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クリッピングを用いる OFDM/TDM のパイロットチャネル推定を用いるときの BER 特性

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あらまし 直交周波数分割多重(OFDM)には高いピーク対平均電力比(PAPR)が発生するという問題がある．筆者らは PAPR を低減するために，OFDM と時分割多重(TDM)を組み合わせた OFDM/TDM を提案した．振幅クリッピングを用いれば更に PAPR を低減できる．本論文では MMSE 周波数領域等化とクリッピングを用いる OFDM/TDM に周波数領域一次および二次線形補間および遅延時間領域窓関数を用いるチャネル推定法を適用している．非線形のチャネルによる OFDM/TDM 信号の平均二乗誤差(MSE)を最小にする重みを導出している．筆者らは計算機シミュレーションにより周波数選択性レイリーフェージングチャネルにおけるビット誤り率(BER)特性を明らかにし，遅延時間領域窓関数を用いるチャネル推定の特性が周波数領域一次線形補間と二次補間の特性より優れていることを示している．

キーワード *OFDM/TDM, MMSE-FDE，パイロットチャネル推定，クリッピング.*

BER Performance of Clipped and Filtered OFDM/TDM with Pilot-assisted Channel Estimation

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Abstract: Orthogonal frequency division multiplexing combined with time division multiplexing (OFDM/TDM) was proposed to overcome the high peak-to-average-power ratio (PAPR) of conventional OFDM. Clipping and filtering can be used to further reduce the PAPR. OFDM/TDM with minimum mean square error (MMSE) frequency-domain equalization (FDE) requires accurate channel estimation (CE). In this paper, pilot-assisted CE with frequency-domain interpolation for clipped and filtered OFDM/TDM with MMSE-FDE is presented. We consider frequency-domain polynomial (i.e., linear and second-order) and delay time-domain interpolation techniques. In the frequency-selective fading channel, frequency-domain linear and second-order polynomial interpolation gives poor performance while good performance is obtained with delay time-domain interpolation. It is shown that as the channel frequency-selectivity increases, OFDM/TDM with MMSE-FDE can obtain larger frequency diversity gain and improve the bit error rate (BER) performance.

Keywords: *OFDM/TDM, MMSE-FDE, pilot-assisted channel estimation, clipping.*

1. Introduction

Recently [1], we proposed OFDM combined with time division multiplexing (OFDM/TDM) to overcome the high PAPR of OFDM [2]. In the OFDM/TDM design, the N_c-point inverse fast Fourier transform (IFFT) time window of conventional OFDM is divided into K slots (which constitutes the OFDM/TDM frame). Within each slot, an OFDM signal with reduced number of subcarriers $N_m=N_c/K$ is transmitted. However, OFDM/TDM cannot completely eliminate the PAPR problem [3]. To further reduce the PAPR to acceptable level, the amplitude clipping and filtering has been introduced into OFDM/TDM [4].

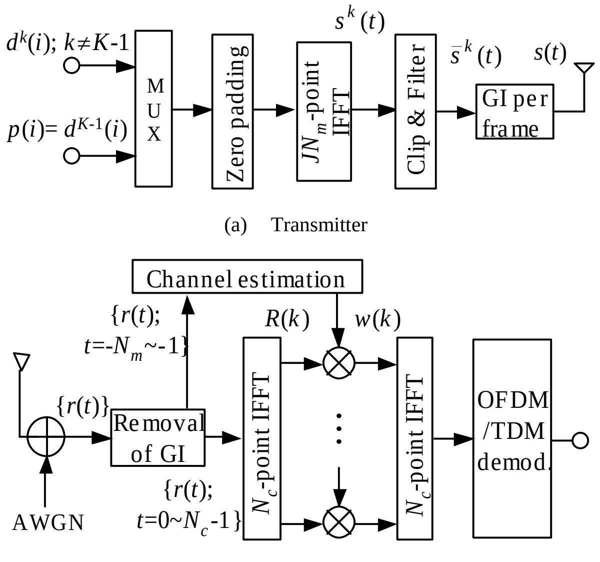
The frequency-domain equalization (FDE) based on minimum mean square error (MMSE) criterion can be applied to OFDM/TDM to improve the bit error rate (BER) performance. MMSE-FDE requires accurate channel estimation (CE). It was shown in [5] that the best set of pilots to be used, are those that are equally spaced. In [6] zero-forcing (ZF) CE and MMSE-CE, together with piecewise-linear or piecewise second-order interpolation are studied. OFDM with pilot-assisted and pilot-aided CE is studied in [7]. Pilot-aided CE for OFDM [5]-[7] cannot be directly applied for OFDM/TDM since, in the receiver, N_c-point FFT over the entire OFDM/TDM frame is applied for FDE. Thus, channel gains necessary for FDE cannot be directly computed from the received OFDM/TDM signal.

In this paper, pilot-assisted CE with frequency-domain interpolation for clipped and filtered OFDM/TDM with MMSE-FDE is presented. Pilot signal is inserted within one reserved slot (i.e., pilot slot) in the OFDM/TDM frame. However, all channel gains required for FDE cannot be obtained (since number of OFDM/TDM subcarriers is $N_m=N_c/K$) and hence, after CE, frequency-domain interpolation is applied to estimate required channel gains. We consider frequency-domain polynomial (i.e., linear and second-order) and delay time-domain interpolation techniques [5]-[7].

The rest of the paper is organized as follows. Section 2 describes clipped and filtered OFDM/TDM system. In Sect. 3, pilot-assisted CE is presented. Section 4 evaluates, by computer simulation, the OFDM/TDM system performance. Section 5 concludes the paper.

2. Clipped and Filtered OFDM/TDM System

The clipped and filtered OFDM/TDM system is illustrated in Fig. 1. Throughout this paper, T_c-spaced discrete time representation is used, where T_c represents FFT sampling period.



(a) Transmitter

(b) Receiver

Figure 1. OFDM/TDM transmitter/receiver.

2.1 Transmit Signal Representation

The N_c-point IFFT time window of conventional OFDM is divided into K slots, i.e., OFDM/TDM frame. A sequence of data-modulated symbols, is divided into blocks with $N_m=N_c/K$ symbols. The k-th block symbol sequence is denoted by $\{d^k(i);\ i=0\sim N_m-1\}$, where $d^k(i)=d(kN_m+i)$ for $k=0\sim K-2$. Then, within the (K-1)th slot (i.e., pilot slot) a pilot signal $\{p(i);\ i=0\sim N_m-1\}$ is inserted. In the following $d^{K-1}(i)=p(i)$ for $i=0\sim N_m-1$. Then, to avoid frequency aliasing, JN_m-point IFFT is applied to generate an interpolated time-domain OFDM signal with N_m subcarriers as

$$s^k(t)=\frac{1}{\sqrt{N_m}}\sum_{i=0}^{N_m-1}d^k(i)\exp\left(j2\pi t\frac{i}{JN_m}\right). \tag{1}$$

for $t=0\sim N_c-1$, where J is over-sampling ratio.

Figure 2 shows the amplitude clipping and filtering block. The amplitude clipping can be written as [4]

$$\hat{s}^k(t)=\begin{cases} s^k(t), & \left|s^k(t)\right|\le\beta \\ \beta\dfrac{s^k(t)}{\left|s^k(t)\right|}, & \text{otherwise}\end{cases} \tag{2}$$

for $t=0\sim JN-1$, where β denotes the clipping level. The clipped signal is transformed into the frequency-domain by applying JN_m-point FFT as

$$\bar{d}^k(i)=\frac{1}{\sqrt{JN_c}}\sum_{t=0}^{JN_c-1}\hat{s}^k(t)\exp\left(-j2\pi i\frac{t}{JN_c}\right) \tag{3}$$

for $i=0\sim JN_c-1$. To suppress the out-of-band (OoB) spectral spreading, the first N_m components are picked up and then, N_m-point IFFT is applied to generate clipped and filtered OFDM signal as

$$\bar{s}^k(t)=\frac{1}{\sqrt{N_m}}\sum_{i=0}^{N_m-1}\bar{d}^k(i)\exp\left(j2\pi t\frac{i}{N_m}\right). \tag{4}$$

Using the Bussgang theorem [8], a clipped and filtered OFDM signal can be expressed as a sum of a useful attenuated input replica and an uncorrelated nonlinear distortion as

$$\bar{s}^k(t)=\sqrt{\frac{2E_s}{T_c}}\left[\alpha s^k(t)+\tilde{s}^k(t)\right], \tag{5}$$

where α and $\tilde{s}^k(t)$, respectively, denote the attenuation constant and clipping noise of the kth slot's clipped and filtered OFDM signal. The attenuation constant α is chosen to minimize MSE term $E\left[\left|\bar{s}^k(t)-\alpha s^k(t)\right|^2\right]$ [8]. For β>7 dB, α→1 while for lower β, α can be well approximated as [8]

$$\alpha=1-\exp\left\{-\beta^2\right\}+\frac{\sqrt{\pi}\beta}{2}erfc\{\beta\}. \tag{6}$$

Finally, the OFDM/TDM signal can be expressed using the equivalent lowpass representation as

$$s(t)=\sum_{k=0}^{K-1}\bar{s}^k(t-kN_m)u(t-kN_m) \tag{7}$$

for $t=0\sim N_c-1$, where $u(t)=1(0)$ for $t=0\sim N_m-1$ (elsewhere) as shown by Fig. 3. After insertion of a N_m-sample GI, the OFDM/TDM signal is multiplied by the power coefficient $\sqrt{2E_s/T_c}$, where E_s is the data-modulated symbol energy and transmitted over a frequency-selective fading channel. Note that the length of GI equals to the slot length (i.e., N_m-samples) and hence, a pilot signal from the (K-1)th slot is inserted into GI as a cyclic prefix at the beginning of the frame as shown by Fig. 3. This frame structure using pilot and GI is similar to the one presented in [9].

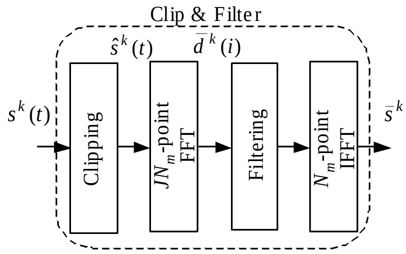


Figure 2. Clipping and filtering.

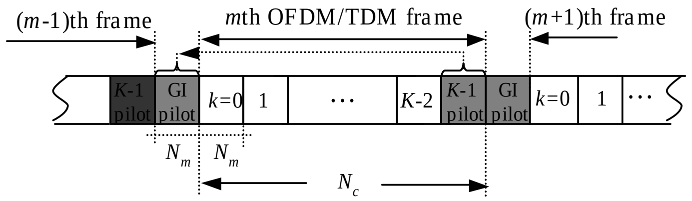


Figure 3. OFDM/TDM frame structure.

2.2 FDE for OFDM/TDM in a Nonlinear Channel

The clipped signal propagates through the channel having the discrete-time channel impulse response $h(t)$ given as

$$h(t)=\sum_{l=0}^{L-1} h_l \delta(t-\tau_l)\,, \tag{8}$$

where h_l and τ_l are the path gain and time delay of the lth path with $\sum_{l=0}^{L-1} E[|h_l|^2]=1$. The received signal can be expressed as

$$r(t)=\sqrt{\frac{2E_s}{T_c}}\sum_{l=0}^{L-1} h_l s(t-\tau_l)+n(t) \tag{9}$$

for t=-N_m~N_c-1, where $n(t)$ is the additive white Gaussian noise (AWGN) process with zero mean and variance $2N_0/T_c$ with N_0 being the single-sided power spectrum density.

After removing the N_m-sample GI, the received signal $\{r(t);\ t=0\sim N_c-1\}$ is decomposed into N_c frequency components $\{R(n);\ n=0\sim N_c-1\}$ for FDE as

$$R(n)=\sqrt{\frac{2E_s}{T_c}}\left[\alpha S(n)H(n)+\tilde{S}(n)H(n)\right]+N(n)\,, \tag{10}$$

where $S(n)$, $H(n)$, $\tilde{S}(n)$ and $N(n)$, respectively, denote the transmitted OFDM/TDM signal component, the channel gain, the clipping noise and the AWGN noise component at the nth frequency. They are given by

$$\begin{cases} S(n)=\dfrac{1}{N_c}\sum_{t=0}^{N_c-1} s(t)\exp\left(-j2\pi n\dfrac{t}{N_c}\right) \\ H(n)=\sum_{l=0}^{L-1} h_l \exp\left(-j2\pi n\dfrac{\tau_l}{N_c}\right) \\ \tilde{S}(n)=\dfrac{1}{N_c}\sum_{t=0}^{N_c-1} \tilde{s}(t)\exp\left(-j2\pi n\dfrac{t}{N_c}\right) \\ N(n)=\dfrac{1}{N_c}\sum_{t=0}^{N_c-1} n(t)\exp\left(-j2\pi n\dfrac{t}{N_c}\right) \end{cases}. \tag{11}$$

One-tap FDE is applied to $R(n)$ as [9]

$$\hat{R}(n)=w(n)R(n) \tag{12}$$

where $w(n)$ is the MMSE equalization weight given by

$$w(n)=\frac{\alpha H^*(n)}{|H(n)|^2\left\{1-e^{-\beta^2}\right\}+\left(\dfrac{E_s}{N_0}\right)^{-1}}. \tag{13}$$

For large β (e.g., β>7 dB), $\alpha\approx 1$ and $\exp(-\beta^2)\approx 0$ and hence, Eq. (13) reduces to the MMSE equalization weight for unclipped system.

3. Channel Estimation

To compute the MMSE weight, estimation of the channel gain $H(n)$ is necessary. A pilot signal from the (K-1)th slot is copied into GI as a cyclic prefix in the beginning of the OFDM/TDM frame as shown by Fig. 3. The previous frame's pilot slot acts as a cyclic prefix for the current frame's pilot (which is the GI). Thus, the channel estimation can be performed using the N_m-sample GI.

To avoid noise enhancement in the channel estimation process, it is desirable that a pilot sequence has constant amplitude in the frequency-domain. On the other hand, large amplitude variations may appear in the time-domain and signal may be distorted due to nonlinear processing leading to poor channel estimates. In this paper, Chu pilot sequence [10] is considered since amplitude in both time- and frequency-domain is constant and hence, clipping will not affect CE. The Chu pilot is given by $p(i)=\cos(\pi i^2/N_c)+j\sin(\pi i^2/N_c)$ for $i=0\sim N_m-1$.

3.1 Reverse Modulation

The N_m-point FFT is applied to decompose the received pilot into N_m frequency components as

$$\tilde{R}(q)=\sum_{t=-N_m}^{-1} r(t)\exp\left(-j2\pi q\frac{t}{N_m}\right), \tag{14}$$

where $q=\lfloor n/K\rfloor$ for $n=0\sim N_c-1$ is the subcarrier of the pilot and $\lfloor x\rfloor$ denotes the largest integer smaller than or equal to x. As shown by Fig. 4, $\{\tilde{R}(q);\ q=0\sim N_m-1\}$ are located at the frequency $n=0, K, 2K,\ldots(N_m-1)K$. The instantaneous channel gain estimate at the qKth frequency is obtained by the reverse modulation as

$$\tilde{H}(q)=\frac{\tilde{R}(q)}{P(q)}. \tag{15}$$

The frequency separation between N_m channel gains obtained by Eq. (15) is K and hence, it is necessary to interpolate between them to obtain the channel gains for all frequencies of $n=0\sim N_c-1$.

3.2 Delay Time-domain Interpolation [11]

For high-resolution delay time-domain interpolation, N_m-point IFFT is performed on $\{\tilde{H}(q);\ q=0\sim N_m-1\}$ to obtain the instantaneous channel impulse response $\{\tilde{h}(t);\ t=0\sim N_m-1\}$ as

$$\tilde{h}(t)=\frac{1}{N_m}\sum_{q=0}^{N_m-1}\tilde{H}(q)\exp\left(j2\pi t\frac{q}{N_m}\right). \tag{16}$$

Then, N_c-point FFT is applied to obtain the interpolated channel gain estimates for all N_c frequency components $\{H_e(n);\ n=0\sim N_c-1\}$ as

$$H_e(n)=\frac{1}{N_c}\sum_{t=0}^{N_m-1}\tilde{h}(t)\exp\left(-j2\pi n\frac{t}{N_c}\right). \tag{17}$$

The above-mentioned time-domain interpolation is equivalent to a frequency-domain interpolation. Using Eqs. (16) and (17), we obtain

$$\begin{aligned}H_e(n)&=\frac{1}{N_m}\sum_{q=0}^{N_m-1}\tilde{H}(q)\sum_{t=0}^{N_m-1}\exp\left[-j2\pi t\frac{Kq-n}{N_c}\right]\\&=\sum_{q=0}^{N_m-1}\tilde{H}(q)\Psi(n,q)\end{aligned} \tag{18}$$

where

$$\begin{aligned}\Psi(n,q)&=\frac{1}{N_m}\frac{\sin\left(\pi\frac{n-Kq}{K}\right)}{\sin\left(\pi\frac{n-Kq}{N_c}\right)}\\&\times\exp\left[j\pi(n-Kq)\frac{N_c-K}{KN_c}\right]\end{aligned} \tag{19}$$

is the frequency-domain interpolation coefficient for $n=0\sim N_c-1$. From Eq. (18), it can be seen that to estimate the channel gains at any frequency, all the N_m pilot components are used.

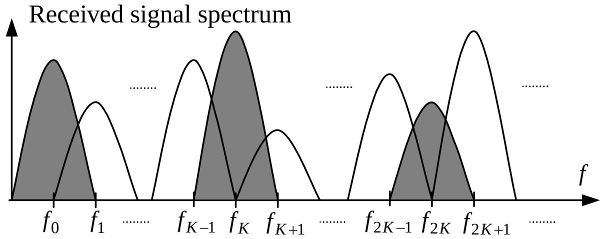


Figure 4. Received signal frequency components.

3.3 Polynomial Interpolation [12]

The channel gain estimate at the nth frequency, $n=qK+i$ ($q, i=0\sim N_m-1$), using linear interpolation is given as

$$\begin{aligned}H_e(n)&=\tilde{H}(qK+i)\\&=\frac{K-i}{K}\tilde{H}(q)+\frac{i}{K}\tilde{H}(q+1)\end{aligned}. \tag{20}$$

The channel gain estimate by second-order interpolation is given as

$$\begin{aligned}H_e(n)&=\tilde{H}(qK+i)\\&=c_0\tilde{H}(q)+c_1\tilde{H}(q+1)+c_2\tilde{H}(q+2)\end{aligned}, \tag{21}$$

where

$$\begin{cases}c_0=\dfrac{(i-K)(i-2K)}{2K^2}\\ c_1=\dfrac{i(2K-i)}{K^2}\\ c_2=\dfrac{i(i-K)}{2K^2}\end{cases}. \tag{22}$$

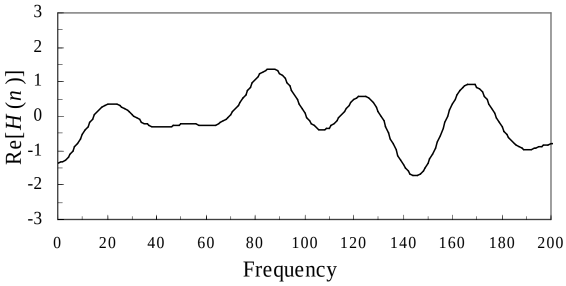
4. Computer Simulations

Simulation parameters are shown in Table 1. We assume QPSK data-modulation with N_c=256 and N_m=16. The propagation channel is T_c-spaced L-path frequency-selective Rayleigh fading channel having uniform power delay profile, where the path gains stay constant at least over one OFDM/TDM frame.

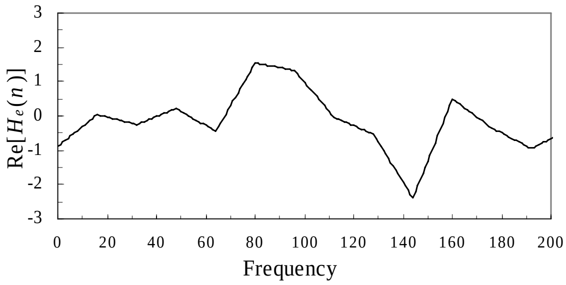
Table 1. Simulation parameters.

Transmitter	Data modulation	QPSK
	No. of IFFT points	N_m=16
	No. of slots	K=16
	Over sampling	J=4
	Frame length	N_c=256
	GI	N_m=16
Channel	L-path frequency-selective Rayleigh fading	
Receiver	No. of FFT points	N_c=256
	FDE	MMSE

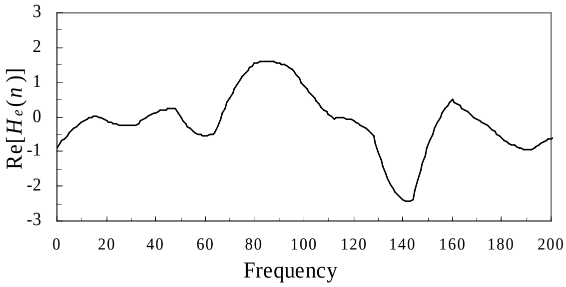
To discuss the accuracy between frequency-domain linear, second-order and delay time-domain interpolation, we illustrate, in Fig. 5, one shot observation of the channel gain estimate $H_e(n)$, with deferent interpolators for L=8. As can be seen from Figs. 5 (a) and (b), by employing frequency-domain linear and second-order interpolation the channel estimate is poor due to strong channel frequency-selectivity. However, for delay time-domain interpolation, the obtained channel estimate is in good agreement with true channel gain. This is because delay time-domain interpolation is high-order interpolation that uses all N_m pilots during interpolation, leading to much better accuracy.



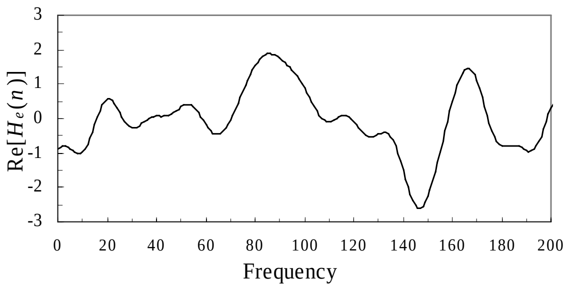
(a) Original channel gain



(b) Linear interpolator



(c) Second-order interpolator



(d) Delay time-domain interpolator

Figure 5. Channel gain after interpolation.

Interpolation accuracy depends on channel frequency-selectivity, which is a function of the number of paths L (i.e., as L increases the channel becomes more frequency-selective and vice versa). Figure 6 shows the channel estimator's MSE defined as MSE=$E[|H_e(n)-H(n)|^2]$, where $E[\cdot]$ denotes the ensemble average operation, as a function of the average signal energy per bit-to-the AWGN power spectrum density ratio E_b/N_0 with L as a parameter. As can be seen from Fig. 6, the MSE with frequency-domain linear and second-order interpolator's increase as L increases. The MSE with the second-order interpolation is worse than MSE with linear interpolation because second-order interpolator produces a slight amplification of noise that is present in $H_e(n)$. Both of the frequency-domain interpolation techniques provide poor MSE performance and cannot be used for frequency-domain interpolation in the case of strong frequency-selective fading. The figure shows that delay time-domain interpolator gives good MSE performance irrespective of L. In the following, we only consider delay time-domain interpolation.

The performance improvement of OFDM/TDM is attributed to the frequency diversity gain achieved by MMSE-FDE [1]. Figure 7 shows the average BER as a function of the E_b/N_0 with L as a parameter. As can be seen from Fig. 7, for BER=10^{-3}, the E_b/N_0 degradation with pilot-assisted CE is about 3.5 dB in comparison to ideal CE irrespective of L. Figure 8 shows the average BER as a function of the E_b/N_0 with and without clipping. It can be seen from the figure that, for average BER=10^{-3}, the E_b/N_0 degradation of CE with (without) clipping in comparison to ideal CE with (without) clipping, is about 3.4 (3.5) dB. In order to further reduce the performance degradation, CE must be improved. This is interesting future study.

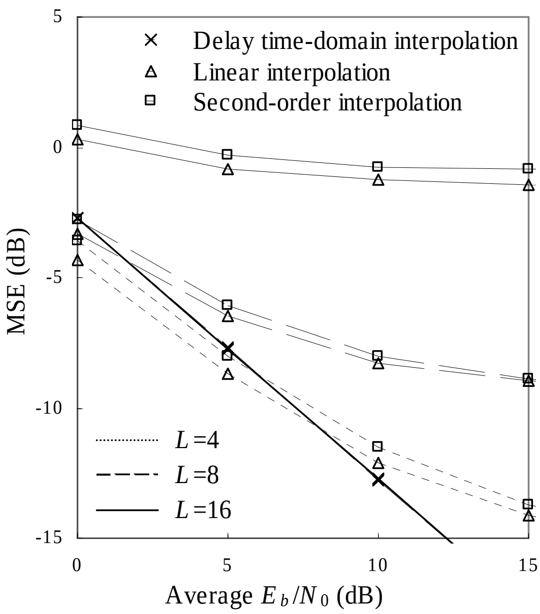


Figure 6. MSE.

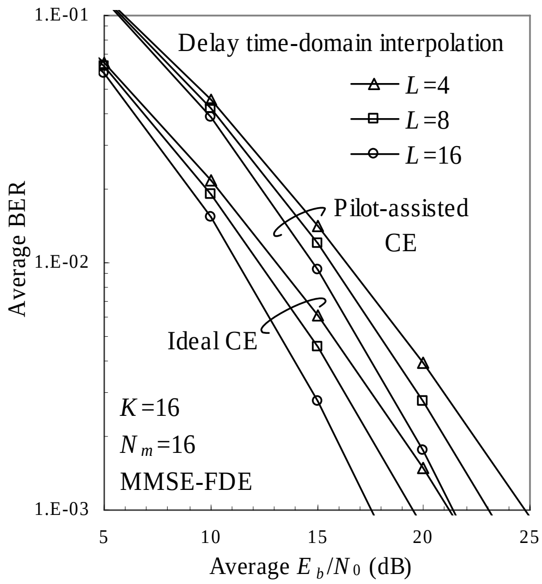


Figure 7. Impact of channel frequency-selectivity.

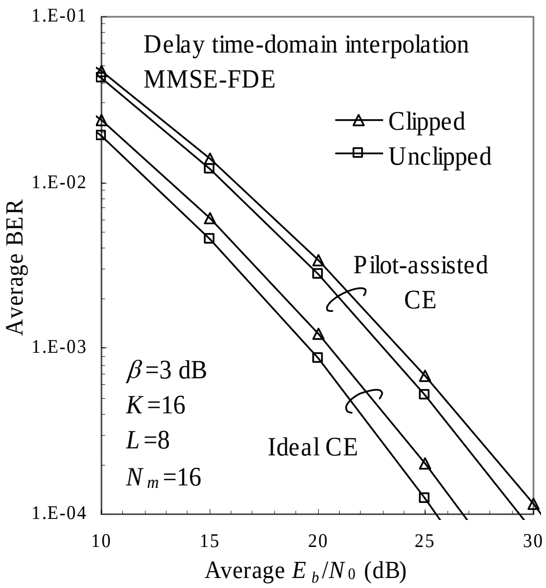


Figure 8. Average BER performance with and without clipping.

5. Conclusions

In this paper, pilot-assisted CE with frequency-domain interpolation for clipped and filtered OFDM/TDM with MMSE-FDE was discussed and its BER performance in a frequency-selective fading channel was evaluated by computer simulation. We considered frequency-domain linear, second-order and delay time-domain interpolation. It was shown that in strong fading environment frequency-domain linear and second-order interpolation achieve poor performance while the best performance is obtained with high-resolution delay time-domain interpolation. This shows that time-domain interpolation for OFDM/TDM with MMSE-FDE is a good combination in strong frequency-selective fading channel.

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