社団法人 電子情報通信学会
THE INSTITUTE OF ELECTRONICS,
INFORMATION AND COMMUNICATION ENGINEERS

信学技報
TECHNICAL REPORT OF IEICE

A Comparative Study on pre-FFT and post-FFT Frequency Domain Adaptive Antenna Array for A Broadband Single-carrier multiple access system

Wei Peng [†] and Fumiyuki Adachi [‡]

Dept. of Electrical and Communication Engineering, Graduate School of Engineering, Tohoku University
6-6-05 Aza-Aoba, Aramaki, Aoba-ku, Sendai, 980-8579 Japan
E-mail: [†] peng@mobile.ecei.tohoku.ac.jp, [‡] adachi @ecei.tohoku.ac.jp

Abstract—In our previous study, we proposed a post FFT frequency domain adaptive antenna array (FD-AAA) algorithm for the uplink detection in the broadband single-carrier multiple access system. In the post-FFT algorithm, the AAA weights on different frequencies are calculated respectively. In this study, we will compare the performance of the proposed algorithm with the pre-FFT type algorithm where the AAA weight vector is calculated before the FFT and all the frequencies share the unique AAA weight. The effects of the distribution of direction of arrival (DOA) spread and three typical DOA models, namely, no spread distribution, uniform distribution and Gaussian distribution will be considered.

Keywords: *pre-FFT, post-FFT, Frequency Domain Equalization, Adaptive Antenna Array*

1. Introduction

To deal with the frequency selectivity [1] caused by multi-path fading channel, and to cancel the multi-user interference in the uplink broadband single-carrier transmission, we proposed a frequency domain adaptive antenna array (FD-AAA) algorithm in our previous study [2]. In the proposed algorithm, the AAA is performed in frequency domain and the AAA weight vector on each frequency will be calculated respectively. In other words, our proposed algorithm is a post-FFT type algorithm. A pre-FFT type algorithm was proposed in [3], it performs the AAA before the FFT and all the frequencies share the unique AAA weight vector. It is assumed in [3] that there is no DOA spread. However, the DOA spread does exist in practical systems. And the motivation of this study is to compare the performance of the proposed algorithm and the pre-FFT type algorithm with the existence of DOA spread. For the DOA distribution, we consider three DOA models, namely, no spread model, uniform distribution model and Gaussian distribution model.

The rest of the paper is organized as follows. The system model will be described in Section 2. The principle of the pre-FFT type algorithm and the proposed FD-AAA algorithm (post-FFT type algorithm) will be described in Section 3. The performances of the pre-FFT and post-FFT algorithms will then be compared by simulation results in Section 4. Finally, the paper will be concluded by Section 5.

2. System Model

A. Uplink transmission

The multi-user single-carrier uplink transmission model is shown in Fig. 1. The BS is equipped with N_r antennas. There are U users and each user has one transmit antenna. A block fading channel between each user and the BS is assumed. In this paper, the symbol-spaced discrete time representation of the signal is used. Assuming $L-$path channel, the impulse response of the channel between user k and the m^{th} antenna of the BS can be expressed as

$$h_{k,m}(\tau)=\sum_{l=0}^{L-1}h_{k,m,l}\delta(\tau-\tau_l)\,, \qquad (1)$$

where $h_{k,m,l}$ and τ_l are the complex-valued path gain and time delay of the l^{th} path, respectively. $h_{k,m,l}$ follows the complex Gaussian distribution and satisfies $\sum_{l=0}^{L-1}E\left\{\left|h_{k,m,l}\right|^2\right\}=1$ where $E\{\cdot\}$ represents the expectation. It is assumed that the time delay is a multiple integer of the symbol duration and $\tau_l=l$. The cyclic-prefixed single-carrier block signal transmission is used. It is assumed that the cyclic prefix (CP) is longer than the maximum path delay of the signal. In the following, we omit the insertion and removal of the CP for the simplicity purpose.

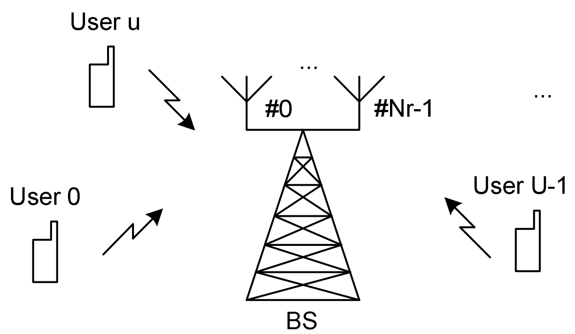

Fig. 1 The system model of multi-user single-carrier uplink transmission.

The baseband equivalent received signal block $\{r_m(t);t=0\sim N_c\}$ of N_c symbols at the m^{th} antenna is given by

$$r_m(t)=\sqrt{\frac{2E_0}{T}}\sum_{l=0}^{L-1}h_{0,m,l}s_0(t-l)+\sqrt{\frac{2E_u}{T}}\sum_{u=1}^{U-1}\sum_{l=0}^{L-1}h_{u,m,l}s_u(t-l)+n_m(t) \quad (2)$$

where $s_u(t)$ and $E_u\,(u=0,\cdots,U-1)$ are the transmitted signal and transmit power from user u, respectively. T is the signal period and $n_m(t)$ is the complex-valued additive white Gaussian noise (AWGN). Let the transmitted signal from the $u=0^{th}$ user be the desired signal, and the transmitted signals from the other users are the interfering signals.

B. DOA distribution

The distribution of DOA from all the users is shown in Fig. 2. We assume three DOA distribution models.

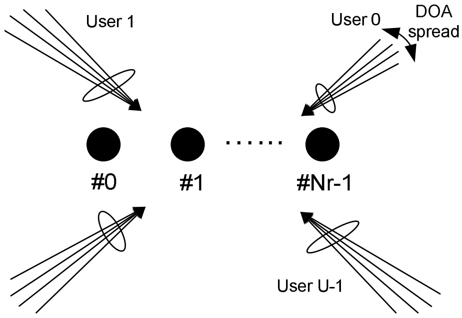

Fig. 2 Distribution of DOA.

1) No DOA spread

No DOA spread is assumed in [3], therefore, the angles of arrival (AOAs) of the u^{th} user can be represented by

$$\angle_{u,l}=\alpha_u;u=0,\cdots,U-1;l=0,\cdots,L-1, \quad (3)$$

where $\angle_{u,l}$ represents the AOA of the l^{th} path of the u^{th} user; α_u is the mean AOA of the u^{th} user.

2) Uniform distributed DOA

In the uniform distribution model, the AOA distribution of the u^{th} user can be represented by

$$P(\angle_{u,l})=\begin{cases}\frac{1}{2\Delta\alpha_u} & \alpha_u-\Delta\alpha_u\le\angle_{u,l}\le\alpha_u+\Delta\alpha_u\\ 0 & otherwise\end{cases}, \quad (4)$$

where $\Delta\alpha_u$ represents the range of DOA spread; $P(\angle_{u,l})$ represents the probability that the AOA equals to $\angle_{u,l}$.

3) Gaussian distributed AOA

In the Gaussian distribution model, the AOA distribution of the u^{th} user can be represented by

$$P(\angle_{u,l})=\frac{1}{\sqrt{2\pi}\Delta\alpha_u}\exp\left(-\frac{(\angle_{u,l}-\alpha_u)^2}{2(\Delta\alpha_u)^2}\right). \quad (5)$$

3. Pre-FFT type algorithm and post-FFT type (FD-AAA) algorithm

A. Pre-FFT type algorithm [3]

The block diagram of the pre-FFT type algorithm is shown in Fig. 3

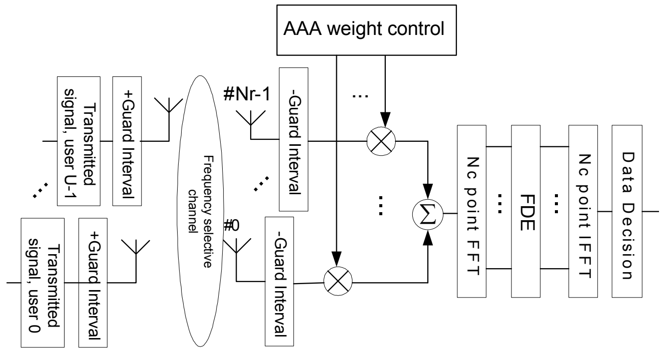

Fig. 3 Block diagram of pre-FFT type transceiver.

Let $y(t)$ represent the signal vector after the AAA combining; Let $\mathbf{w}_{pre}$ represent the weight vector, $\mathbf{w}_{pre}=\left[w_{pre-0},\cdots,w_{pre-N_r-1}\right]^T$ where superscript T means transpose. $y(t)$ can be written as

$$\begin{aligned}y(t)&=\mathbf{w}_{pre}^T\mathbf{r}(t)\\&=\sqrt{\frac{2E_0}{T}}\mathbf{w}_{pre}^T\sum_{l=0}^{L-1}\mathbf{h}_{0,l}s_0(t-l)\\&+\sum_{u=1}^{U-1}\sqrt{\frac{2E_u}{T}}\mathbf{w}_{pre}^T\sum_{l=0}^{L-1}\mathbf{h}_{u,l}s_u(t-l)+\mathbf{w}_{pre}^T\mathbf{n}(t)\end{aligned}, \quad (6)$$

where $\mathbf{r}(t)=\left[r_0(t),\cdots,r_{N_r-1}(t)\right]^T$, $\mathbf{h}_{0,l}=\left[h_{0,0,l},\cdots,h_{0,N_r-1,l}\right]^T$ and $\mathbf{n}(t)=\left[n_0(t),\cdots,n_{N_r-1}(t)\right]^T$. In this study, we follow [3] and use the normalized least mean square (NLMS) method to calculate the AAA weight vector. The AAA weight vector can be achieved iteratively by

$$\begin{cases}\mathbf{w}'_{pre}(n)=\mathbf{w}_{pre}(n-1)+2\mu\cdot e^*(n)\cdot\frac{\mathbf{r}(n)}{\|\mathbf{r}(n)\|^2},\\ \mathbf{w}_{pre}(n)=\frac{\mathbf{w}'_{pre,n}}{\|\mathbf{w}'_{pre,n}\|}\end{cases} \quad (7)$$

where $\mathbf{w}_{pre}(n-1)$ is the weight vector achieved in the previous iteration; N_c is the block size and $e(n)$ is the error function defined as

$$\begin{aligned} e(t) = &\sum_{l=0}^{L-1} \sqrt{\frac{2E_0}{T}} \mathbf{w}_{pre}^T (n-1) \mathbf{h}_{0,l} s_0(t-l) \\ &- \mathbf{w}_{pre}^T (n-1) \mathbf{r}(t) \end{aligned}. \tag{8}$$

The output of AAA combining, $y(t)$, is then transformed to the frequency domain signal by FFT transform, given by

$$\begin{aligned} Y(k) &= \sum_{t=0}^{N_c-1} y(t) \exp\left(-j2\pi k \frac{t}{N_c}\right) \\ &= \sum_{t=0}^{N_c-1} \left(\sum_{u=0}^{U-1} \sqrt{\frac{2E_u}{T}} \mathbf{w}_{pre}^T \sum_{l=0}^{L-1} \mathbf{h}_{u,l} s_u(t-l) \right) \exp\left(-j2\pi k \frac{t}{N_c}\right) \\ &+ \sum_{t=0}^{N_c-1} \mathbf{w}_{pre}^T \mathbf{n}(t) \exp\left(-j2\pi k \frac{t}{N_c}\right) \\ &= \hat{H}_0(k) S_0(k) + \sum_{u=1}^{U-1} \hat{H}_u(k) S_u(k) + \hat{N}(k) \end{aligned}, \tag{9}$$

where

$$\begin{cases} S_u(k) = \sqrt{\dfrac{2E_u}{T}} \displaystyle\sum_{t=0}^{N_c-1} s_u(t) \exp\left(-j2\pi k \frac{t}{N_c}\right) \\ H_{u,m}(k) = \displaystyle\sum_{l=0}^{L-1} \sum_{t=0}^{N_c-1} h_{u,m,l} \exp\left(-j2\pi k \frac{t}{N_c}\right) \\ \hat{H}_u(k) = \displaystyle\sum_{m=0}^{N_r-1} w_{pre-m} H_{u,m}(k) \\ N_m(k) = \displaystyle\sum_{t=0}^{N_c-1} n_m(t) \exp\left(-j2\pi k \frac{t}{N_c}\right) \\ \hat{N}(k) = \displaystyle\sum_{m=0}^{N_r-1} w_{pre-m} N_m(k) \end{cases}.$$

The first item is the desired signal component, the second is the MUI and the third one is the noise component. In the next, $Y(k)$ will be fed into the frequency domain equalizer and frequency domain equalization (FDE) will be performed so that

$$\tilde{Y}(k) = w_{FDE}(k) \cdot \hat{Y}(k). \tag{10}$$

The FDE weight $w_{FDE}(k)$ can be calculated following the minimum mean square error (MMSE) rule [4] given by

$$w_{FDE}(k) = \frac{\hat{H}_0^*(k)}{\left|\hat{H}_0(k)\right|^2 + \left[E_0/P_0\right]^{-1}}, \tag{11}$$

where P_0 is the power of interference plus noise; the superscript * represents the complex conjugate operation. The next step after the FDE will be the inverse transform of the equalized frequency domain signal to the time domain by IFFT transform for data decision as

$$\tilde{d}(t) = \frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{Y}(k) \exp\left(-j2\pi k \frac{t}{N_c}\right). \tag{12}$$

B. FD-AAA algorithm (post-FFT algorithm) [2]

The frequency domain representation of (2) is given by

$$R_m(k) = H_{0,m}(k) S_0(k) + \sum_{u=1}^{U-1} H_{u,m}(k) S_u(k) + N_m(k). \tag{13}$$

The received signal vector $\mathbf{R}(k)$ is then expressed as

$$\mathbf{R}(k) = \mathbf{H}_0(k) S_0(k) + \sum_{u=1}^{U-1} \mathbf{H}_u(k) S_u(k) + \mathbf{N}(k), \tag{14}$$

where $\mathbf{H}_u(k) = \left[H_{u,0}(k) \quad H_{u,1}(k) \quad \cdots \quad H_{u,N_r-1}(k)\right]^T$ and $\mathbf{N}(k) = \left[N_0(k) \quad N_1(k) \quad \cdots \quad N_{N_r-1}(k)\right]^T$.

The transmission system model with the post-FFT algorithm is shown in Fig. 2.

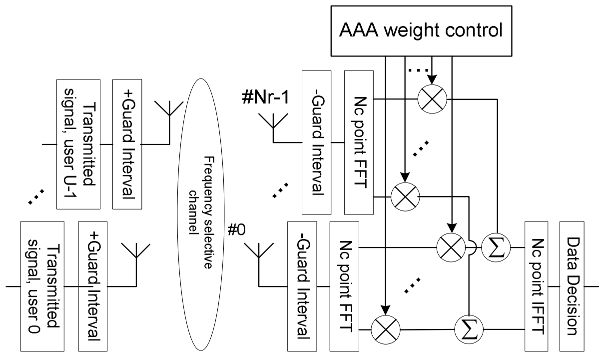

Fig. 4 The FD-AAA single-carrier uplink transmission.

AAA is performed on each frequency as

$$\tilde{R}(k) = \mathbf{W}_{FD-AAA}^T(k) \mathbf{R}(k), \tag{15}$$

where $\mathbf{W}_{FD-AAA}(k) = \left[W_{FD-AAA,0}(k), \cdots, W_{FD-AAA,N_r-1}(k)\right]^T$ and $\mathbf{R}(k) = \left[R_0(k), \cdots, R_{N_r-1}(k)\right]^T$. The AAA weight that minimizes the mean squared error (MSE) between $\tilde{R}(f)$

and the transmitted signal $S_0(k)$ is the Wiener solution which is given by [8]

$$\mathbf{W}_{FD-AAA}(k)=\mathbf{C}_{rr}^{-1}(k)\mathbf{C}_{rd}(k), \tag{16}$$

where

$$\begin{aligned}\mathbf{C}_{rr}(k)&=E\left\{\mathbf{R}^*(k)\mathbf{R}(k)\right\}\\&=\mathbf{A}_0^*(k)\mathbf{A}_0(k)+\sum_{u=1}^{U-1}\mathbf{A}_u^*(k)\mathbf{A}_u(k)+N_0\mathbf{I}\end{aligned}, \tag{17}$$

and

$$\mathbf{C}_{rd}(k)=E\left\{\mathbf{R}^*(k)S_0(k)\right\}=\mathbf{A}_0^*(k). \tag{18}$$

In (17), $\mathbf{A}_u(k)$ represents the signal propagation vector [6] of user u, N_0 represents the noise power and $\mathbf{I}$ is an $N_r \times N_r$ standard matrix.

After performing AAA, the time domain signal block estimate is obtained by N_c-point IFFT as

$$\hat{d}(t)=\frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{R}(k)\exp\left(-j2\pi k\frac{t}{N_c}\right) \tag{19}$$

for data decision.

4. Simulation Result

In this section, the performance of the proposed post-FFT type FD-AAA algorithm is compared with that of the pre-FFT type algorithm by simulations. The parameters used in the simulation are listed in Tab. 1. The number of users is the same as the number of receive antennas at the BS. Four users are assumed and their mean AOAs are set as 30°, 140°, 220° and 270° with the desired user's signal coming from a mean AOA of 30°. No spread, uniform and Gaussian DOA distributions are used and $\Delta\alpha_u$ for uniform distribution and Gaussian distribution varies between 4° and 8°. The AAA weight vector to initialize the AAA weight calculation is $[1,0,0,0]^T$ and the step size μ is 1/32 [3].

Table I Simulation Parameters

Modulation		QPSK
Number of antennas N_r		4
Number of users U		4
Mean AOAs of users		30°(desired user), 140°, 220°, 270°
DOA distribution		No DOA spread
		Uniform $\Delta\alpha_u=4°$, 8°
		Gaussian $\Delta\alpha_u=4°$, 8°
Channel	Channel model	Frequency selective block Rayleigh fading
	Number of paths L	16
	Power delay profile	Uniform
Signal to noise ratio (SNR)		0dB~20dB
N_c		256
Step size μ		1/32

First, no spread DOA distribution is used. The comparison between the post-FFT type algorithm and the pre-FFT type algorithm is shown in Fig. 6. It can be observed that when no DOA spread exists, the post-FFT type algorithm and the pre-FFT type algorithm almost achieve the same performance. The slight performance difference between them is due to the difference between the AAA weight vector achieved in (7) and the optimal AAA weight vector.

Next, the effects of the uniform DOA distribution on the performance of the post-FFT type and pre-FFT type algorithms are shown in Fig. 7. $\Delta\alpha_u=4°$ and 8° are used. It can be observed that the performances of the post-FFT type algorithm with $\Delta\alpha_u=4°$ or 8° are exactly the same as the one with no DOA spread. The reason is that the DOA spread in time domain will be mapped into the frequency domain by the FFT transform, and no DOA spread exists on each frequency. Therefore, whether there exists the DOA spread, or to what extent the DOA spreads will not affect the performance. However, the performance of the pre-FFT algorithm degrades significantly and error floor occurs even for the small $\Delta\alpha_u=4°$. And the performance degradation becomes more significant when $\Delta\alpha_u$ increases. It is pointed out in [3] that the pre-FFT type algorithm is proposed with the assumption that there is no DOA spread. The reason that the DOA spread degrades the performance is due to the interference cancelling ability of the antenna array. An antenna array with N_r element can generate one beam toward the desired the signal and N_r-1 nulls toward the interfering signals by the weight vector. In other words, the antenna array can only cancel the interfering signals coming from N_r-1 directions. When DOA spreads, on one hand, only part of the desired signal can be used and the remaining part becomes interference; on the other hand, only part of the interfering signals can be canceled. The performance degrades as the result of the residual interference, as shown in Fig. 5 taking a two-element antenna array as an example.

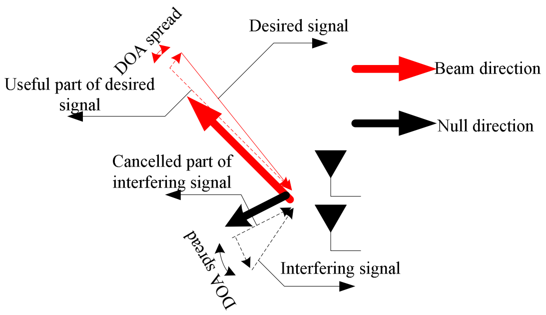

Fig. 5 The residue interference due to the DOA spread.

Next, we consider the Gaussian distributed DOA. The performance comparison of the post-FFT type algorithm and the pre-FFT algorithms is shown in Fig.8. The performance degradation can also be observed for pre-FFT type algorithm when the DOA spread is Gaussian distributed and also the error floor occurs. In addition, it can be observed that with the same $\Delta\alpha_u$, the performance degradation due to the Gaussian distributed DOA spread is less significant than that due to the uniform distributed DOA spread. This is because that the residue interference caused by the Gaussian distributed DOA spread is less than that of the uniform distributed DOA spread.

The results shown in Figs. 6-8 tell us that the pre-FFT type algorithm cannot be used even when the DOA spreads slightly. However, the post-FFT type algorithm, which is proposed by the authors in [2], can always be used and the distribution of DOA spread will not affect its performance.

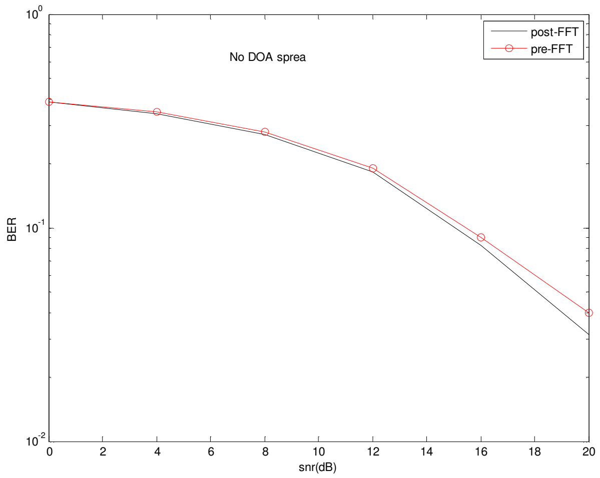

Fig. 6 Performance comparison with no spread DOA distribution.

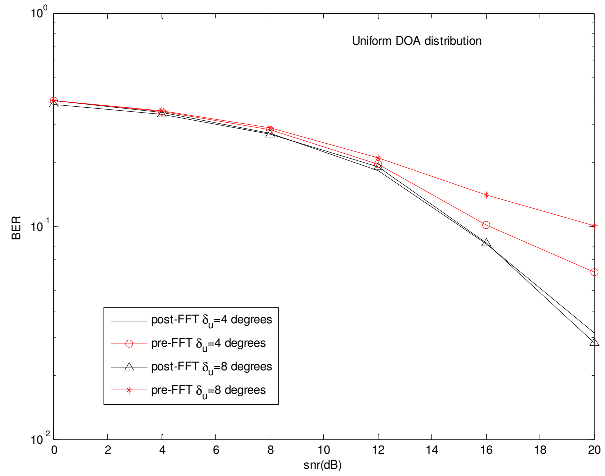

Fig. 7 Performance comparison with uniform distributed DOA.

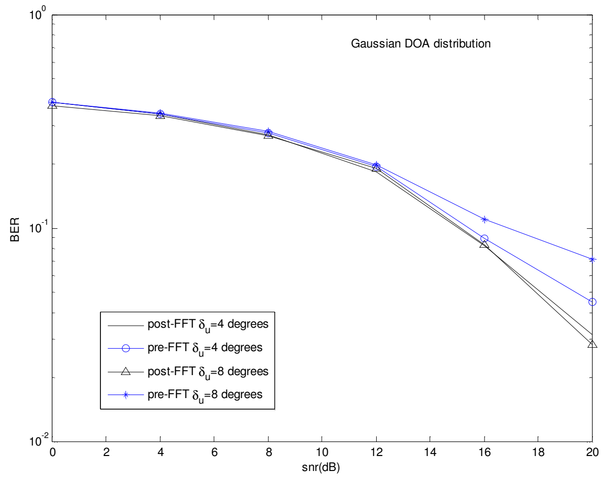

Fig. 8 Performance comparison with Gaussian distributed DOA.

5. Conclusion

In this paper, we compare the post-FFT type algorithm and pre-FFT type algorithm which are used to perform the interference cancellation in the uplink single-carrier transmission under a frequency selective fading channel. To study the effect of DOA spread on the two algorithms, three DOA distribution models have been used. It has been shown that the performance of the post-FFT type algorithm is not affected by the DOA spread while the performance of the pre-FFT type algorithm degrades significantly when DOA spread exists. The degradation caused by the Gaussian distributed DOA spread is less significant than that caused by the uniform distributed DOA spread when they have the same $\Delta\alpha_u$ and the degradation of both becomes more significant when $\Delta\alpha_u$ increases.

References

[1] J. G. Proakis, Digital Communications, fourth edition, New York: McGraw Hill, 2001.

[2] W. Peng and F. Adachi, "Frequency Domain Adaptive Antenna Array Algorithm for Single-carrier Uplink Transmission," IEEE conference PIMRC 2009, pp. 1-5, Sept. 2009.

[3] B. W. Kang, K. Takeda and F. Adachi, "Performance of Single-Carrier Frequency-Domain Adaptive Antenna Array," IEEE conference VTC Fall 2007, pp. 491-495, Sept. 2007.

[4] F. Adachi, H. Tomeba and K. Takeda, "Frequency-Domain Equalization for Broadband Single-Carrier Multiple Access," IEICE Transactions on Communications, col. E92-B, No. 5, pp. 1441-1456, May 2009.

[5] J. H. Winters, "Signal Acquisition and Tracking with Adaptive Arrays in Wireless Systems," IEEE Conference VTC 1993, pp. 85-88, May 1993.