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周波数選択性フェージングチャネルにおけるクリッピングを用いるOFDM/TDMの伝送特性

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あらまし: 直交周波数分割多重(OFDM)には高いピーク対平均電力比(PAPR)が発生するという問題がある．筆者らはPAPRを低減するために，OFDMと時分割多重(TDM)を組み合わせたOFDM/TDMを提案した．振幅クリッピングを用いれば更にPAPRを低減できる．しかし，この方法では誤り率(BER)特性が劣化するばかりか，スペクトルが広がる．このため，フィルタリングが必要である．本論文ではクリッピングによるPAPR低減と帯域外スペクトル増加とのトレードオフについて検討している．計算機シミュレションにより，クリッピングとフィルタリングを用いるOFDM/TDMはOFDMよりクリッピングレベルと所要E_b/N_0を低減できること示している．

Transmission Performance of Clipped and Filtered OFDM/TDM in A Frequency-selective Fading Channel

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Abstract: Recently, we proposed OFDM combined with time division multiplexing (OFDM/TDM) to alleviate the high peak-to-average power ratio (PAPR) problem of OFDM. To further reduce the PAPR, amplitude clipping can be used. However, the bit error rate (BER) performance degrades and the out-of-band (OoB) power spectrum grows. Therefore, filtering is necessary to suppress the OoB power growth, but the PAPR will regrow. In this paper, by computer simulation, we study PAPR and OoB radiation trade-off of amplitude clipped and filtered OFDM/TDM. We discuss about how, and by how much, clipping affects the OFDM/TDM transmission in terms of PAPR, the power spectrum density (PSD) and the BER degradation. It is shown that OFDM/TDM can be used to reduce the clipping level and the required E_b/N_0 for the given BER in comparison to OFDM with slight increase in OoB emission.

Keywords: *OFDM, time division multiplexing, frequency-domain equalization, frequency-selective fading.*

1. Introduction

Orthogonal frequency division multiplexing (OFDM), which is robust against multipath fading, has a behavior similar to that of a Gaussian random process. This yields a drawback of having a large amplitude dynamic range, i.e., a large peak-to-average power ratio (PAPR) [1]. Recently, we proposed OFDM combined with time division multiplexing (OFDM/TDM) to overcome the high PAPR of OFDM [2]. In OFDM/TDM design, the inverse fast Fourier transform (IFFT) time window (OFDM/TDM frame) for conventional OFDM is divided into K slots. An OFDM signal with reduced number of subcarriers (N_m=N_c/K) is transmitted during each time slot. However, OFDM/TDM cannot completely eliminate the PAPR problem [2].

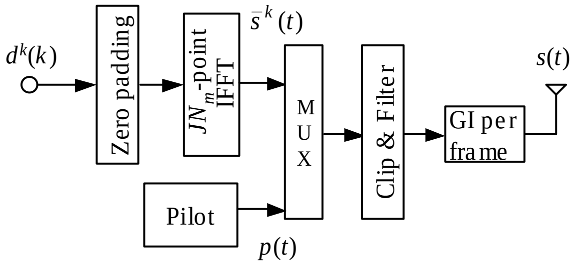
To further reduce the PAPR to acceptable level, the amplitude clipping can be applied [3]-[4]. However, the amplitude clipping causes signal distortion and the BER performance degrades. In addition to the bit error rate (BER) degradation, the out-of-band (OoB) power spectrum grows [4]. Bandpass filtering can be used to suppress the OoB power growth, but the PAPR regrows [4], [5]. Therefore, the joint amplitude clipping and filtering technique is necessary to suppress OoB power growth.

In this paper, a trade-off between the PAPR reduction, OoB radiation and the BER performance is discussed. We discuss about how, and by how much, clipping affects the OFDM/TDM transmission performance in terms of PAPR, the power spectral density (PSD) and the BER degradation. The PAPR benefit of OFDM/TDM is evaluated by complementary cumulative distribution function (CCDF). The BER performance of clipped and filtered OFDM/TDM with MMSE-FDE in a frequency-selective fading channel is evaluated by computer simulation.

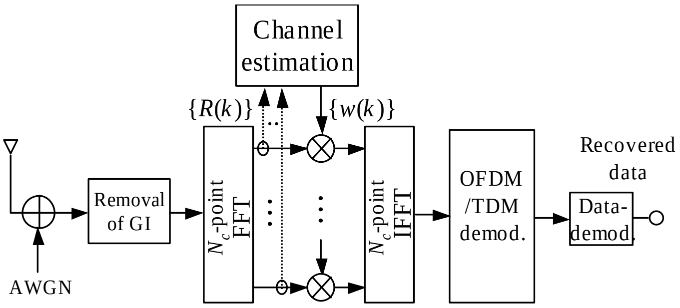
The remainder of this paper is organized as follows. Section 2 presents the principle of clipped and filtered OFDM/TDM transmission system. In Sect. 3, the transmission performance of clipped and filtered OFDM/TDM with MMSE-FDE in a frequency-selective Rayleigh fading channel is evaluated by computer simulation. Section 4 concludes the paper.

2. Amplitude Clipping and Filtering for OFDM/TDM Signals

The OFDM/TDM transmission system model is illustrated in Fig. 1. Throughout this paper, T_c-spaced discrete time representation is used, where T_c represents the fast Fourier transform (FFT) sampling period.



(a) Transmitter



(b) Receiver

Figure 1. OFDM/TDM transmitter and receiver.

2.1. Transmit Signal Representation

The OFDM/TDM signal might sometimes exceed the amplifier's saturation level. In order to reduce the dynamic range of the OFDM/TDM signal, $s(t)$, amplitude clipping can be used. A sequence of N_c data-modulated symbols, $\{d(i);\ i=0\sim N_c-1\}$ with $E[|d(i)|^2]=1$, is divided into K blocks with $N_m=N_c/K$ symbols. The k-th block symbol sequence is denoted by $\{d^k(i);\ i=0\sim N_m-1\}$, where $d^k(i)=d(kN_m+i)$ for $k=0\sim K-1$. Then, JN_m-point IFFT is applied to generate an interpolated time-domain OFDM signal with N_m subcarriers as

$$s^k(t)=\frac{1}{\sqrt{N_m}}\sum_{i=0}^{N_m-1} d^k(i)\exp\left(j2\pi t\frac{i}{JN_m}\right). \tag{1}$$

for $t=0\sim JN_m-1$, where J is oversampling ratio (in this paper $J=8$).

The OFDM/TDM signal is passed through the amplitude clipping and filtering block. Figure 2 shows amplitude clipping and filtering process [3]. The amplitude clipping can be written as [6]-[7]

$$\hat{s}^k(t)=\begin{cases} s^k(t), & \left|s^k(t)\right|\le\beta \\ \beta\dfrac{s^k(t)}{\left|s^k(t)\right|}, & \text{otherwise}\end{cases} \tag{2}$$

for $t=0\sim JN_m-1$, where β denotes the amplitude of the clipping level. As a result of this operation, the maximum peak power is suppressed to the clipping level.

A bandpass filtering is applied to suppress the OoB power spreading as shown in Fig. 2. The clipped signal is transformed back into the frequency-domain by applying JN_m-point FFT. While the first N_m elements of the FFT output are the new data symbols that correspond to the clipped signal, the other part of the output vector contains only inter-modulation products that would appear as OoB power spreading. To eliminate the power spreading, the first N_m frequency components are picked up and then, N_m-point IFFT is applied to generate the OFDM transmit signal as

$$\bar{s}^k(t)=\frac{1}{\sqrt{N_m}}\sum_{i=0}^{N_m-1}\bar{d}^k(i)\exp\left(j2\pi t\frac{i}{N_m}\right). \tag{3}$$

The OFDM/TDM signal can be expressed using the equivalent lowpass representation as

$$s(t)=\sum_{k=0}^{K-1}\bar{s}^k(t-kN_m)u(t-kN_m) \tag{4}$$

for $t=0\sim N_c-1$, where $u(t)=1(0)$ for $t=0\sim N_m-1$ (elsewhere). After insertion of guard interval (GI) the OFDM/TDM signal is multiplied by the power coefficient $\sqrt{2E_s/T_c}$, where E_s is the data-modulated symbol energy. Note that in this paper we assume ideal power amplification, i.e., transmit signal is not distorted by power amplifier.

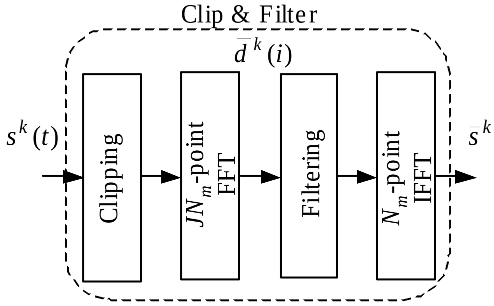


Figure 2. Clipping and filtering.

2.2. Received Signal Representation

The clipped signal propagates through the channel having the discrete-time channel impulse response $h(t)$ given as

$$h(t)=\sum_{l=0}^{L-1}h_l\delta(t-\tau_l), \tag{5}$$

where h_l and τ_l are the path gain and time delay of the lth path with $\sum_{l=0}^{L-1}E[|h_l|^2]=1$. The received signal can be expressed as

$$r(t)=\sqrt{\frac{2E_s}{T_c}}\sum_{l=0}^{L-1}h_l s(t-\tau_l)+n(t) \tag{6}$$

for $t=-N_g\sim N_c-1$, where $n(t)$ is the additive white Gaussian noise (AWGN) process with zero mean and variance $2N_0/T_c$ with N_0 being the single-sided power spectrum density.

After removing the GI, the received signal $\{r(t); t=0\sim N_c-1\}$ is decomposed into N_c frequency components $\{R(n); n=0\sim N_c-1\}$ by applying N_c-point FFT as

$$\begin{aligned} R(n) &= \frac{1}{N_c}\sum_{t=0}^{N_c-1} r(t)\exp\left(-j2\pi n\frac{t}{N_c}\right) \\ &= \sqrt{\frac{2E_s}{T_c}}S(n)H(n)+N(n) \end{aligned}, \qquad (7)$$

where $S(n)$, $H(n)$ and $N(n)$, respectively, denote the transmitted OFDM/TDM signal component, the channel gain and the AWGN noise component at the nth frequency. They are given by

$$\begin{cases} S(n) = \dfrac{1}{N_c}\displaystyle\sum_{t=0}^{N_c-1} s(t)\exp\left(-j2\pi n\frac{t}{N_c}\right) \\ H(n) = \displaystyle\sum_{l=0}^{L-1} h_l\exp\left(-j2\pi n\frac{\tau_l}{N_c}\right) \\ N(n) = \dfrac{1}{N_c}\displaystyle\sum_{t=0}^{N_c-1} n(t)\exp\left(-j2\pi n\frac{t}{N_c}\right) \end{cases}. \qquad (8)$$

2.3. FDE and OFDM/TDM Demodulation

One-tap FDE is applied to $R(n)$ as [8]

$$\begin{aligned} \hat{R}(n) &= w(n)R(n) \\ &= \sqrt{\frac{2E_s}{T_c}}S(n)\hat{H}(n)+\hat{N}(n) \end{aligned} \qquad (9)$$

with

$$\begin{cases} \hat{H}(n) = w(n)H(n) \\ \hat{N}(n) = w(n)N(n) \end{cases}, \qquad (10)$$

where $w(n)$ denotes the MMSE equalization weight for the nth frequency given by [9]

$$w(n) = \frac{H^*(n)}{|H(n)|^2+2\sigma^2}, \qquad (11)$$

where σ^2 denotes the noise power.

By applying N_c-point IFFT to $\{\hat{R}(n); n=0\sim N_c-1\}$, we obtain the time-domain OFDM/TDM signal $\{\hat{r}(t); t=0\sim N_c-1\}$, which can be expressed as

$$\hat{r}(t) = \sum_{n=0}^{N_c-1}\hat{R}(n)\exp\left(j2\pi t\frac{n}{N_c}\right). \qquad (12)$$

The decision variable for the ith data symbol of the kth OFDM signal can be obtained by applying N_m-point FFT as [2]

$$\hat{d}^k(i) = \frac{1}{N_m}\sum_{t=kN_m}^{(k+1)N_m-1}\hat{r}(t)\exp\left(-j2\pi i\frac{t-kN_m}{N_m}\right) \qquad (13)$$

for $i=0\sim N_m-1$ and $k=0\sim K-1$.

3. Simulation Results

The computer simulation conditions are given in Table 1. We assume an OFDM/TDM frame size of $N_c=256$ samples, GI length of $N_g=32$ samples and ideal coherent QPSK data modulation/demodulation. As the propagation channel, we assume an $L=16$-path block Rayleigh fading channel with uniform power delay profile, where $\{h_l; l=0\sim L-1\}$ are zero-mean independent complex Gaussian variables. It is assumed that the time delay of the lth path is $\tau_l=l$ samples (i.e., the maximum delay difference is less than the GI length since $L\leq N_g$). The normalized Doppler frequency $f_DT_s=7\times10^{-4}$ (where $1/T_s=1/[T_c(1+N_g/N_c)]$ is the transmission symbol rate); $f_DT_s=7\times10^{-4}$ corresponds to a mobile terminal moving speed of 55 km/h for the 5 GHz carrier frequency and the transmission data rate of $1/T_c=100$ Msymbols/s. Ideal noise and channel estimation is assumed. Note that $K=1$ corresponds to conventional OFDM system.

Table 1. Simulation parameters.

Transmitter	Data modulation	QPSK
	No. of IFFT points	$N_m=256/K$
	No. of slots	$K=1\sim64$
	Frame length	$N_c=256$
	GI	$N_g=32$
	Oversampling	$J=8$
Channel	$L=16$-path frequency-selective block Rayleigh fading	
Receiver	No. of FFT points	$N_c=256$, $N_m=256/K$
	FDE	MMSE
	Channel estimation	Ideal CE

3.1 PAPR

The PAPR benefit of OFDM/TDM is evaluated by CCDF. In OFDM/TDM, the PAPR is defined as the maximum instantaneous peak power over an OFDM/TDM frame normalized by the ensemble average power. The PAPR of the observed frame is defined as

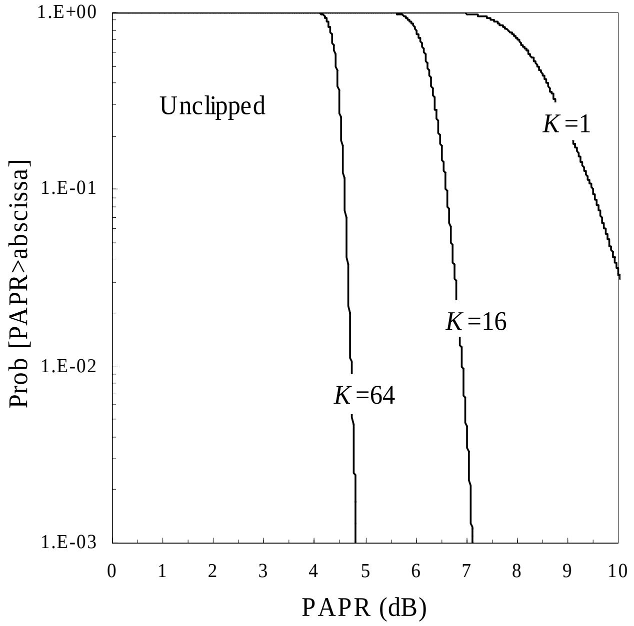
$$PAPR = \frac{\max\{|s(t)|^2\}_{t=0\sim N_c-1}}{E\{|s(t)|^2\}}, \qquad (14)$$

where $E\{|s(t)|^2\}$ is the ensemble average of the transmitted OFDM/TDM signal power. Figure 3 illustrates the CCDF of the PAPR distribution as a function of K obtained by computer simulation for three cases; (*a*) without clipping, (*b*) clipping, and (*c*) clipping and filtering.

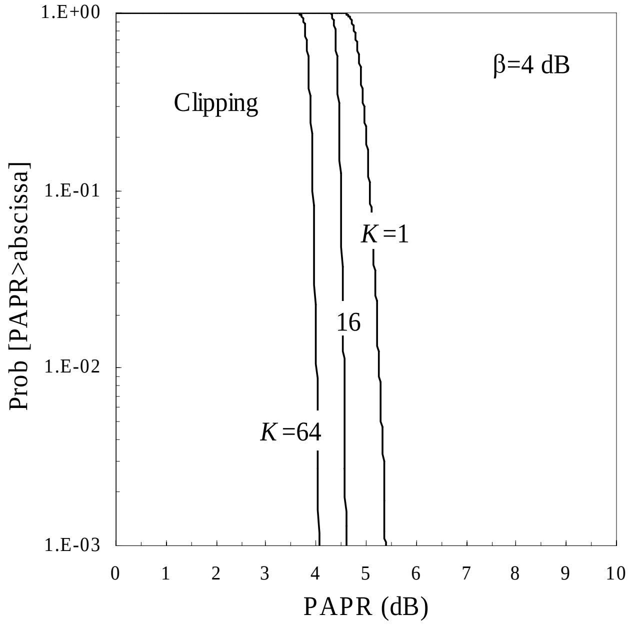
It can be seen from Fig. 3(a) that the $\mathrm{PAPR}_{10\%}$ level (i.e., which the PAPR of OFDM/TDM exceeds with a probability of 10%), OFDM/TDM achieves lower PAPR than OFDM by about 2.9 (4.8), for $K=16$ (64). For unclipped system, the $\mathrm{PAPR}_{10\%}$ level is about 9.5 for $K=1$, 6.6 dB for $K=16$ and 4.6 dB for $K=64$.

Figure 3(b) shows that the $\mathrm{PAPR}_{10\%}$ is about 5 for $K=1$, 4.4 dB for $K=16$ and 4 dB for $K=64$. Thus, for $K=16$ (64), OFDM/TDM achieves lower PAPR than OFDM by about 0.6 (1.2) dB when $\beta=4$ dB.

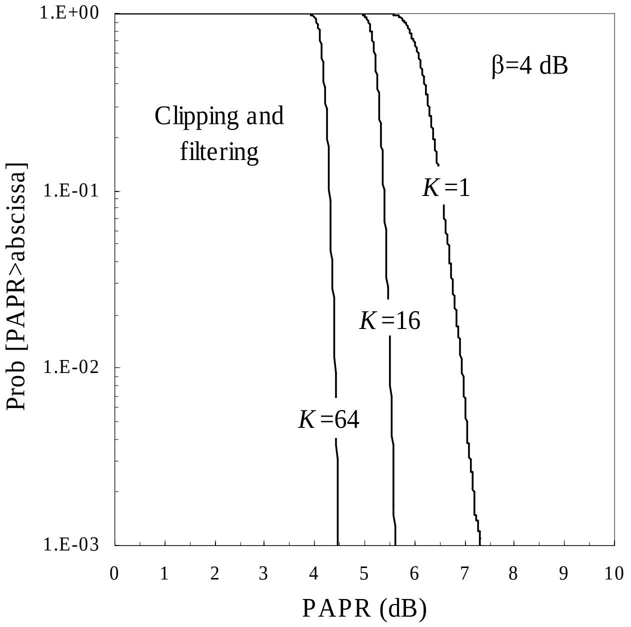
Finally, it can be seen from Fig. 3(c) that filtering will regrow the PAPR; the $\mathrm{PAPR}_{10\%}$ is about 6.5 for $K=1$, 5.4 dB for $K=16$ and 4.3 dB for $K=64$. From the above discussion on PAPR, it is clear that filtering less affects the PAPR for larger *K*. This clearly shows the advantage of OFDM/TDM over conventional OFDM in terms of PAPR when amplitude clipping and filtering is used.



(a) Unclipped



(b) Clipping; β=4 dB.



(c) Clipping and filtering; β=4 dB.

Figure 3. Distribution of PAPR.

3.2 PSD

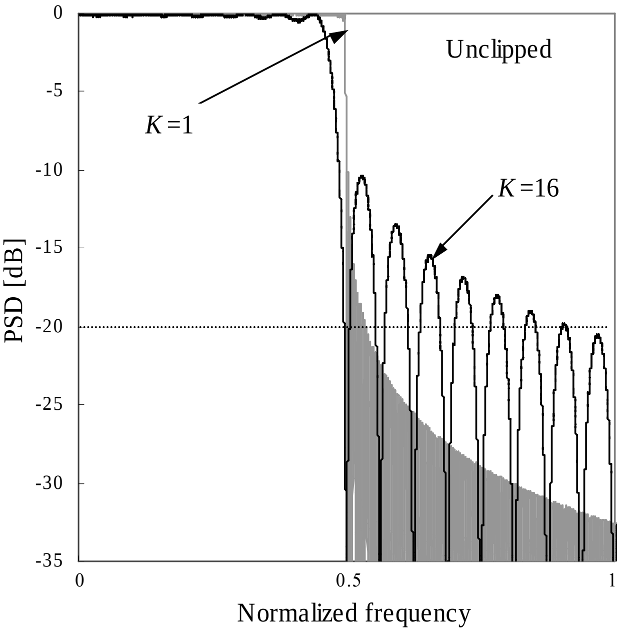
Figure 4 shows the PSD's of (*a*) unclipped, (*b*) clipped, and (*c*) clipped and filtered OFDM/TDM and OFDM systems.

It can be seen from Fig. 4(a) that PSD of unclipped OFDM/TDM is larger than unclipped OFDM. This is because OFDM/TDM signals have discontinuity in their waveforms in the OFDM/TDM frame and cause a higher order spectral spreading. However, PSD improvement of OFDM is paid with very high PAPR level (see Fig. 3(a)).

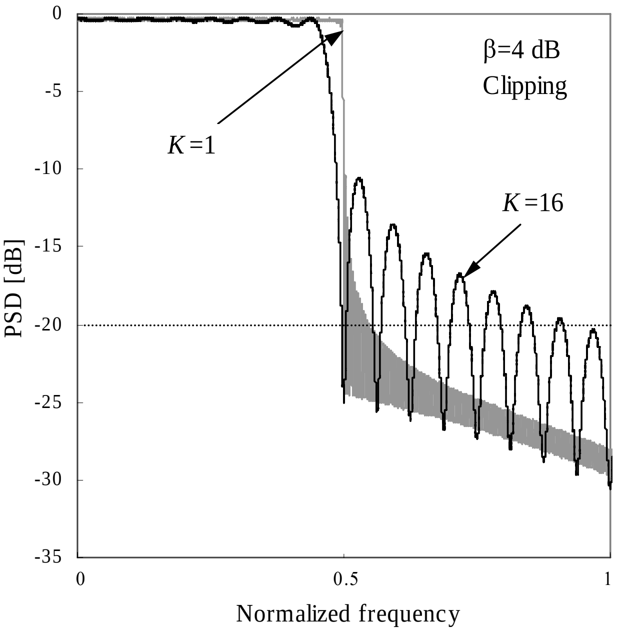
Figure 4(b) shows that, for β=4 dB, the PSD of OFDM slightly grows while for OFDM/TDM is almost not affected by clipping (e.g., see -20 dB PSD level). However, the OFDM/TDM signals have lower PAPR in comparison to OFDM as shown in Fig. 3(b).

It can be seen from Fig. 4(c) that PSD with clipping and filtering is almost the same as for unclipped system (see Fig. 4(a)). However, in this case the PAPR of OFDM significantly grows about 2 dB, while the PAPR of OFDM/TDM grows only about 0.9 (0.4) dB for *K*=16 (64) (see Fig. 3(c)).

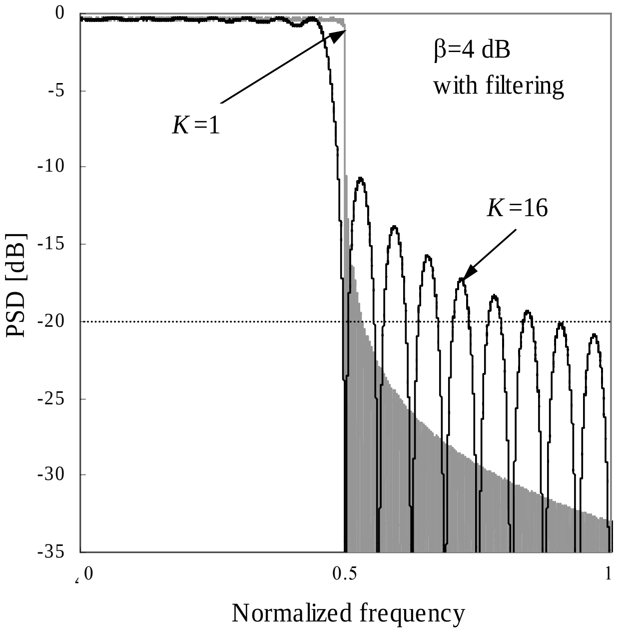
Therefore, it is important to state that this slight PSD performance improvement does not seem to justify the added complexity for implementing a bandpass filter.



(a) Unclipped



(b) Clipping; β=4 dB



(b) Clipping and filtering; β=4 dB

Figure 4. PSD performance.

3.3 Impact of Clipping Level

Figure 5 shows the impact of amplitude clipping level β on the average BER performance for the E_b/N_0=20 dB ($E_b/N_0=0.5(E_s/N_0)\times(1+N_g/N_c)$). The figure shows that OFDM/TDM can be used to reduce the required amplitude clipping level β while achieving the better BER than OFDM. For example, if the required BER=10^{-3} for the average E_b/N_0=20 dB, OFDM (K=1) cannot achieve this performance irrespective of β. Hence, to achieve BER=10^{-3} and reduce the clipping level β, we can use OFDM/TDM. When K increases from 16 to 32, the amplitude clipping level β can be reduced from 7 to 1 dB for BER=10^{-3}, respectively. Note that K=64 can achieve BER=10^{-3} irrespective to β. This is because as K increases, the PAPR of the OFDM/TDM signal reduces and the signal is less degraded in the clipping process.

This shows that OFDM/TDM can be used to significantly reduce the clipping level β with the small increase in PSD (see Fig. 4(b)).

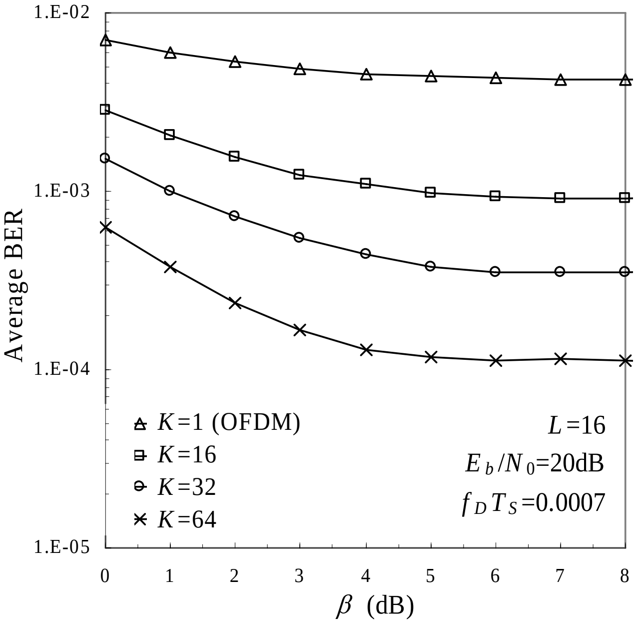


Figure 5. Impact of clipping level β.

Figure 6 shows the impact of OFDM/TDM parameter K on the required E_b/N_0. The figure shows that OFDM/TDM can be used to reduce the required E_b/N_0 in comparison to conventional OFDM for the given BER. The OFDM/TDM with K=16 (64) reduces the required E_b/N_0 for about 6 (9.5), 6.5 (10.3) and 6.1 (9.2) dB when β=0, 4 and 8 dB, respectively, in comparison to OFDM. It can be seen from Fig. 6 that OFDM/TDM for β>4 dB approaches β=∝, while the conventional OFDM for β=4 dB still requires additional 1 dB of E_b/N_0 to achieve performance with β=∝ (β=∝ corresponds to no clipping).

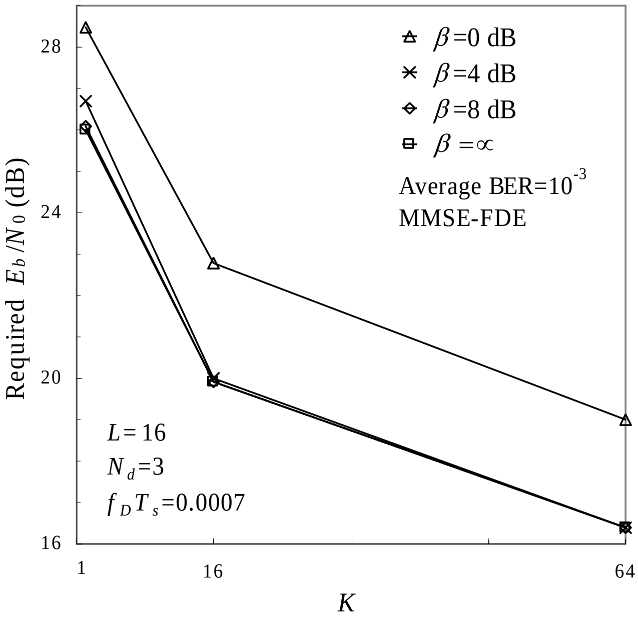


Figure 6. Reduction of required E_b/N_0 vs. K.

4. Conclusions

A trade-off between the PAPR reduction, OoB radiation and the BER performance was discussed; the OFDM/TDM reduces the PAPR and improves the BER with slight increase in PSD in comparison to OFDM. It was shown that the PAPR of OFDM/TDM signal after amplitude clipping and filtering is lower than OFDM with slight increase in PSD. The clipped and filtered OFDM/TDM can be used to reduce the amplitude clipping level β while achieving the same or better BER performance than clipped and filtered OFDM. Moreover, a slight PSD performance improvement when filtering is used does not seem to justify the added complexity for implementing the bandpass filter.

References

[1] R. D. J. van Nee, R. Prasad, and R. van Nee, *OFDM for wireless multimedia communications*, Artech House, January 2000.

[2] H. Gacanin, S. Takaoka and F. Adachi, "Bit error rate analysis of OFDM/TDM with frequency-domain equalization," IEICE Trans. Commun., Vol. E89-B, No. 2, pp.509-519, Feb. 2006.

[3] J. Armstrong, “New OFDM peak-to-average power reduction scheme,” Proc. of IEEE Vehicular Technology Conference (VTC), Vol. 1, pp. 756-760, Sept. 2001.

[4] H. Ochiai and H. Imai, “Performance analysis of deliberately clipped OFDM signals,” IEEE Trans. Commun., Vol. 50, No. 1, Jan. 2002.

[5] X. Li and L. J. Cimini, Jr., “Effects of clipping and filtering on the performance of OFDM,” IEEE Comm. Letters, Vol. 2, pp. 131-133, May 1998.

[6] P. Boonsrimuang, K. Mori, T. Paungma and H. Kobayashi, “Proposal of clipping and inter-modualtion noise mitigation method for OFDM signal in non-linear channel,” IEICE Trans. Commun., Vol. E88-B, No. 2, pp. 427-435, Feb. 2005.

[7] T. Fujii and M. Nakagawa, “Adaptive clipping level control for OFDM peak power reduction using clipping and filtering,” IEICE Trans on Fundam., Vol. E85-A, No. 7, pp.1647-1655, July 2002.

[8] D. Falconer, S.L. Ariyavisitakul, A. Benyamin-Seeyar, and B. Eidson, “Frequency-domain equalization for single-carrier broadband wireless systems,” IEEE Commun. Mag., Vol. 40, pp. 58-66, April 2002.

[9] S. Hara and R. Prasad, *Multicarrier Techniques for 4G Mobile Communications,* Artech House, June 2003.