## 2ホップバーチャルセルラーネットワークにおける品質劣化確率

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**あらまし** 超高速無線通信を実現するときに問題となる送信電力の増大を解決するためにバーチャルセルラネットワーク(VCN)が提案されている. VCN では, コアネットワークへのゲートウェイとなる中央無線ポートから送信された信号は,分散配置された無線ポートで受信され,移動端末へと無線マルチホップ通信によって転送される. 本論文では2ホップ VCN を扱う. 2ホップ VCN では中央無線ポートと移動端末とが無線ポートを経由して, 2ホップで通信する.本論文では、無線ポートの配置法が品質劣化確率へ与える影響を理論検討および計算機シミュレーションにより明らかにしている.

キーワード バーチャルセルラネットワーク,2ホップネットワーク,品質劣化確率,カバレッジ.

### Outage Probablity of a 2-Hop Virtual Cellular Network

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**Abstract**— A wireless multi-hop virtual cellular network (VCN) was recently proposed to avoid the large peak transmit power, resulting from the high speed data transmissions expected in next generation mobile communication systems. In a multi-hop VCN, a central port (CP) which is a gateway to the core network, communicates with a mobile terminal (MT) via wireless ports (WPs) in the downlink. In this paper, 2-hop VCN, which is a special case the of multi-hop VCN where a CP communicates with MT directly or via one WP using 2 hops only, is proposed. The impact of the distribution of WPs on the outage probability is evaluated by numerical analysis and computer simulation.

Keywords: virtual cellular network, 2-hop network, outage probability, coverage extension.

#### 1. Introduction

Recently the mobile communication services are shifting from voice transmissions to data transmissions. Therefore, there is an increasing demand of high speed data transmissions in the next generation mobile communication systems. Ignoring the shadowing loss and multipath fading, the energy per bit-to-AWGN power spectrum density ratio  $E_b/N_0$  is given by [1], [2]

$$\frac{E_b}{N_0} = \left(\frac{P_{t,BS} / B}{N_0}\right) r_0^{-\alpha}, \qquad (1)$$

where  $P_{t,BS}$  is the transmit power, B is the bit rate,  $r_0$  is the cell radius,  $\alpha$  is the path loss exponent.

From Eq. (1), as the bit rate *B* increases, the transmit power should be increased in order to satisfy the required  $E_b/N_0$ . Therefore, with the conventional network architecture, keeping the same transmit power in next generation mobile systems will result in decrease of coverage of the Base Station (BS).

In order to extend the coverage of the BS using the same transmit power, different network architecture should be used. One example of such network architecture is the multi-hop Virtual Cellular Network (VCN) [4], [5]. In the multi-hop VCN, a central port (CP) which is a gateway to the core network, communicates with a mobile terminal (MT) directly or indirectly via wireless ports (WPs) distributed inside the cell. A 2-hop VCN is the simplest case of the multi-hop VCN where the CP connects with MT directly or via one WP using 2 hops only. Fig. 1 illustrates the coverage extension of a 2-hop VCN.





Fig. 1: 2-hop Virtual Cellular Network.

To realize the 2-hop VCN, the distribution of WPs by which the coverage of CP can be effectively extended with least number of WPs, should be found out. The main objective of this paper is to evaluate the impact of different distribution patterns of WPs on the outage probability by numerical analysis and computer simulation. The rest of the paper is organized as follows. Several distribution patterns of WPs are considered in Sect. 2. In Sect. 3 numerical analysis of outage probability of a single cell is done. In Sect. 4, the impact of distribution patterns of WPs on the outage probability is evaluated by numerical analysis and computer simulation. Finally the paper is concluded in Sect. 5.

#### 2. Distribution of WPs

We assume that at low data rate, the CP covers a circular cell of radius  $r_0$  satisfying an allowable outage probability. As the data rate increases, the coverage of the CP satisfying the allowable outage probability decreases from  $r_0$  to  $r_0$ ' as shown by the big circle and small circle in Fig. 2. K number of WPs should be distributed such that the CP can reach the MTs outside its coverage  $r_0$ ' and thus increase its coverage back to  $r_0$ . The transmit powers of the CP and the WPs are assumed to be the same. Therefore the coverage of the CP and the WPs are the same. Below, the proposed methods of distribution of the WPs are explained followed by their illustrating figures.

#### Method A

To give relaying assistance whenever needed by CP to communicate with the MTs outside the coverage of CP, the WPs are placed, equidistant from each other at the edge of the coverage of CP.

#### Method B

Using method A, when K gets bigger, the coverage of one WP overlaps with the coverage of other WPs largely. Therefore, there may be not much improvement in the outage probability, despite increasing the number of WPs. In method B, in order to decrease the overlapped coverage area of the WPs compared to method A, the WPs are randomly distributed outside the coverage of CP.

#### Method C

In method C, WPs are randomly distributed outside the coverage of CP and other WPs. Method D

In method D, the WPs are randomly distributed such that the coverage of one does not overlap with the coverage of CP and other WPs. However, when K increases, the WPs cannot satisfy this condition. In this case, those WPs that cannot satisfy this condition are distributed using method C again.



#### 3. Numerical Analysis

In this section the numerical analysis of outage probability at a certain position in a single cell with and without the WPs is done. Moreover, the total outage probability of a single cell is evaluated.

#### Outage Probability of a cell without WP

A circular cell of radius  $r_0$  is considered with CP in its center. The received power at MT from CP  $P_{r,MT-CP}$  is given by [1], [2], [3]

$$P_{r,MT-CP} = P_{r,0} r_1^{-\alpha} 10^{\frac{-\alpha}{10}},$$
(2)

where  $P_{r,r_0} = P_{t,CP} r_0^{-\alpha}$  is the received power at the cell edge with  $P_{t,CP}$  denoting the transmit power of CP,  $\alpha$  is the path loss exponent,  $\eta_1$  is the shadowing loss of standard deviation  $\sigma$  between CP-MT link,  $r_1 = r_{CP-MT}/r_0$  is the normalized distance between CP and MT.  $r_1$  can be expressed in terms of the coordinates of MT as

$$r_{1} = \frac{r_{CP-MT}}{r_{0}} = \sqrt{\left(\frac{x_{CP} - x}{r_{0}}\right)^{2} + \left(\frac{y_{CP} - y}{r_{0}}\right)^{2}},$$
 (3)

where  $(x_{CP}, y_{CP})$  and (x, y) are the coordinates of CP and MT respectively.

We define 
$$\gamma_{req} = \left(\frac{E_b}{N_0}\right)_{req} = \left(\frac{P_{req}/B}{N_0}\right)$$
 as the required

 $E_b/N_0$ , where  $P_{req}$  is the required received signal power. The outage probability  $P(r_1)$  at distance  $r_1$  is given by

$$P(r_{1}) = \operatorname{Prob}\left[\frac{P_{rMT,CP} / B}{N_{0}} < \gamma_{reg}\right] = \operatorname{Prob}\left[\chi_{CP} r_{1}^{-\alpha} 10^{-\frac{\eta_{1}}{10}} < 1\right], (4)$$

where  $\chi_{CP} = \frac{P_{r,r_0} / B}{N_0} \times \frac{1}{\gamma_{req}}$  is the ratio of received  $E_b/N_0$  at the cell edge transmitted by CP and required  $E_b/N_0$ .

condition for  $\chi_{CP} r_1^{-\alpha} 10^{-\eta_1/10} < 1$ The is  $\eta_1 > \eta_{req,1} = 10(\log \chi_{CP} - \alpha \log r_1)$ . The probability density function (pdf) of  $\eta_1$  is given by

$$p(\eta_1) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{\eta_1^2}{2\sigma^2}\right].$$
 (5)

Therefore, the outage probability  $P(r_1)$  at  $r_1$  is given by

$$P(r_{1}) = \int_{\eta_{req,1}}^{\infty} \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{\eta_{1}^{2}}{2\sigma^{2}}\right] d\eta_{1} = \frac{1}{2} \operatorname{erfc}\left[\frac{\eta_{req,1}}{\sqrt{2\sigma}}\right].$$
(6)

Next, the total outage probability of the cell  $P_{total}(r_1)$  is given by [2]

$$P_{total}(r_{1}) = \frac{1}{\pi} \int_{0}^{1} P(r_{1}) 2\pi r_{1} dr_{1}$$
  
= 
$$\int_{0}^{1} erfc \left[ \frac{10}{\sqrt{2}\sigma} \{ \log \chi_{CP} - \alpha \log r_{1} \} \right] r_{1} dr_{1}$$
 (7)

The above integration is numerically evaluated using Simpson's approximation [7], [8].

### Outage Probability of a cell with one WP

A WP is placed inside the cell at distance  $r_{CP-WP}$ from the CP. The CP communicates with the MT directly or via WP. The outage probability of CP-MT link  $P_{CP}$  is given by Eq.(6).The outage probability of the CP-WP-MT link  $P_{relay}$  is given by [5]

$$P_{relay} = \operatorname{Prob}\left[\left(\chi_{CP}r_{2}^{-\alpha}10^{\frac{\eta_{2}}{10}} < 1\right)\operatorname{or}\left(\chi_{WP}r_{3}^{-\alpha}10^{\frac{\eta_{3}}{10}} < 1\right)\right]$$
  
= 
$$\operatorname{Prob}\left(\chi_{CP}r_{2}^{-\alpha}10^{\frac{\eta_{2}}{10}} < 1\right) + \operatorname{Prob}\left(\chi_{WP}r_{3}^{-\alpha}10^{\frac{\eta_{3}}{10}} < 1\right) , (8)$$
  
$$-\operatorname{Prob}\left(\chi_{CP}r_{2}^{-\alpha}10^{\frac{\eta_{2}}{10}} < 1\right) \times \operatorname{Prob}\left(\chi_{WP}r_{3}^{-\alpha}10^{\frac{\eta_{3}}{10}} < 1\right)$$
  
$$= P(r_{2}) + P(r_{3}) - P(r_{2}) \times P(r_{3})$$

where  $r_2$  is the normalized distance between CP and WP and  $r_3$  is the normalized distance between WP and MT, given by

$$r_{2} = \frac{r_{CP-WP}}{r_{0}} = \sqrt{\left(\frac{x_{CP} - x_{WP}}{r_{0}}\right)^{2} + \left(\frac{y_{CP} - y_{WP}}{r_{0}}\right)^{2}},$$
 (9)

$$r_{3} = \frac{r_{WP-MT}}{r_{0}} = \sqrt{\left(\frac{x_{WP} - x}{r_{0}}\right)^{2} + \left(\frac{y_{WP} - y}{r_{0}}\right)^{2}},$$
 (10)

where  $(x_{CP}, y_{CP})$ ,  $(x_{WP}, y_{WP})$  and (x, y) are the coordinates of CP, WP and MT respectively.  $\eta_2$  and  $\eta_3$  are the shadowing losses of standard deviation  $\sigma$  between CP-WP link and WP- WT link, respectively, and  $\chi_{WP} = \frac{P_{t,WP} r_0^{-\alpha} / B}{N_0} \times \frac{1}{\gamma_{req}}$ , where  $P_{t,WP}$  is the transmit

power of WP. Similarly to Eq. (5),  $P(r_2)$  and  $P(r_3)$  are given by

$$P(r_2) = \frac{1}{2} \operatorname{erfc}\left[\frac{\eta_{req,2}}{\sqrt{2}\sigma}\right],\tag{11}$$

$$P(r_{3}) = \frac{1}{2} \operatorname{erfc}\left[\frac{\eta_{req,3}}{\sqrt{2}\sigma}\right], \qquad (12)$$

where  $\eta_{req,2} = 10 \{ \log \chi_{CP} - \alpha \log r_2 \}$  and

 $\eta_{req,3} = 10 \{ \log \chi_{WP} - \alpha \log r_3 \}.$ 

Therefore, the outage probability at (x,y) is given by

$$P(x, y) = P_{CP} \times P_{relay} = P(r_1) \{ P(r_2) + P(r_3) - P(r_2) \times P(r_3) \}.$$
 (13)

Next, the total outage probability of the cell  $P_{total}(x,y)$  is given by[2]

$$P_{total}(x,y) = \frac{1}{\pi} \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} P(x,y) dx dy .$$
 (14)

Here, the WP can be positioned such that there is no outage between the CP-WP link, i.e.  $P(r_2)=0$ . By doing so, the outage probability P(x,y) at (x,y) and the total outage probability  $P_{total}(x,y)$  can be modified as

$$P(x, y) = P_{CP} \times P_{relay} = P(r_1) \times P(r_3), \qquad (15)$$

$$P_{total}(x,y) = \frac{1}{\pi} \int_{-1}^{1} \frac{\sqrt{1-x^2}}{\sqrt{1-x^2}} P(r_1) \times P(r_3) dx dy .$$
 (16)

The above integration is also numerically evaluated using Simpson's approximation [7], [8]. The plotted results will be shown in the next section.

#### 4. Computer Simulation

In this section, the simulation results are discussed. Comparison between the numerical and simulated results is also made. Monte-Carlo simulation was conducted to evaluate the outage probability in a single cell environment. A circular cell of normalized radius  $r_0$  with CP in its center is considered. An MT is generated randomly inside the cell. K WPs are generated as explained in Sect. 2. The links between CP and WPs are always considered such that there is no outage between them as mentioned at the end of Sect. 3.

The simulation parameters are as follows. Path loss exponent  $\alpha$  is assumed to be 3.5, standard deviation  $\sigma$  of shadowing loss is assumed to be 7 and the allowable outage probability is assumed to be 0.1. The transmit powers of CP and WPs are considered to be the same.

#### A. Impact of the ratio of received $E_b/N_0$ at the cell edge and required $E_b/N_0$ ( $\chi_{CP}$ ) on the total outage probability

From Eq. (1), increase in the transmission rate is equivalent to the decrease in  $\chi_{CP}$ . Fig. 3 shows the impact of  $\chi_{CP}$  on the total outage probability of the cell. As the  $\chi_{CP}$  decreases, the total outage probability of the cell increases. It is seen that the numerical and simulated results are very close.



Fig. 3: Impact of  $\chi_{CP}$  on outage probability

From Fig. 3, for an allowable outage probability to be 0.1,  $\chi_{CP}$  should be 2.75. As the transmission rate increases 10 times, which is equivalent to decreases in  $\chi_{CP}$  by 10 times ( $\chi_{CP} = 0.275$ ), the outage probability increases more than 0.45. Moreover, when the transmission rate increases 100 times, which is equivalent to decreases in  $\chi_{CP}$  by 100 times ( $\chi_{CP} = 0.0275$ ), the outage probability increases to almost 0.8.

# B. Decrease in coverage of CP with higher data transmission rate

As stated earlier, when the transmission rate is 100 times higher, the outage probability increases to almost 0.8. Therefore CP's coverage area has to be decreased to satisfy the allowable total outage probability of 0.1. Here, we investigated the coverage of the CP when  $\chi_{CP}$  is 0.0275. Fig. 4 shows the outage probability as a function of the normalized radius. From Fig. 4, when the transmission rate becomes 100 times higher, the coverage of the CP decreases to a circle of radius  $r_0 / r_0 = 0.265$  from a circle of radius 1. It is also seen that the numerical and simulated results are very close.



Fig. 4: Coverage of CP at transmission rate 100 times higher

# C. Impact of different distribution patterns of WPs on the outage probability

To extend the coverage of CP, K WPs are distributed using the proposed methods in Sect. 2. Fig. 4 shows the impact of these methods on the total outage probability of the cell. Since the positions of the WPs are random in method B,C and D, the numerical evaluation is only done for method A. From Fig. 5, it is seen that the number of WPs required to satisfy the allowable outage probability of 0.1 by methods A, B, C and D are nearly equal to 39, 15, 12 and 11, respectively. The smallest number of WPs required to satisfy the allowable outage of 0.1 is when method D is used. In method D, WPs are distributed such that their coverage do not overlap as far as possible and when this is not possible they are distributed such that one WP does not fall under the coverage of another WP. Among the 4 methods, it is seen that by method D the coverage of the CP can be increased, satisfying the allowable outage probability, with least number of WPs. Therefore, method D is the most efficient one among these methods.



Fig. 5: Impact of different distribution of WPs on outage probability

#### 5. Conclusion

In this paper the outage probability of 2-hop VCN with different distribution patterns of WPs is calculated by numerical analysis and computer simulation. It can be concluded that by distributing the WPs efficiently, such that their coverages do not overlap and when this is not possible by distributing the new WP such that it does not fall under the coverage of others (Method D), the coverage of CP can be extended even in very high speed data transmission.

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