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Low-PAPR Transmit Filter based on Minimization of Instantaneous Transmit Power Variation for Single-Carrier Transmission

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Abstract A modification of transmit filtering in single-carrier (SC) transmission yields an improvement of system performance in aspects of either peak-to-average power ratio (PAPR) or error probability. In this paper, such a novel transmit filtering is introduced in order to reduce PAPR, while maintaining the error probability similar to conventional transmit filter. The filter is determined based on minimization of variance of instantaneous transmit power (VIP), while the excess-bandwidth transmission is also introduced in order to obtain the frequency diversity. The degree of excess-bandwidth is controlled by the roll-off factor. PAPR performance of the proposed filter is evaluated for various values of roll-off factor α, and is also compared to square-root raised cosine filter, which is claimed as conventional filter. Proposed filter shapes for each roll-off factor are also illustrated. Effect of roll-off factor on PAPR performance is also discussed.

Keyword Single-carrier (SC) transmission, transmit filter, peak-to-average power ratio (PAPR)

1. Introduction

High-speed and high-quality are the main requirements for the next generation mobile network [1]. However, frequency-selective fading produced by multipath propagation with time delays, severely degrades system performance in terms of error probability [2]. Orthogonal frequency division multiplexing (OFDM) is a good transmission technique that is robust against frequency-selective fading, but its high peak-to-average power ratio (PAPR) property is the main drawback [3]. On the other hand, single-carrier (SC) transmission [4] has lower PAPR, while the effect from frequency-selective fading channel can be mitigated by using frequency-domain equalization (FDE) [5].

Transmit filter is typically used in SC transmission for limiting the transmission bandwidth. Square-root raised cosine filter [6] is one of generally-used transmit filters. However, system performance also changes when filter roll-off factor changes since the roll-off factor controls the excess bandwidth. In aspect of PAPR, a certain value of filter roll-off factor gives very low PAPR [7]. In aspect of bit error rate (BER), the excess bandwidth can inherit additional frequency diversity gain as long as the original spectrum can be recovered, hence BER is improved [8]. The increasing of transmission bandwidth by changing filter roll-off factor also decreases spectrum efficiency. Besides filter roll-off factor, any modification on transmit filter such as filter shape also affect system performance. In this paper, PAPR reduction is our main objective rather than BER improvement.

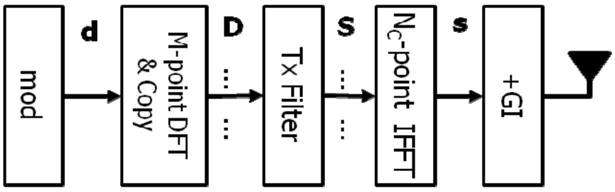
There exist literatures which proposed PAPR reduction algorithms for both OFDM and SC schemes. Some literatures proposed new filter shapes by either improving conventional filters or deriving from Nyquist prototype such as [9-12]. Slimane [13] has shown PAPR performance of various conventional filters for OFDM transmission. Nevertheless, most of proposed filter cannot guarantee the lowest PAPR since they are not optimum, and this also indicates possibility to determine a filter which provide lower PAPR than existing filters. On the other hand, Falconer [14] suggested that the variance of instantaneous transmit power (VIP) relates to PAPR, and also proposed a low-PAPR precoder based on minimization of VIP for OFDM scheme. Even though our objective is to determine a filter instead of precoder, proposed algorithm in [14] is still usable with some modification. In addition, the filter roll-off factor is still not applied in [14].

In this paper, we determine a new transmit filter for SC transmission by minimizing VIP. PAPR performance of proposed filter better compares to square-root raised cosine filter. Roll-off factor is also applied to achieve excess-bandwidth transmission. On the receiving side, an FDE based on the minimum mean-square error criterion (MMSE-FDE) is applied to combat inter-symbol interference (ISI). Spectrum combining [15] is also introduced to recover the original spectrum and then obtain additional frequency diversity gain.

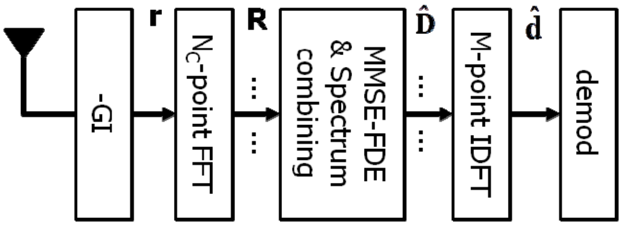
This paper is organized as follows. The SC transmission system model is introduced in Section 2. Low-PAPR filtering based on minimization of VIP is presented in Section 3. Section 4 shows the simulation results, and Section 5 concludes the paper.

2. Transmission System Model

Figure 1 illustrates SC transmission system model considered in this paper, while transmission is indicated as block transmission of M symbols over available N_c subcarriers. In this system, transmit filter is modified in order to reduce PAPR. On the other hand, we also employ MMSE-FDE and spectrum combining, as indicated in [8], for obtaining additional frequency diversity gain. In addition, transmission is conducted over frequency-selective fading channel, and hence, guard interval is required.



(a) Transmitter



(b) Receiver

Fig.1 SC transmission system model.

2.1. Transmitter

First, we have a block of M modulated symbol $\mathbf{d}$, where $\mathbf{d} = [d(0), d(1), ..., d(M-1)]^T$. The block $\mathbf{d}$ is transformed to frequency domain and the copied to entire N_c-point (N_c = 2M). Prior to this, a matrix $\mathbf{E}_M$ is introduced for the operation, which is a modification of discrete Fourier transform (DFT) matrix by repeating vertically, that is

$$\mathbf{E}_M = \begin{bmatrix} \mathbf{F}_M \\ \mathbf{F}_M \end{bmatrix}, \tag{1}$$

where $\mathbf{F}_M$ is M-point DFT matrix, which is

$$\mathbf{F}_M = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{\frac{-j2\pi(1)(1)}{M}} & \cdots & e^{\frac{-j2\pi(1)(M-1)}{M}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{\frac{-j2\pi(M-1)(1)}{M}} & \cdots & e^{\frac{-j2\pi(M-1)(M-1)}{M}} \end{bmatrix}. \tag{2}$$

In addition, we also determine frequency-domain signal vector $\mathbf{D} = \mathbf{E}_M\mathbf{d}$.

Transmit filter is generally used for limiting the signal bandwidth. Square-root raised cosine filter is an example of Nyquist filter, which can be referred as conventional filter. In this paper, transmit filter is introduced by a matrix $\mathbf{H}_T$. $\mathbf{H}_T$ is $N_c \times N_c$ diagonal matrix which the first J elements of diagonal contains filter coefficients. In addition, $J = (1+\alpha)M$ while α is roll-off factor, and $\mathbf{H}_T$ can be illustrated as

$$\mathbf{H}_T = \begin{bmatrix} H_T(\frac{-J}{2}) & & & \mathbf{0} \\ & \ddots & & \\ & & H_T(\frac{J}{2}-1) & \\ \mathbf{0} & & & \mathbf{0} \end{bmatrix}. \tag{3}$$

In case of square-root raised cosine filter [6], a set of filter coefficients $\{H_T(-J/2), ..., H_T(J/2-1)\}$ is

$$H_T(k) = \begin{cases} 1, & 0 \le |k| < \frac{1-\alpha}{2}M \\ \cos[\frac{\pi}{2\alpha M}(|k| - \frac{1-\alpha}{2}M)], & \frac{1-\alpha}{2}M \le |k| < \frac{1+\alpha}{2}M \\ 0, & otherwise \end{cases} \tag{4}$$

Instead of conventional filter, we determine a new set of filter coefficients $\{H_T(-J/2), ..., H_T(J/2-1)\}$ for (3) which gives lower PAPR. The method of determination will be discussed later.

After that, N_c-point inverse DFT (IFDT) matrix $\mathbf{F}_{N_c}^H$ is applied for transforming the filtered signal back to time domain. Before adding guard interval, transmit time-domain signal $\mathbf{s} = [s(0), ..., s(N_c - 1)]^T$ after passing through all processes in (1) and (3) can be described as in (5), and the transmission algorithm is illustrated as shown in Fig.2.

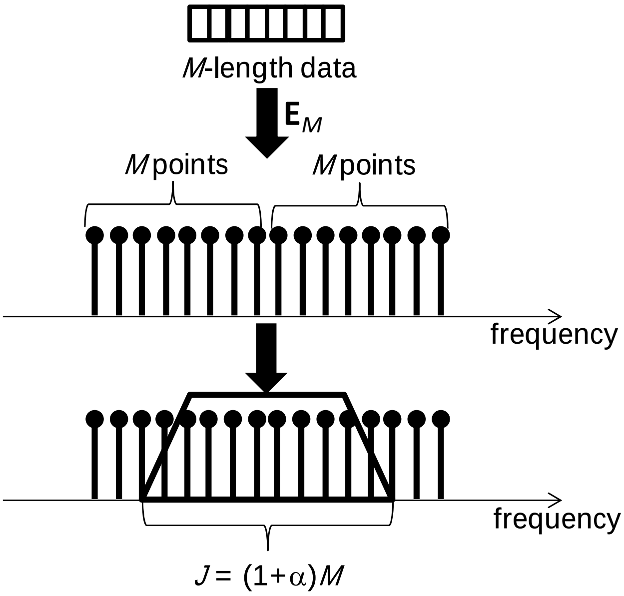


Fig.2 Transmit filtering with excess bandwidth.

$$\mathbf{s} = \mathbf{F}_{N_c}^H \mathbf{H}_T \mathbf{D} = \mathbf{F}_{N_c}^H \mathbf{H}_T \mathbf{E}_M \mathbf{d}. \tag{5}$$

2.2. Receiver

The transmission is conducted under independent L-path block Rayleigh fading channel [2]. Regarding to this, the channel impulse response can be expressed as

$$h(\tau)=\sum_{l=0}^{L-1}h_l\delta(\tau-\tau_l)\,. \tag{6}$$

where h_l and τ_l are complex-valued path gain and time delay of lth path, respectively. The received signal vector after removing guard interval, $\mathbf{r}=[r(0),...,r(N_c-1)]^T$ can also be expressed as

$$\mathbf{r}=\sqrt{\frac{2E_s}{T_s}}\mathbf{hs}+\mathbf{n}\,, \tag{7}$$

where E_s and T_s represent symbol energy and symbol duration, respectively. Transmit signal vector $\mathbf{s}$ is obtained from (5). A vector $\mathbf{n}$ represents zero-mean Gaussian noise. In addition, channel impulse response matrix $\mathbf{h}$ is a circular matrix which is expressed as

$$\mathbf{h}=\begin{bmatrix} h_0 & & & h_{L-1} & \cdots & h_1 \\ h_1 & \ddots & & & \ddots & \vdots \\ \vdots & & h_0 & \mathbf{0} & & h_{L-1} \\ h_{L-1} & & h_1 & \ddots & & \\ & \ddots & \vdots & & \ddots & \\ \mathbf{0} & & h_{L-1} & \cdots & \cdots & h_0 \end{bmatrix}. \tag{8}$$

Next, received signal $\mathbf{r}$ is transformed into frequency domain by using N_c-point DFT matrix $\mathbf{F}_{N_c}$, obtaining frequency-domain received signal $\mathbf{R}$ as

$$\begin{aligned}\mathbf{R} &= \sqrt{\frac{2E_s}{T_s}}\mathbf{F}_{N_c}\mathbf{hs}+\mathbf{F}_{N_c}\mathbf{n} \\ &= \sqrt{\frac{2E_s}{T_s}}\mathbf{F}_{N_c}\mathbf{hF}_{N_c}^{H}\mathbf{H}_T\mathbf{D}+\mathbf{F}_{N_c}\mathbf{n}\,. \\ &= \sqrt{\frac{2E_s}{T_s}}\mathbf{H}_C\mathbf{H}_T\mathbf{D}+\mathbf{N}_C\end{aligned} \tag{9}$$

Here, we also define $\mathbf{H}_C=\mathbf{F}_{N_c}\mathbf{hF}_{N_c}^{H}$ as a diagonal matrix determining frequency-domain channel response with respect to each frequency index.

In this paper, we also employ MMSE-FDE with spectrum combining as same as in [8] and [15]. Hence, the frequency-domain desired signal at the receiver can be expressed as $\hat{\mathbf{D}}=\mathbf{WR}$. Time-domain desired signal vector $\hat{\mathbf{d}}$ is obtained after transforming $\hat{\mathbf{D}}$ into time domain by using M-point IDFT matrix $\mathbf{F}_M^H$. Here, the operation matrix $\mathbf{W}$ for MMSE-FDE and spectrum combining is determined as in the following equations. We also define $\hat{\mathbf{H}}=\mathbf{H}_C\mathbf{H}_T$.

$$\mathbf{W}=\begin{bmatrix} W(0) & \mathbf{0} \\ & \ddots \\ \mathbf{0} & W\left(\frac{N_c}{2}-1\right) \\ W\left(\frac{N_c}{2}\right) & \mathbf{0} \\ & \ddots \\ \mathbf{0} & W(N_c-1) \end{bmatrix}, \tag{10}$$

$$W(k)=\frac{\hat{H}^*(k)}{\sum_{g=0}^{1}\left|\hat{H}(k\bmod M+gM)\right|^2+(E_s/N_0)^{-1}}\,. \tag{11}$$

3. Low-PAPR Filtering

As previously mentioned, [14] suggested that VIP implies PAPR characteristic and also introduced a method to find an optimum precoding matrix which minimizes VIP for OFDM scheme. In this paper, [14] is referenced as a guideline to determine an appropriate set of filter coefficients which contributes low PAPR for SC transmission. Filter roll-off factor is also approachable. Low-PAPR filter determination can be done by firstly determining VIP and starting precoder, then optimizing the precoder, and finally obtaining the set of filter coefficients from the optimum precoding matrix.

3.1 VIP and Precoding Matrix Determination

We consider an SC block transmission (same as indicated in Section 2) of M-length block of QPSK modulated symbols. For simplicity, we also define a matrix $\mathbf{X}$ as an overall transmit operation matrix, which means $\mathbf{X}=\mathbf{F}_{N_c}^{H}\mathbf{H}_T\mathbf{E}_M$. Then transmit signal vector $\mathbf{s}$ is rewritten as $\mathbf{s}=\mathbf{Xd}$, and each element in transmission block $\mathbf{s}=[s(0),..., s(N_c-1)]^T$ is described as

$$s(n)=\sum_{m=0}^{M-1}x_{nm}d(m)\,, \tag{12}$$

where x_{nm} is an element in $\mathbf{X}$ at the nth row and mth column. Therefore, instantaneous transmit power is $|s(n)|^2$, and VIP σ^2 can be expressed by averaging over a block of N_c samples as follow

$$\begin{aligned}\sigma^2 &= \frac{1}{N_c}\sum_{n=0}^{N_c-1}E\left[\left||s(n)|^2-E[|s(n)|^2]\right|^2\right] \\ &= \frac{1}{N_c}\sum_{n=0}^{N_c-1}E\left[|s(n)|^4\right]-P_{avg}^2\end{aligned} \tag{13}$$

In (13), P_{avg} represents average transmit power of $\mathbf{s}$. Substitute (12) into (13) yields the definition of VIP as follow.

$$\sigma^2=\frac{1}{N_c}\left(\sum_{n=0}^{N_c-1}\left(2\left[\sum_{m=0}^{M-1}|x_{nm}|^2\right]^2-\left[\sum_{m=0}^{M-1}|x_{nm}|^4\right]\right)-P_{avg}^2\right) \tag{14}$$

VIP in (14) is similar to one determined from OFDM transmission since they both are derived from overall transmit operation matrix $\mathbf{X}$. However, overall transmit operation matrix in this paper and in [14] are different due to different transmission scheme. This implies filtering algorithm needs to be modified in order to be compatible with SC transmission.

On the other hand, a precoding matrix is also defined. Even though the main objective of this paper is to obtain filter coefficients, we determine a precoder in order to make the optimization simpler. A precoding matrix $\mathbf{P}$, whose dimension is $N_c \times M$, is defined as $\mathbf{P} = \mathbf{H}_T \mathbf{E}_M$. After the optimization, optimum filter coefficients matrix $\mathbf{H}_{T,opt}$ is obtained from $\mathbf{H}_{T,opt} = \mathbf{P}_{opt} \mathbf{E}_M^{+}$, where $\mathbf{A}^{+}$ represents Moore-Penrose pseudoinverse operation of matrix $\mathbf{A}$. Note that the square magnitude of each column vector of precoding matrix, $|\mathbf{p}_m|^2$, is one due to power constraint.

3.2 Minimization of VIP

Regarding to the precoding matrix **P** and VIP, as indicated in (14), the objective function of constrained minimization is expressed as

$$\arg\min_{\mathbf{P}} \sigma^2 \quad s.t. |\mathbf{p}_m|^2 = 1. \tag{15}$$

The objective function in (14) and (15) are non-convex. Alternatively, we minimize the objective function numerically by employing gradient search with respect to the real and imaginary parts of $\mathbf{p}_m$. The gradient of (14) is expressed as

$$\nabla_{\mathbf{p}_m} \sigma^2 = \frac{4}{N_c} \sum_{n=0}^{N_c-1} \left[2\sum_{q=0}^{M-1} |x_{nq}|^2 - |x_{nm}|^2 \right] x_{nm} \mathbf{e}_n, \tag{16}$$

where

$$\mathbf{e}_n = \frac{1}{\sqrt{N_c}} \left[1, \exp(-j2\pi \tfrac{(1)(n)}{N_c}), \ldots, \exp(-j2\pi \tfrac{(J-1)(n)}{N_c}) \right]^T \tag{17}$$

Gradient search is done iteratively for each column vector of precoding matrix $\mathbf{p}_m$, $m = 0, \ldots, M-1$. At the tth iteration, gradient search of each column vector is done by

$$\begin{aligned} \tilde{\mathbf{p}}_m[t+1] &= \mathbf{p}_m[t] - \gamma \nabla_{\mathbf{p}_m[t]} \sigma^2, \\ \mathbf{p}_m[t+1] &= \frac{\tilde{\mathbf{p}}_m[t+1]}{|\tilde{\mathbf{p}}_m[t+1]|} \end{aligned} \tag{18}$$

where γ means step-size parameter.

One important thing for iterative gradient search algorithm is finding the appropriate starting point. DFT is typically selected as a starting precoder, which refers to unmodified SC transmission (without excess-bandwidth transmission). In this paper, a starting precoder is constructed by $\mathbf{P} = \mathbf{H}_T \mathbf{E}_M$, where $\mathbf{H}_T$ square-root is raised cosine filter. Iterative minimization on square-root raised cosine filter requires less iterations compared to rectangular window.

4. Simulation Results

Simulation parameters are summarized follow. We assume block transmission consisting of M = 256 QPSK modulated symbols. The number of available fast Fourier transform (FFT) point is $N_c = 2M = 512$. The transmission is conducted under 16-path block Rayleigh fading. Perfect channel estimation and zero timing offset are also assumed in this simulation model. Guard interval length is assumed as cyclic prefix where N_g = 32. Such particular values of filter roll-off factor, i.e., α = 0, 0.25, 0.5, 0.75, and 1 are evaluated for both square-root raised cosine filter and proposed filter. As mentioned in the previous parts, we expect lower PAPR compared to conventional filters, while the increasing of filter roll-off factor gives better error probability.

4.1 Filter Coefficients

Gradient search algorithm mentioned in (16) and (18) is done iteratively at the transmitting side, where the starting precoder is determined from square-root raised cosine filter. The sets of optimal transmit filter coefficients $\{H_T(-J/2), \ldots, H_T(J/2-1)\}$ for different filter roll-off factors are discovered after 100 iterations, while step-size parameter γ is set to be 1. Transmit filters for each roll-off factors are shown in Fig.3. Note that the number of iterations and step-size parameter are chosen by trial and error. One thing that should be mentioned is the optimal filter coefficients are changed if the modulation scheme is changed. This is because derived VIP for each modulation scheme is different (see [14]).

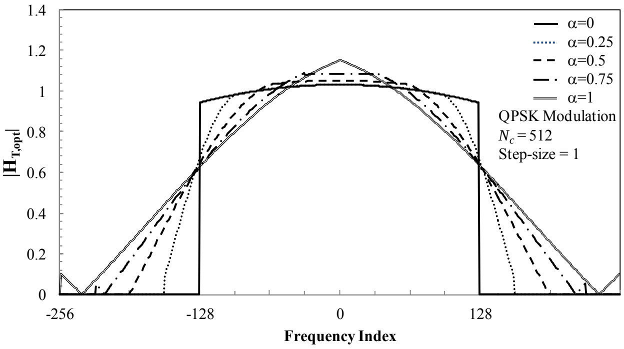


Fig.3 Proposed filter coefficients.

4.2 PAPR

PAPR over a block of transmission is defined as

$$PAPR = \frac{\max\{|s(n)|^2\}}{E[|s(n)|^2]}, n = 0, \ldots, N_c - 1. \tag{19}$$

We use the complementary cumulative distribution function (CCDF) as the indicator of PAPR performance. Figure 4 shows the CCDF of PAPR of both conventional filter (i.e., square-root raised cosine filter) and proposed filter, with particular values of filter roll-off factor. As previously mentioned the objective function, which is (14), is non-convex and hence provides many points of local minima. Even though gradient search cannot guarantee global minimum, we can still reduce the PAPR from conventional filter.

At a place where probability of occurrence equals 0.1%, called $\mathrm{PAPR}_{0.1\%}$, approximately 0.3dB reduction is obtained when α=0, while about 1.3dB reduction is possible when α=0.75. Note that expanding the filter bandwidth until such a particular value can reduce PAPR, as seen that the lowest PAPR can be achieved when α is

0.5 for conventional filter, and 0.75 for proposed filter, respectively.

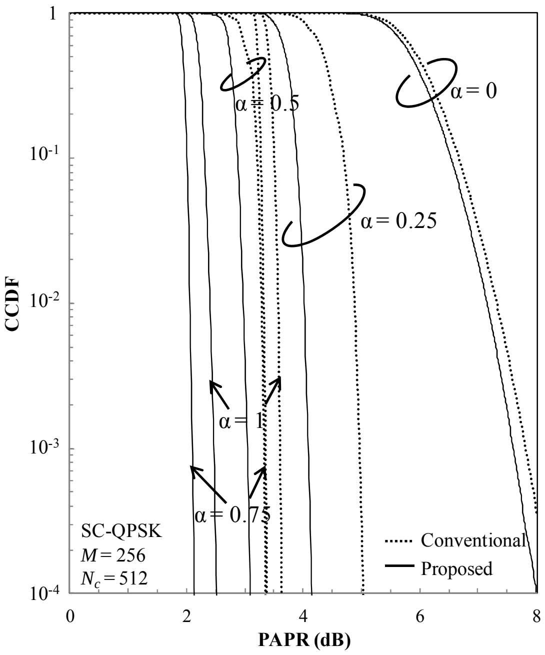


Fig.4 Cumulative distribution function (CCDF) of PAPR.

4.3 BER Performance

BER performance of both conventional filter and proposed filter with various values of filter roll-off factor α is shown in Fig.5 as a function of average received bit energy-to-noise power spectrum density ratio $E_b / N_0 = (E_s / N_0)(1 + N_g / N_c)/2$. With the aid of MMSE-FDE and spectrum combining, we can achieve better BER performance when roll-off factor increases. Using MMSE-FDE with spectrum combining gives additional frequency diversity gain, which is inherited from excess-bandwidth transmission. It is also noticed that BER performance of proposed filter is similar to one from square-root raised cosine filter. This means using the proposed transmit filter can reduce PAPR of transmitted signal while BER performance is similar to one from conventional filter.

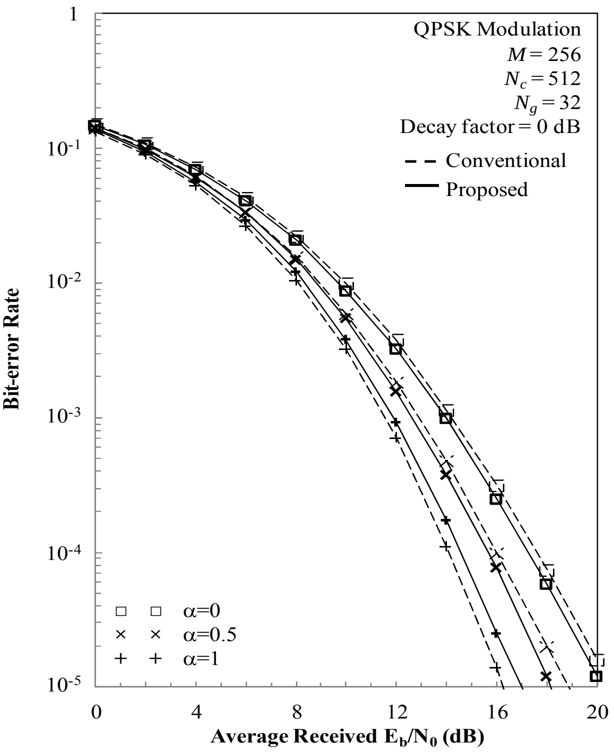


Fig.5 BER performance of conventional and proposed filter.

5. Conclusion

In this paper, a novel low-PAPR filtering technique based on the minimization of VIP at the transmitting side for SC transmission has been examined. Excess-bandwidth transmission is also achievable by introducing the filter roll-off factor to the proposed filter. A combination of proposed filter at the transmitter and MMSE-FDE with spectrum combining at the receiver provides lower PAPR, while additional frequency diversity gain is still obtained. Simulation results confirmed that proposed filter gives better PAPR performance. BER performance is also similar to one from the square-root raised cosine filter. It was shown that better BER performance is achieved by increasing the filter roll-off factor as well.

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