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Hybrid Frequency Reuse for Cellular MIMO Systems with Multi-user Diversity

Wei Peng[†] and Fumiyuki Adachi[‡]

Dept. of Electrical and Communication Engineering, Graduate School of Engineering, Tohoku University
6-6-05 Aza-Aoba, Aramaki, Aoba-ku, Sendai, 980-8579 Japan
E-mail: [†]peng@mobile.ecei.tohoku.ac.jp, [‡]adachi @ecei.tohoku.ac.jp

Abstract—In the cellular MIMO systems, the system capacity can be increased with the presence of multi-user diversity (MUD). In this paper, a hybrid frequency reuse algorithm is proposed to realize efficient frequency reuse factor (FRF) allocation for cellular MIMO systems with MUD. As a result, the system capacity will be increased greatly. The system capacities by using the proposed algorithm are theoretically analyzed and the effectiveness of hybrid frequency reuse will be testified by numerical results..

Keywords: *hybrid frequency reuse, MUD, MIMO, downlink*

1. Introduction

Multiple input multiple output (MIMO) [1, 2] technology has been widely adopted because of its capability to improve the bandwidth efficiency or bit error rate (BER) performance without extra bandwidth or power consumption. Recently, the system capacity of cellular MIMO systems has attracted much interest [3-5]. In the cellular systems, to efficiently utilize the limited bandwidth, the same carrier frequency is reused at spatially separated base stations (BSs). By taking the frequency reuse factor (FRF) into consideration, a hybrid frequency reuse algorithm for the downlink cellular MIMO systems was proposed by the authors [6]. This hybrid frequency reuse algorithm adaptively chooses the FRF from {1, 3} to maximize the cellular capacity when single user is considered. It has been shown that the hybrid frequency reuse algorithm in [6] can effectively improve the system capacity. At the same time, the cellular capacity can also benefit from multi-user diversity (MUD) [7-8]. It is reported in [9] that by using the dirty paper approach, the sum capacity (i.e., the sum of capacities on each link between the base station (BS) and the mobile station (MS)) can be achieved. Therefore, it is of interest how the cellular capacity can be increased if both MUD and hybrid frequency reuse can both be adopted and how to realize the hybrid frequency reuse algorithm for multi-user cellular MIMO systems.

In this paper, a multi-user hybrid frequency reuse algorithm is proposed for the downlink cellular MIMO systems. The cellular capacity using the proposed algorithm is theoretically analyzed. The effectiveness of the proposed algorithm is testified by the numerical results. The rest of paper is organized as follows. Section 2 describes the system model for multi-user downlink cellular MIMO systems. Hybrid frequency reuse algorithm will then be proposed in Section 3. The cellular capacity will be analyzed in Section 4. Numerical results will be shown in Section 5 and finally Section 6 will conclude the paper.

2. System Model

Consider a downlink cellular system. The number of transmit antennas at the base station (BS) is N_t and each user is using single antenna. Dirty paper pre-coding [10] is assumed and therefore the number of active users in each transmission will be limited to N_t as well. In the cellular circumstance, there exists co-channel interference (CCI) from the neighboring cells due to the frequency reuse. The number of co-channel cells to be considered depends on the cellular structure. As shown in Fig. 1, when FRF=1 where all the cells are using the same frequency, for the BS of interest, the BSs in the first-tier and second-tier neighboring cells are considered as co-channel BSs and B=18. When FRF > 1 (3, 4, 7, etc), only the BSs in the first-tier neighboring cells are considered as co-channel BSs and B=6.

Assuming N_t active users, the received signal vector $\mathbf{y}=[y_1,\cdots,y_{N_t}]^T$ with y_k representing the k^{th} user received signal is given by

$$\begin{aligned}\mathbf{y} &= \mathbf{HLx}+\sum_{j=1}^{B}\mathbf{H}_j\mathbf{L}_j\mathbf{x}_j+\mathbf{n}, \\ &= \mathbf{HLx}+\mathbf{v}\end{aligned} \tag{1}$$

where $x=[x_1,\cdots,x_{N_t}]^T$ is the transmit signal vector with x_k representing the transmit signal from the k^{th} antenna; superscript T represents transpose; $\mathbf{H}=[\mathbf{h}_1,\cdots,\mathbf{h}_{N_t}]$ is an $N_t\times N_t$ channel matrix ($\mathbf{h}_k$ represents the k^{th} column vector of **H**), whose elements are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance; $\mathbf{L}$ is an $N_t\times N_t$ diagonal matrix representing the effects of path loss and shadowing loss. The k^{th} diagonal element of $\mathbf{L}$ is $\sqrt{d_{0,k}^{-\alpha}10^{-\xi/10}}$ where $d_{0,k}$ represents the distance between BS and the k^{th}

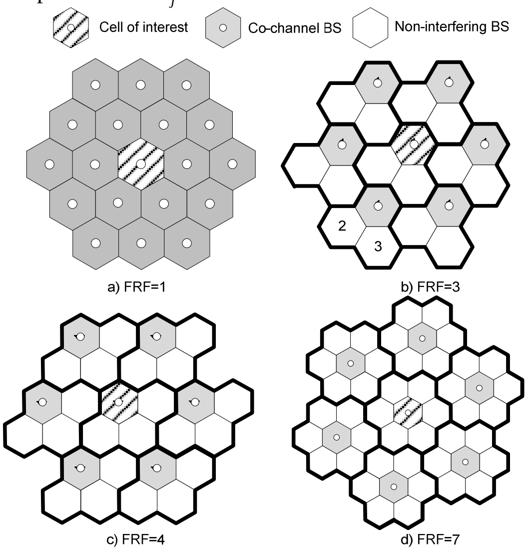
active user; α and ξ represent the path loss exponent and shadowing loss in dB, respectively. To simplify the analysis, in this study, $\xi = 0$ (no shadowing loss) is assumed. $\mathbf{n} = [n_1, \cdots, n_{N_t}]^T$ is the additive white Gaussian noise (AWGN) vector with variance σ_n^2. $\mathbf{H}_i$, $\mathbf{L}_i$ and $\mathbf{x}_i$ represent the channel matrix, the matrix of path loss and the transmit signal vector from the j^{th} co-channel BS, respectively.

Let $\mathbf{H} = \mathbf{TQ}$ be a QR decomposition so that $\mathbf{T}$ is an $N_t \times N_t$ lower triangular matrix and $\mathbf{Q}$ is an $N_t \times N_t$ unitary matrix. The unitary matrix $\mathbf{Q}^*$ is applied to the modulated vector $\mathbf{u}$ to obtain the transmit vector as $\mathbf{x} = \mathbf{Q}^*\mathbf{u}$, where superscript * represents conjugate transpose. To avoid interference between the active users, dirty paper approach is adopted at the BS. At the same time, the transmit power is controlled at the BS [11] so that $\left|t_{k,k}u_k\right|^2 = \left\|\mathbf{h}_k\right\|^2 \cdot P_t \big/ N_t$ where $t_{k,k}\left(k = 1, \cdots, N_t\right)$ is the k^{th} diagonal element of matrix $\mathbf{T}$, $\left\|\mathbf{h}_k\right\|^2 = \sum_{i=1}^{N_t}\left|h_{k,i}\right|^2$ is the squared norm of $\mathbf{h}_k$ and P_t is the total transmit power from the BS (the details of the dirty paper coding and transmit power control will not be discussed for brevity).

In (1), $\mathbf{v}$ represents the interference plus noise term, where in this study, Gaussian approximation is used to model the CCI [3]. Under this assumption, $\mathbf{v}$ can be treated as equivalent AWGN vector with zero mean and variance σ_v^2 given by

$$\sigma_v^2 = \sum_{j=1}^{B} d_j^{-\alpha} E\left\{\left\|\mathbf{x}_j\right\|^2\right\} + \sigma_n^2 , \tag{2}$$

where d_j is the distance from the j^{th} co-channel BS to the BS of interest and $E\left\{\left\|\mathbf{x}_j\right\|^2\right\}$ is the average total transmit power of $\mathbf{x}_j$.



a) FRF=1 b) FRF=3 c) FRF=4 d) FRF=7

Fig. 1 The cellular structure and CCI

3. Hybrid Frequency Reuse Algorithm

A. Multi-user scheduling

Let K represent the number of users in total. When $K \geq N_t$, MUD can be achieved through multi-user scheduling. Although the system capacity can be maximized by optimal permutation on the transmit antennas at the BS, how to perform such permutation is still a problem to be solved yet. Therefore, no permutation at the BS is assumed in this study and the scheduling process will be carried out sequentially on the transmit antennas. By taking consideration of both performance and fairness, scheduling will be based on the instantaneous channel condition of each user and their long term statistics. Let $\left\{s_1, \cdots, s_{N_t}\right\}$ be the set of selected users. $\left\{s_1, \cdots, s_{N_t}\right\}$ are selected by the following criterion:

$$\begin{aligned} s_1 &= \underset{s_1 \in \{1,\cdots,K\}}{\arg\max} \left\|\mathbf{h}_1 \left| s_1 \right.\right\| \Big/ E\left\{\left\|\mathbf{h}_1 \left| s_1 \right.\right\|\right\} \\ s_2 &= \underset{s_2 \in \{1,\cdots,K\}, s_2 \neq s_1}{\arg\max} \left\|\mathbf{h}_2 \left| s_2 \right.\right\| \Big/ E\left\{\left\|\mathbf{h}_2 \left| s_2 \right.\right\|\right\} \\ &\vdots \\ s_{N_t} &= \underset{\substack{s_{N_t} \in \{1,\cdots,K\}, \\ s_{N_t} \neq s_1,\cdots,s_{N_t-1}}}{\arg\max} \left\|\mathbf{h}_{N_t} \left| s_{N_t} \right.\right\| \Big/ E\left\{\left\|\mathbf{h}_{N_t} \left| s_{N_t} \right.\right\|\right\} \end{aligned} , \tag{3}$$

where $\mathbf{h}_k \left| s_k \right.$ represents the channel vector $\mathbf{h}_k$ when user s_k is selected. $E\{\cdot\}$ represent expectation.

B. Hybrid frequency reuse algorithm

It has been shown in [6] that for single user cellular MIMO systems, the maximum capacity can be achieved by FRF 1 for the users located near the BS while the system capacity will be maximized by FRF 3 for the users located near the cell boundary. Based on this, a hybrid FRF allocation algorithm can be realized by adaptively allocate FRF from {1, 3} to maximize the system capacity, as shown in Fig. 2.

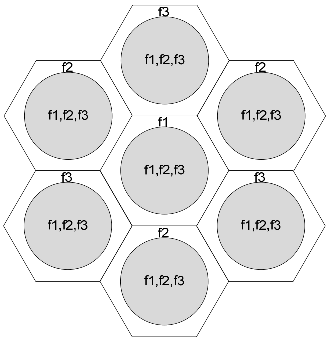


Fig. 2 Hybrid FRF algorithm

In the above figure, $\left\{f_1, f_2, f_3\right\}$ is a set of three frequencies. For each user, FRF 1 or FRF 3 will be used according to its position within the cell. When the user is near the cell center, $\{f_1, f_2, f_3\}$ will all be used (FRF=1). Otherwise, when the user is near the cell boundary, only one frequency, f_1 or f_2 or f_3, will be used (FRF=3). The FRF for each user is determined separately. As a result, during one transmission, it is possible that a part of the active users are using FRF 1 and the others are using FRF 3.

The hybrid FRF algorithm for the multi-user downlink cellular MIMO systems can be realized by the following steps:

Step 1. Each user does channel estimation and feedback the channel state information (CSI) to the BS.

Step 2. Select N_t active users according to the criterion in (3).

Step 3. Perform dirty-paper pre-coding and transmit power control at the BS to generate the transmit vector.

Step 4. Determine the FRF for each user to maximize the capacity.

The criterion to determine FRF for each user in Step 4 will be explained later in Section 4.

4. Cellular Capacity Analysis

It is assumed that all the BSs are using the same total transmit power, i.e., $E\left\{\left\|\mathbf{x}_j\right\|^2\right\} = E\left\{\left\|\mathbf{x}\right\|^2\right\} = P_t$. Without MUD, the capacity (bps/Hz) on the k^{th} link (the link between BS and user k) is given by [12]

$$c_k = \log\left(1 + SINR_k\right), \tag{4}$$

where $SINR_k$ is the signal to interference plus noise ratio (SINR) of user k. Given the FRF F, $SINR_k$ can be calculated by

$$SINR_k\left(F\right) = \frac{\left|t_{k,k} x_k\right|^2 l_{k,k}^2}{\sigma_v^2} \approx \begin{cases} \dfrac{1}{N_t} \cdot \dfrac{\left\|\mathbf{h}_k\right\|^2 \cdot r_0^{-\alpha} \cdot P_t / \sigma_n^2 \cdot \varepsilon_k^{-\alpha}}{r_0^{-\alpha} \cdot P_t / \sigma_n^2 \sum\limits_{i=1}^{18} \left(d_i / r_0\right)^{-\alpha} + 1}, F = 1 \\ \dfrac{1}{N_t} \cdot \dfrac{\left\|\mathbf{h}_k\right\|^2 \cdot r_0^{-\alpha} \cdot P_t / \sigma_n^2 \cdot \varepsilon_k^{-\alpha}}{r_0^{-\alpha} \cdot P_t / \sigma_n^2 \sum\limits_{i=1}^{6} \left(d_i / r_0\right)^{-\alpha} + 1}, F > 1 \end{cases}, \tag{5}$$

where $l_{k,k}$ is the k^{th} diagonal element of $\mathbf{L}$; r_0 represents the cell radius and $\varepsilon_k^{-\alpha} = d_{0,k} / r_0$ with $d_{0,k}$ representing the distance of active user k from its BS. When FRF>1, the bandwidth allocated is reduced by a factor of F or the total bandwidth necessary is increased F times. Therefore, the cellular capacity (bps/Hz/BS) should be normalized by the bandwidth and is given by

$$c_{k,hybrid} = \frac{1}{F_k} \log\left(1 + SINR_k\left(F_k\right)\right), \tag{6}$$

where F_k is the FRF allocated to user k. According to (6), F_k can be determined by

$$F_k = \arg\max_{F_k \in \{1,3\}} \left(\frac{1}{F_k} \log\left(1 + SINR_k\left(F_k\right)\right)\right). \tag{7}$$

Let ω be the norm of the channel vector between BS and a random user. It is obvious that ω followings the chi-squared distribution with $2N_t$ degrees of freedom. The probability density function (p.d.f.) and cumulative density function (c.d.f.) of ω can then be given as [13]

$$p_\omega\left(\omega\right) = \omega^{N_t - 1} \exp\left(-\omega\right) / \left(N_t - 1\right)! \tag{8}$$

and

$$P_\omega\left(\omega\right) = 1 - \exp\left(-\omega\right) \sum_{k=0}^{N_t - 1} \frac{\omega^k}{k!}. \tag{9}$$

Given a randomly selected user k, whose distance from the BS is $d_{0,k}$, the instantaneous capacity for the link between BS and $d_{0,k}$ is calculated by averaging (6) over the distribution of ω as

$$C_{k,hybrid} = \int_0^\infty c_{k,hybrid}\, p_\omega\left(\omega\right) d\omega. \tag{10}$$

The ergodic cellular capacity can then be evaluated by

$$C = \int_0^{r_0} \cdots \int_0^{r_0} \sum_{k=1}^{N_t} C_{k,hybrid} f\left(d_{0,1}\right) \cdots f\left(d_{0,N_t}\right) d_{d_{0,N_t}} \cdots d_{d_{0,1}}. \tag{11}$$

where $f\left(d_{0,k}\right)$ is the p.d.f. of $d_{0,k}$. When MUD is available, let $\Omega_k = \left\|\mathbf{h}_k \left|s_k\right.\right\|$. According to (3), the k^{th} selected user obtains the maximum of $\left\|\mathbf{h}_k \left|s_k\right.\right\| / E\left\{\left\|\mathbf{h}_k \left|s_k\right.\right\|\right\}$ where $s_k \in \{1, \cdots, K\}$, $s_k \neq s_1, \cdots, s_{k-1}$. Since $E\left\{\left\|\mathbf{h}_k \left|s_k\right.\right\|\right\}$ is a user-independent constant, the p.d.f. of Ω_k can then be obtained by using the conclusions of order statistics [14]

$$p_{\Omega_i}\left(\Omega_k\right) = \left(K - k + 1\right)\left[P_\omega\left(\Omega_k\right)\right]^{K-k} p_\omega\left(\Omega_k\right), \tag{12}$$

The cellular capacity with MUD can then be evaluated by substitution of (12) into (10) ~ (11) as

$$C_{k,MUD}=\int_0^{\infty}c_{k,hybrid}\,p_{\Omega_k}(\Omega_k)\,d\Omega_k\,, \tag{13}$$

and

$$C_{MUD}=\int_0^{r_0}\cdots\int_0^{r_0}\sum_{k=1}^{N_t}C_{k,MUD}\,f(d_{0,1})\cdots f(d_{0,N_t})\,d_{d_{0,N_t}}\cdots d_{d_{0,1}}\,. \tag{14}$$

5. Numerical and Simulation Result

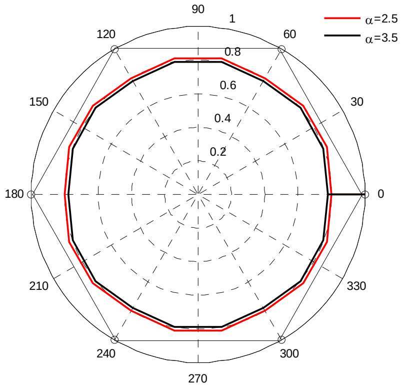
In this section, the performance of the proposed hybrid frequency reuse algorithm will be studied by numerical results. It is assumed that all the users are uniformly located within a cell. The parameters used to generate the numerical results are listed in Table 1.

Table I Parameters

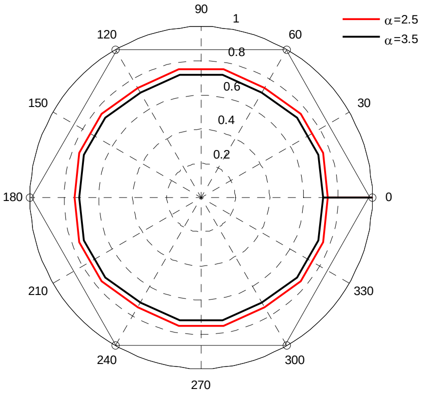
Number of antennas (N_t)	2~6
Number of users (K)	4~20
Path loss exponent α	2.5, 3.5
Received SNR at the cell boundary $P_t\cdot r_0^{-\alpha}/\sigma_n^2$	20dB, 30dB

A. *The effect of hybrid FRF algorithm*

At first, the performance of the hybrid FRF algorithm is studied. According to (7), the FRF allocation within a cell is shown in Fig. 3.



(a) $P_t\cdot r_0^{-\alpha}/\sigma_n^2=10dB$



(b) $P_t\cdot r_0^{-\alpha}/\sigma_n^2=30dB$

Fig. 3 FRF allocation within a cell.

The cellular capacities with and without MUD ((11) and (14)) are compared with that of the systems using FRF 1 scheme. The results are shown in Fig. 4 and Fig. 5 separately. It is assumed that $N_t=4$ and $K=20$, the received signal-to-noise power ratio (SNR) at the cell boundary $P_t\cdot r_0^{-\alpha}/\sigma^2=20dB$. It is seen that the cellular capacity can be effectively increased by the hybrid FRF allocation algorithm. It can be further observed by comparing Figs. 4 and 5 that the cellular capacity increases with the use of MUD; this will be studied in more detail in the following.

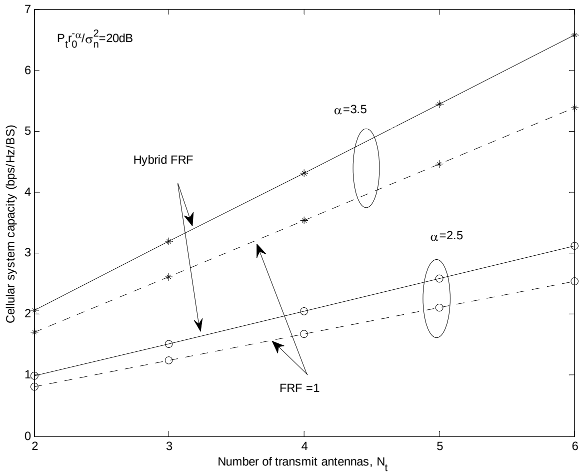


Fig. 4 The effect of hybrid FRF algorithm, without MUD.

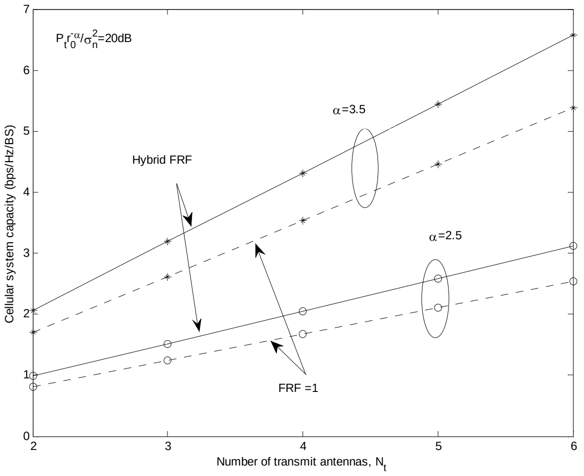


Fig. 5 The effect of hybrid FRF algorithm, with MUD.

Next, the effect of MUD is studied. The number *K* of users increases from 4 to 20. The cellular capacity with MUD in (14) is shown in Fig. 6. It is assumed that the number of transmit antennas at the BS is fixed at 4. The corresponding cellular capacity without MUD given in (11) is also shown as a reference. It can be observed that the use of MUD improves the cellular capacity and that the cellular capacity increases as the K increases. However, the additional increase will become less significant when K is large. In other words, the MUD will saturate.

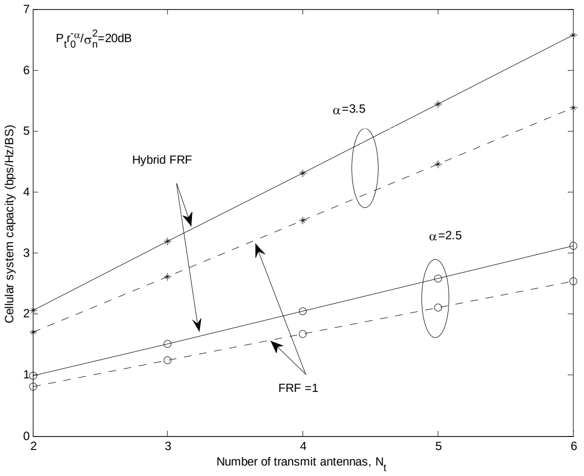


Fig. 6 The effect of MUD.

6. Conclusion

This paper has proposed a multi-user hybrid FRF algorithm for cellular MIMO systems to improve the cellular capacity. The capacity performance of the proposed algorithm has been theoretically analyzed and presented by numerical results. It has been observed that on one hand, the proposed algorithm can greatly improve the cellular capacity; on the other hand, the gain brought by MUD will saturate when the number of users increases.

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