1\}$, where $d_q^k(i)=d_q(kN_m+i)$ with $\left|d_q^k(i)\right|=1$. $N_m$-point IFFT is applied to each data block to generate a sequence of $K$ OFDM signals with $N_m=N_c/K$ subcarriers as shown in Fig. 2. The OFDM/TDM signal of the $q$th modulated-data frame can be expressed using equivalent lowpass representation as [2]

$$s_q(t)=s_q^{\lfloor t/N_m \rfloor}(t-kN_m) \tag{1}$$

for $t=0\sim N_c-1$, where $\lfloor x \rfloor$ represents the largest integer smaller than or equal to $x$ and $s_q^{\lfloor t/N_m \rfloor}(t)$ is the OFDM signal with $N_m$ subcarriers, given by

$$s_q^{\lfloor t/N_m \rfloor}(t)=\sum_{i=0}^{N_m-1} d_q^{\lfloor t/N_m \rfloor}(i)\exp\left[j2\pi t\frac{i}{N_m}\right] \tag{2}$$

for $t=0\sim N_m-1$.

### 2.2. Frame-by-Frame STTD Encoding

Unlike STTD encoding for conventional OFDM, the STTD encoding for OFDM/TDM is performed on each frequency component [5]. In the even OFDM/TDM frame interval, the STTD encoded OFDM/TDM signals, to be transmitted from the first and second antennas, are given as [5]

$$\begin{cases}\text{Antenna } 0: s_e(t)\\ \text{Antenna } 1: s_o(t)\end{cases} \quad \text{for } t=0\sim N_c-1.$$

In the odd OFDM/TDM frame interval, the STTD encoded signals, to be transmitted from the first and second antennas, are given as [5]

$$\begin{cases}\text{Antenna } 0: -s_o^*(N_c-t)\\ \text{Antenna } 1: s_e^*(N_c-t)\end{cases} \quad \text{for } t=N_c\sim 2N_c-1.$$

After inserting the GI at the beginning of the each STTD encoded OFDM/TDM signal frame, the signal is multiplied by the power coefficient $\sqrt{E_s/T_cN_m}$, where $E_s$ and $T_c$ represent the symbol energy and the sampling period, respectively, and transmitted over a frequency-selective fading channel.

### 2.3. Joint STTD Decoding and Antenna Diversity Reception Based on MMSE Criterion

We assume an $L$-path frequency-selective block-fading channel, where the path gains remain constant during the period of two frame intervals. STTD encoded received signals are decomposed by $N_c$-point FFT into $N_c$ frequency components as

$$\begin{cases}R_{e,m_r}(n)=H_{0,m_r}(n)S_e(n)+H_{1,m_r}(n)S_o(n)+N_{e,m_r}(n)\\ R_{o,m_r}(n)=H_{1,m_r}(n)S_e^*(n)-H_{0,m_r}(n)S_o^*(n)+N_{o,m_r}(n)\end{cases}, \tag{3}$$

where $S_{e(\text{or } o)}(n)$ is the $n$th frequency component of the transmitted OFDM/TDM signal given as $N_c$-point FFT of Eq. (1). $H_{m_t,m_r}(n)$ and $N_{e(\text{or } o),m_r}(n)$ denote the channel gain between the $m_t$-th transmit antenna and the $m_r$-th receive antenna and the AWGN noise component, respectively, given as

$$\begin{cases}H_{m_t,m_r}(n)=\sqrt{\dfrac{E_s}{T_cN_m}}\displaystyle\sum_{l=0}^{L-1} h_{m_t,m_r}^l \exp\left(-j2\pi n\frac{\tau_l}{N_c}\right)\\ N_{e(\text{or } o),m_r}(n)=\displaystyle\sum_{t=0}^{N_c-1} n_{e(\text{or } o),m_r} \exp\left(-j2\pi n\frac{t}{N_c}\right)\end{cases}, \tag{4}$$

where $n_{e,m_r}(t)$ and $n_{o,m_r}(t)$ represent the independent additive white Gaussian noise (AWGN) processes for the even and odd OFDM/TDM frame intervals having zero mean and variance $2N_0/T_c$ with $N_0$ being the single-sided power spectrum density. $h_{m_t,m_r}^l$ and $\tau_l$ in Eq. (4) are the path gain and the time delay of the $l$th path corresponding to the $m_t$-th transmitter antenna and the $m_r$-th receiver antenna, respectively.

The joint STTD decoding and antenna diversity reception is carried out on each frequency component as:

$$\begin{cases}\hat{R}_e(n)=\displaystyle\sum_{m_r=0}^{N_r-1}\left\{w_{0,m_r}^*(n)R_{e,m_r}(n)+w_{1,m_r}(n)R_{o,m_r}^*(n)\right\}\\ \hat{R}_o(n)=\displaystyle\sum_{m_r=0}^{N_r-1}\left\{w_{1,m_r}^*(n)R_{e,m_r}(n)-w_{0,m_r}(n)R_{o,m_r}^*(n)\right\}\end{cases}, \tag{5}$$

where $w_{0(\text{or } 1),m_r}(n)$ is the MMSE weight corresponding to the 0-th (1-th) transmit antenna and the $m_r$-th receive antenna, given by [6]

$$\begin{cases}w_{0,m_r}(n)=\dfrac{H_{0,m_r}(n)}{\displaystyle\sum_{m_t=0}^{1}\sum_{m_r=0}^{N_r-1}\left|H_{m_t,m_r}(n)\right|^2+\left(\frac{1}{2}\frac{E_s}{N_0}\right)^{-1}}\\ w_{1,m_r}(n)=\dfrac{H_{1,m_r}(n)}{\displaystyle\sum_{m_t=0}^{1}\sum_{m_r=0}^{N_r-1}\left|H_{m_t,m_r}(n)\right|^2+\left(\frac{1}{2}\frac{E_s}{N_0}\right)^{-1}}\end{cases}. \tag{6}$$

By applying $N_c$-point IFFT to $\{\hat{R}_{e(\text{or } o)}(n);\ n = 0 \sim N_c - 1\}$, the time-domain OFDM/TDM signal $\hat{r}_{e(\text{or } o)}(t)$ is recovered as

$$\hat{r}_{e(\text{or } o)}(t) = \frac{1}{N_c}\sum_{n=0}^{N_c-1}\hat{R}_{e(\text{or } o)}(n)\exp\left[j2\pi t\frac{n}{N_c}\right] \quad (7)$$

for $t$=0~$N_c$-1. Then, $N_m$-point FFT is applied to $\hat{r}_{e(\text{or} o)}(t)$ to obtain

$$\hat{d}^{k}_{e(\text{or } o)}(i) = \frac{1}{N_m}\sum_{t=kN_m}^{(k+1)N_m-1}\hat{r}_{e(\text{or } o)}(t-kN_m)\exp\left[-j2\pi i\frac{t}{N_m}\right] \quad (8)$$

for $i$=0~$N_m$-1 and $k$=0~$K$-1, which is the decision variable for data demodulation of the even and odd OFDM/TDM frames.

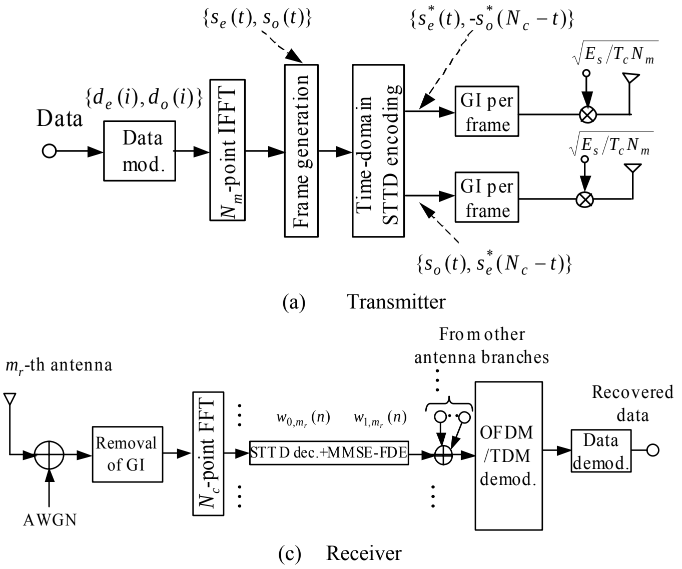


Figure 1. OFDM/TDM transmitter/receiver structure.

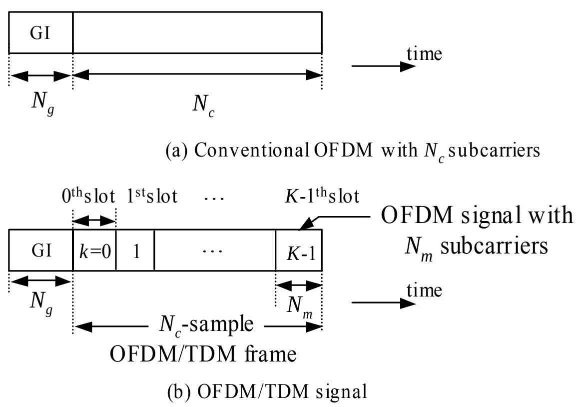


Figure 2. OFDM/TDM frame structure.

## 3. BER Analysis

The conditional BER is derived based on the Gaussian approximation of the ISI and then, the theoretical average BER performance is evaluated by Monte-Carlo numerical computation method. In the following analysis, we assume ideal CE and block fading (i.e., the path gains remain constant over two signaling intervals).

Substituting Eq. (5) into Eq. (7), we obtain

$$\begin{aligned}\hat{r}_e(t) &= s_e(t)\left[\frac{1}{N_c}\sum_{n=0}^{N_c-1}\sum_{m_r=0}^{N_r-1}\left(\hat{H}_{0,m_r}(n)+\hat{H}_{1,m_r}(n)\right)\right] \\ &+ \frac{1}{N_c}\sum_{t'=0}^{N_c-1}s_e(t')\left[\begin{array}{l}\sum_{n=0}^{N_c-1}\sum_{m_r=0}^{N_r-1}\left(\hat{H}_{0,m_r}(n)+\hat{H}_{1,m_r}(n)\right)\\ \times\exp\left(j2\pi n\frac{t-t'}{N_c}\right)\end{array}\right], \\ &+ \sum_{m_r=0}^{N_r-1}\left[\hat{n}_{e,m_r}(n)+\hat{n}_{o,m_r}(n)\right]\end{aligned} \quad (9)$$

where

$$\begin{cases}\hat{H}_{0,m_r}(n) = H_{0,m_r}(n)w^*_{0,m_r}(n)\\ \hat{H}_{1,m_r}(n) = H^*_{1,m_r}(n)w_{1,m_r}(n)\end{cases}. \quad (10)$$

The first term in Eq. (9) represents the desired signal component, the second term is the ISI component and the third term is the noise component. Based on the Gaussian approximation of the ISI, the sum of ISI and noise due to the AWGN is treated as a new zero-mean complex-valued Gaussian noise.

Since different data symbols are independent, i. e., $E\left[d^k(i)d^{k*}(i')\right] = \delta(i-i')$, the ISI variance is given as

$$2\sigma^2_{ISI} = \frac{E_s}{T_cN_c}\sum_{m_r=0}^{N_r-1}\sum_{n=0}^{N_c-1}\left|\begin{array}{l}\hat{H}_{0,m_r}(n)+\hat{H}_{1,m_r}(n)\\ -\frac{1}{N_c}\sum_{m=0}^{N_c-1}\left[\hat{H}_{0,m_r}(m)+\hat{H}_{1,m_r}(m)\right]\end{array}\right|^2\left|\Psi(n)\right|^2 . \quad (11)$$

where

$$\Psi(n) = \frac{1}{N_m}\sum_{t=kN_m}^{(k+1)N_m-1}\exp\left[-j2\pi t\frac{iK-n}{N_c}\right]. \quad (12)$$

The noise variance is given by

$$2\sigma^2_{AWGN} = \frac{2N_0}{T_c}\frac{1}{N_c}\sum_{m_r=0}^{N_r-1}\sum_{n=0}^{N_c-1}\left[\left(\left|w_{0,m_r}(n)\right|^2+\left|w_{1,m_r}(n)\right|^2\right)\left|\Psi(n)\right|^2\right]. \quad (13)$$

Hence, the variance $2\sigma^2$ of the ISI plus noise is given as

$$2\sigma^2=\frac{N_0}{T_c}\frac{1}{N_c}\sum_{m_r=0}^{N_r-1}\sum_{n=0}^{N_c-1}\left[\frac{1}{2}\frac{E_s}{N_0}\left|\begin{array}{l}\hat{H}_{0,m_r}(n)+\hat{H}_{1,m_r}(n)\\ -\frac{1}{N_c}\sum_{m=0}^{N_c-1}\begin{bmatrix}\hat{H}_{0,m_r}(m)\\+\hat{H}_{1,m_r}(m)\end{bmatrix}\end{array}\right|^2 +\left[\left|w_{0,m_r}(n)\right|^2+\left|w_{1,m_r}(n)\right|^2\right]\right]\left|\Psi(n)\right|^2. \quad (14)$$

We assume all "1" transmission (i.e., $d^k(i)=(1+j1)/\sqrt{2}$ ) without loss of generality and quaternary phase shift keying (QPSK) data-modulation. Since the ISI can be assumed to be circularly symmetric, the conditional BER for the given set of $\{H_{m_t,m_r}(n)\}$ can be expressed as [7]

$$p_b\left(\frac{E_s}{N_0},\{H_{m_r,m_t}(n)\}\right)=\frac{1}{2}erfc\left[\sqrt{\frac{1}{4}\gamma\left(\frac{E_s}{N_0},\{H_{m_r,m_t}(n)\}\right)}\right], \quad (15)$$

where $erfc[x]=(2/\sqrt{\pi})\int_x^\infty \exp(-t^2)dt$ is the complementary error function and $\gamma(E_s/N_0, \{H_{m_t,m_r}(n)\})$ is the conditional SINR defined as

$$\gamma\left(\frac{E_s}{N_0},\{H_{m_t,m_r}(n)\}\right)= \frac{\frac{E_s}{N_0}\left|\frac{1}{N_c}\sum_{m_r=0}^{N_r-1}\sum_{n=0}^{N_c-1}\left[\hat{H}_{0,m_r}(n)+\hat{H}_{1,m_r}(n)\right]\right|^2}{\frac{1}{K}\sum_{m_r=0}^{N_r-1}\sum_{n=0}^{N_c-1}\left[\frac{1}{2}\frac{E_s}{N_0}\left|\begin{array}{l}\hat{H}_{0,m_r}(n)+\hat{H}_{1,m_r}(n)\\ -\frac{1}{N_c}\sum_{m=0}^{N_c-1}\left[\hat{H}_{0,m_r}(m)+\hat{H}_{1,m_r}(m)\right]\end{array}\right|^2 +\left|w_{0,m_r}(n)\right|^2+\left|w_{1,m_r}(n)\right|^2\right]\left|\Psi(n)\right|^2}. \quad (16)$$

The theoretical average BER is numerically evaluated by averaging Eq. (15) over $\{H_{m_t,m_r}(n)$ ; $n$=0~$N_c$-1}.

## 5. Numerical and Simulation Results

The simulation conditions are given in Table 1. We assume an OFDM/TDM frame size of $N_c$=256 samples, GI length of $N_g$=32 samples and ideal coherent QPSK data modulation. As the propagation channel, an $L$=16-path block Rayleigh fading channel having a uniform power delay profile is considered. It is assumed that the time delay of the $l$th path is $\tau_l=l$ samples (i.e., the maximum delay difference is less than the GI length since $L\leq N_g$).

Table 1 Simulation conditions

| | | |
|---|---|---|
| Transmitter | Data modulation | QPSK |
| | Transmit diversity | STTD ($N_t$=2) |
| | No. of IFFT points | $N_m$=256/$K$ |
| | No. of slots per frame | $K$=1~256 |
| | Frame length | $N_c$=256 |
| | GI | $N_g$=32 |
| Channel | Frequency -selective block Rayleigh fading | |
| | No. of paths | $L$=16 |
| Receiver | No. of FFT points | $N_c$=256, $N_m$=256/$K$ |
| | No. of receive antennas | $N_r$=1, 2 and 4 |
| | FDE | MMSE |
| | Channel estimation | Ideal |

The evaluation of the theoretical average BER is done by Monte-Carlo numerical computation method as follows. A set of path gains $\{h^l_{m_t,m_r}(n)$ ; $l$=0~$L$-1} is generated for obtaining $\{H_{m_t,m_r}(n)$ ; $n$=0~$N_c$-1} using Eq. (4) and then $\{w_{m_t,m_r}(n)$ ; $n$=0~$N_c$-1} is computed using Eq. (6). The conditional BER for the given average received $E_s/N_0$ is computed using Eqs. (15) and (16). This is repeated a sufficient number of times to obtain the theoretical average BER of Eq. (17). Also presented below are the computer simulation results for the OFDM/TDM signal transmission to confirm the validity of the theoretical analysis.

The theoretical average BER performance is plotted with $K$ as a parameter, in Fig. 3, for joint STTD and receive antenna diversity with MMSE-FDE, as a function of the average received bit energy-to-the AWGN power spectrum density ratio $E_b/N_0$, which is given by $E_b/N_0$=0.5($E_s/N_0$)(1+$N_g/N_c$)/$N_t$. From Fig. 3 it can be seen that the proposed joint STTD encoding for OFDM/TDM with MMSE-FDE takes advantage of channel frequency-selectivity and improves the BER performance due to enhanced frequency diversity effect as $K$ increases. Reason for this performance improvement in comparison to the conventional OFDM is due to STTD encoding for OFDM/TDM, which is performed over several conventional OFDM signals. From Fig. 3, it is observed good agreement between theoretical and simulation results.

The performance improvement of OFDM/TDM is attributed to both spatial and frequency diversity gains. The frequency diversity gain depends on the channel frequency-selectivity and the channel frequency-selectivity depends on the decay factor β; as β increases, the channel frequency-selectivity becomes weaker and thus, the BER performance of OFDM/TDM becomes a function of β. Figure 4 shows the impact of channel frequency-selectivity on the OFDM/TDM with $K$=16 using joint STTD and $N_r$-branch receive antenna with MMSE-FDE for β=0 dB and 8 dB. For β=8 dB, the MMSE-FDE antenna diversity gain becomes smaller due to weaker channel frequency-selectivity and the BER performance degrades. When $K$=16, antenna diversity gain of about 3.8, 2.1, 1.1 dB is achieved for the average BER=$10^{-4}$ over β=8 dB case when $N_r$=1, 2 and 4 with β=0 dB, respectively. From Fig. 4 it is again seen that theoretical results agree well with the simulation results.

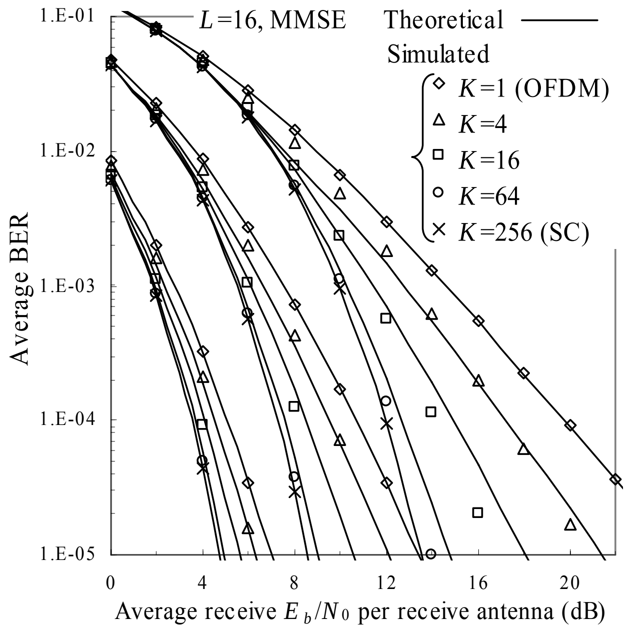


Figure 3. Average BER performance.

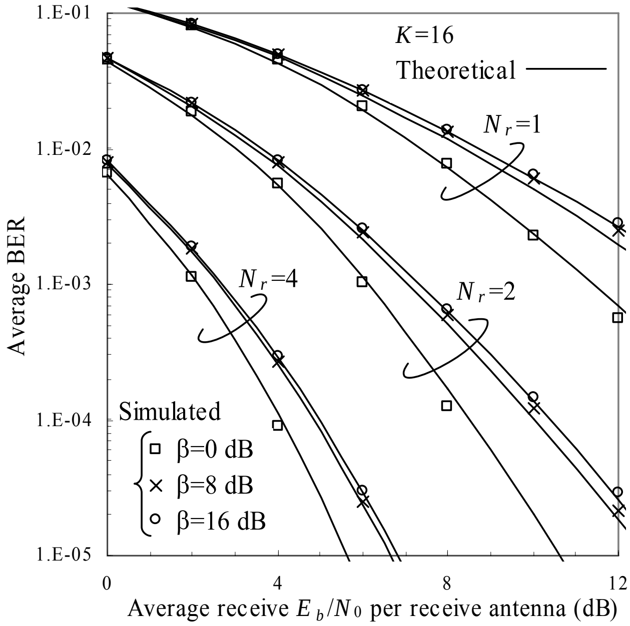


Figure 4. Impact of channel decay factor β.

## 6. Conclusions

In this paper, a theoretical BER analysis was developed for joint frequency-domain STTD and antenna receive diversity based on MMSE criterion for OFDM/TDM in a frequency-selective fading channel. Unlike STTD encoding/decoding for conventional OFDM, where only spatial diversity gain can be achieved, the STTD encoding/decoding for OFDM/TDM is performed on each OFDM/TDM frequency component on a frame-by-frame basis and achieves both spatial and frequency diversity gains. The STTD-encoded BER performance of OFDM/TDM in a frequency-selective block Rayleigh fading channel was evaluated by Monte-Carlo numerical computation using the derived conditional BER and by computer simulation. The theoretical results were compared with the computer simulation results and a fairly good agreement was observed. It was shown that proposed joint frequency-domain STTD and antenna diversity reception based on MMSE criterion improves the BER performance of OFDM/TDM in comparison to conventional OFDM since both spatial and frequency diversity gains can be achieved.

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