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# Excess-Bandwidth Joint Transmit/Receive Frequency-Domain Equalization for Single-Carrier Transmission

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**Abstract** A modification of transmit filtering in single-carrier (SC) transmission contributes to an improvement of system performance in terms of peak-to-average power ratio (PAPR) and bit error rate (BER). Using minimum mean-square error based frequency-domain equalization (MMSE-FDE) at the receiver can combat the problem of inter-symbol interference (ISI) arising from channel frequency-selectivity. In this paper, instead of conventional transmit and receive filters, we propose an excess-bandwidth joint transmit/receive FDE. Transmit FDE weight (Tx-FDE) is determined jointly with receive FDE (Rx-FDE) in a suboptimal approach. Excess-bandwidth transmit filtering also leads to the additional frequency diversity gain when MMSE-FDE with spectrum combining is used at the receiver. Performance evaluation of the proposed transmission scheme is shown in terms of PAPR, BER, and system throughput, with the variation of transmit roll-off factor.

**Keyword** Single-carrier (SC) transmission, transmit filter, frequency-domain equalization (FDE)

## 1. Introduction

Broadband wireless channel is characterized as a frequency-selective fading channel, in which the inter-symbol interference (ISI) degrades the system performance in terms of bit-error rate (BER) significantly [1]. Orthogonal frequency division multiplexing (OFDM), is robust against frequency-selective fading, but its high peak-to-average power ratio (PAPR) property is the main drawback [2]. On the other hand, single-carrier (SC) transmission [3] provides lower PAPR than OFDM, while using minimum mean-square error based frequency-domain equalization (MMSE-FDE) [4] can combat with ISI.

Although MMSE-FDE is able to improve the BER performance [5], the performance is still far from matched-filter bound. This is because the existence of residual ISI. Joint transmit/receive MMSE-FDE (joint Tx/Rx MMSE-FDE) [6], [7], whose FDE weights are jointly calculated based on MMSE criterion, is introduced as a good method to reduce residual ISI with lower computational complexity.

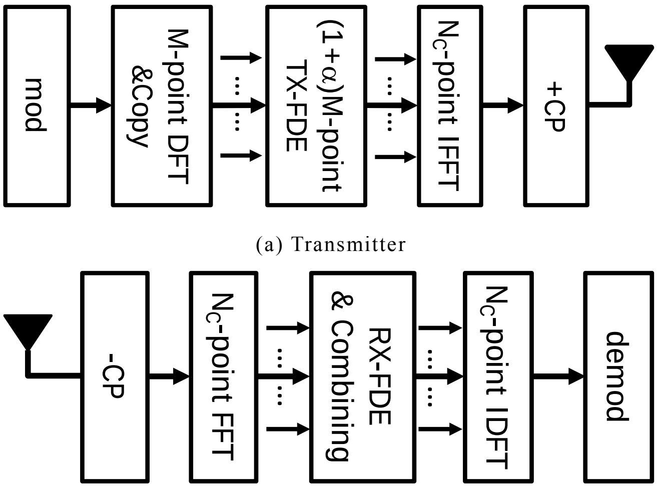
In SC transmission, transmit filtering is typically applied in order to limit the transmission bandwidth. The square-root Nyquist filter, such as square-root raised cosine filter [8], is one of generally-used transmit filter. It is known that when the filter roll-off factor changes, PAPR [9] and BER [10] also change. Excess-bandwidth transmission, controlled by filter roll-off factor, can inherit additional frequency diversity gain. In [11], a combination of excess-bandwidth transmission and spectrum combining is introduced in order to improve BER performance. However, increasing transmission bandwidth leads to the decreasing of spectrum efficiency. This implies that there exist trade-off among PAPR, BER, and spectrum efficiency. In this paper, improvement on BER performance is our main focus.

Hence, in this paper, we propose an excess-bandwidth joint Tx/Rx MMSE-FDE. Transmit FDE (Tx-FDE) weight is calculated jointly with receive FDE (Rx-FDE) weight in a suboptimal approach. The additional frequency diversity gain is also obtained from a combination of excess-bandwidth filtering and MMSE-FDE equipped with spectrum combining. As mentioned earlier, any modification to transmit filter alters PAPR performance, and therefore, it is also important to observe PAPR performance of proposed scheme.

This paper is organized as follows. The transmission system model is introduced in Sect. 2. A suboptimal set of Tx-FDE and Rx-FDE weights are derived in Sect. 3. Sect. 4 shows the simulation results, and Sect. 5 concludes the paper.

## 2. Transmission System Model

Figure 1 illustrates SC transmission system model considered in this paper, while transmission is indicated as block transmission of $M$ symbols over available $N_c$ discrete frequency points. Conventional transmit filter is replaced by Tx-FDE. Rx-FDE is also equipped at the receiving side. We also employ excess-bandwidth transmission and spectrum combining, as indicated in [11], for obtaining additional frequency diversity gain. In addition, transmission is conducted over frequency-selective fading channel, and hence, cyclic prefix (CP) insertion is required.



(a) Transmitter

(b) Receiver

**Fig.1** SC transmission system model using joint Tx/Rx MMSE-FDE.

## 2.1. Transmitter

First, we have a block of $M$ modulated symbol $\mathbf{d}$, where $\mathbf{d}=[d(0),d(1),...,d(M-1)]^T$. The block $\mathbf{d}$ is transformed to frequency domain and copied to entire $N_c$-point (we let $N_c$ = $2M$ for simplicity). Prior to this, a matrix $\mathbf{E}_M$ is introduced for the operation, which is a row-repeated discrete Fourier transform (DFT) matrix, that is

$$\mathbf{E}_M = \begin{bmatrix} \mathbf{F}_M \\ \mathbf{F}_M \end{bmatrix}, \tag{1}$$

where $\mathbf{F}_M$ is $M$-point DFT matrix, which is

$$\mathbf{F}_M = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{\frac{-j2\pi(1)(1)}{M}} & \cdots & e^{\frac{-j2\pi(1)(M-1)}{M}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{\frac{-j2\pi(M-1)(1)}{M}} & \cdots & e^{\frac{-j2\pi(M-1)(M-1)}{M}} \end{bmatrix}. \tag{2}$$

In addition, we also determine frequency-domain signal vector $\mathbf{D} = \mathbf{E}_M\mathbf{d}$.

Tx-FDE is introduced by a matrix $\mathbf{W}_t$. $\mathbf{W}_t$ is $N_c \times N_c$ diagonal matrix which the first $J$ elements of diagonal contains Tx-FDE weight. The process of calculating Tx-FDE weight will be described in the next section. In addition, $J = (1+\alpha)M$ while α is roll-off factor, and $\mathbf{W}_t$ can be illustrated as

$$\mathbf{W}_t = \begin{bmatrix} \mathbf{0} & & & \mathbf{0} \\ & W_t(\frac{N_c-J}{2}) & & \\ & & \ddots & \\ & & & W_t(\frac{N_c+J}{2}-1) \\ \mathbf{0} & & & \mathbf{0} \end{bmatrix}. \tag{3}$$

After that, $N_c$-point inverse DFT (IFDT) is applied for transforming the filtered signal back to time domain. Before CP insertion, transmit time-domain signal $\mathbf{s}=[s(0),s(1),...,s(N_c-1)]^T$ after passing through all processes in (1) and (3) can be described as in (4), and the transmission algorithm is illustrated as shown in Figure 2.

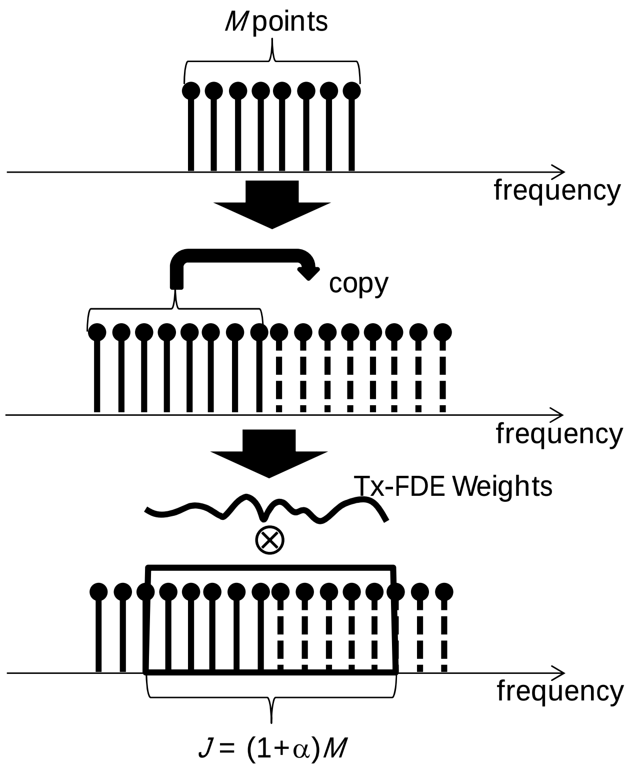


**Fig.2** Transmit algorithm using Tx-FDE with excess bandwidth.

$$\mathbf{s} = \mathbf{F}_{N_c}^H \mathbf{W}_t \mathbf{D} = \mathbf{F}_{N_c}^H \mathbf{W}_t \mathbf{E}_M \mathbf{d}\,. \tag{4}$$

## 2.2. Receiver

The transmission is conducted under independent $L$-path block Rayleigh fading channel [1]. In more details, the channel impulse response can be expressed as

$$h(\tau) = \sum_{l=0}^{L-1} h_l \delta(\tau - \tau_l)\,, \tag{5}$$

where $h_l$ and $\tau_l$ are complex-valued path gain and time delay of $l$-th path, respectively. The received signal vector after removing CP, $\mathbf{r}=[r(0),r(1),...,r(N_c-1)]^T$ can also be expressed as

$$\mathbf{r} = \sqrt{\frac{2E_s}{T_s}}\mathbf{hs} + \mathbf{n}\,, \tag{6}$$

where $E_s$ and $T_s$ represent symbol energy and symbol duration, respectively. Transmit signal vector $\mathbf{s}$ is obtained from (4). A vector $\mathbf{n}$ represents zero-mean Gaussian noise. In addition, a matrix $\mathbf{h}$ is a circular matrix representing time-domain channel impulse response, which is expressed as

$$\mathbf{h} = \begin{bmatrix} h_0 & & & h_{L-1} & \cdots & h_1 \\ h_1 & \ddots & & & \ddots & \vdots \\ \vdots & & h_0 & \mathbf{0} & & h_{L-1} \\ h_{L-1} & & h_1 & \ddots & & \\ & \ddots & \vdots & & \ddots & \\ \mathbf{0} & & h_{L-1} & \cdots & \cdots & h_0 \end{bmatrix}. \quad (7)$$

Next, received signal $\mathbf{r}$ is transformed into frequency domain by using $N_c$-point DFT, obtaining frequency-domain received signal $\mathbf{R}$ as

$$\begin{aligned} \mathbf{R} &= \sqrt{\frac{2E_s}{T_s}}\mathbf{F}_{N_c}\mathbf{hs} + \mathbf{F}_{N_c}\mathbf{n} \\ &= \sqrt{\frac{2E_s}{T_s}}\mathbf{F}_{N_c}\mathbf{hF}_{N_c}^H\mathbf{W}_t\mathbf{D} + \mathbf{F}_{N_c}\mathbf{n} \\ &= \sqrt{\frac{2E_s}{T_s}}\mathbf{HW}_t\mathbf{D} + \mathbf{N}_C \end{aligned} . \quad (8)$$

Here, we also define $\mathbf{H} = \mathbf{F}_{N_c}\mathbf{hF}_{N_c}^H$ as a diagonal matrix determining frequency-domain channel response with respect to each frequency index.

Rx-FDE is equipped. In this paper, Rx-FDE is determined based on MMSE criterion, together with Tx-FDE. Spectrum combining [11] is also included. Hence, the frequency-domain desired signal at the receiver can be expressed as $\hat{\mathbf{D}} = \mathbf{W}_r\mathbf{R}$. Time-domain desired signal vector $\hat{\mathbf{d}}$ is obtained after transforming $\hat{\mathbf{D}}$ into time domain by using $M$-point IDFT. Here, the operation matrix $\mathbf{W}_r$ for Rx-FDE and spectrum combining, with the dimension of $M \times N_c$, is determined as in the following equation.

$$\mathbf{W}_r = \begin{bmatrix} W_r(0) & & \mathbf{0} & W_r(\frac{N_c}{2}) & & \mathbf{0} \\ & \ddots & & & \ddots & \\ \mathbf{0} & & W_r(\frac{N_c}{2}-1) & \mathbf{0} & & W_r(N_c-1) \end{bmatrix}. \quad (9)$$

## 3. Suboptimal Transmit and Receive FDE Weights

In this section, Tx-FDE and Rx-FDE weights are derived jointly based on MMSE criterion. However, direct optimization is very difficult to achieve since it involves two parameters, which are $\mathbf{W}_t$ and $\mathbf{W}_r$. We instead optimize the mean-square error (MSE) in a suboptimal approach by determining $\mathbf{W}_r$ first, and then use the derived $\mathbf{W}_r$ to determine $\mathbf{W}_t$ [7].

### 3.1 Receive FDE Weight

To derive Rx-FDE, we consider Rx-FDE weight matrix $\mathbf{W}_r$ without any consideration on $\mathbf{W}_t$. Derivation of $\mathbf{W}_r$ begins with error vector between frequency-domain transmit signal and equivalent receive signal, represented by $\mathbf{e} = [e(0), \ldots, e(N_c - 1)]^T$, which is

$$\begin{aligned} \mathbf{e} &= \hat{\mathbf{D}} - \sqrt{\frac{2E_s}{T_s}}\mathbf{D} \\ &= \sqrt{\frac{2E_s}{T_s}}(\mathbf{W}_r\mathbf{HW}_t - \mathbf{I}_{N_c})\mathbf{D} + \mathbf{W}_r\mathbf{N} \end{aligned} . \quad (10)$$

Then, the MSE, denoted by $e$, is consequently expressed as $e = tr[E(\mathbf{ee}^H)]$, where $tr[\cdot]$ represents trace operation. To find $\mathbf{W}_r$ that minimize $e$, we set the derivative of $e$ equals zero, i.e., $\partial(e)/\partial\mathbf{W}_r = 0$, obtains

$$\mathbf{W}_r = \frac{(\mathbf{HW}_t)^H}{\mathbf{HW}_t(\mathbf{HW}_t)^H + \left(\frac{E_s}{N_0}\right)^{-1}\mathbf{I}_{N_c}} . \quad (11)$$

Note that the entire matrixes in (11) are diagonal. After doing some calculation and rearrange the terms in $\mathbf{W}_r$ in order to match with (9), Rx-FDE weight corresponding to each frequency index $W_r(k)$ can be concluded as

$$W_r(k) = \frac{H^*(k)W_t^*(k)}{\sum_{g=0}^{1}\left|H(k \bmod M + gM)W_t(k \bmod M + gM)\right|^2 + \left(\frac{E_s}{N_0}\right)^{-1}} . \quad (12)$$

We can observe that Rx-FDE weight in (12) is similar to one in [11], with the Rx-FDE weight matrix $\mathbf{W}_r$ also includes spectrum combining.

### 3.2 Transmit FDE Weight

After we derive Rx-FDE weight, as shown in section 3.1, we use it as information to determine Tx-FDE weight. $\mathbf{W}_r$ is substituted back into error vector function, as referred in (10). Together with the trace operation, MSE value $e$ is expressed as

$$e = \frac{1}{E_s/N_0} tr\left[E\left(\left\{\mathbf{HW}_t\mathbf{W}_t^H\mathbf{H}^H + \left(\frac{E_s}{N_0}\right)^{-1}\mathbf{I}_{N_c}\right\}^{-1}\right)\right] . \quad (13)$$

According to Fig.3, transmission is conducted over $J = (1+\alpha)M$ points instead of entire available DFT-point ($N_c$). Hence, Tx-FDE weight is calculated only for $J$ points, where the rest are left as zero. With this condition, and all matrixes in (13) are diagonal, (13) can be simplified as

$$e = \frac{1}{J}\sum_{k=0}^{J-1}\frac{\left(\frac{E_s}{N_0}\right)^{-1}}{|H(k)|^2|W_t(k)|^2 + \left(\frac{E_s}{N_0}\right)^{-1}} . \quad (14)$$

To obtain Tx-FDE weight which minimize MSE, objective function can be described as follow.

$$\arg\min_{\{W_t(k)\}} \quad e = \frac{1}{J}\sum_{k=0}^{J-1}\left(\frac{\left(\frac{E_s}{N_0}\right)^{-1}}{|H(k)|^2|W_t(k)|^2+\left(\frac{E_s}{N_0}\right)^{-1}}\right). \tag{15}$$

$$s.t. \quad tr[\mathbf{W}_t\mathbf{W}_t^H] = M$$

Note that the power constraint $tr[\mathbf{W}_t\mathbf{W}_t^H] = M$ should be included in the process of minimization of MSE. Optimization is done by using Lagargian method [12]. After doing some manipulations, Tx-FDE weight as a function of frequency index $k$ can be expressed as

$$W_t(k) = \max\left[\left\{\sqrt{\frac{(E_s/N_0)^{-1/2}}{\sqrt{\kappa}|H(k)|}-\frac{(E_s/N_0)^{-1}}{|H(k)|^2}}\right\}, 0\right]. \tag{16}$$

Here, κ is selected to satisfy the transmit power constraint, which is $tr[\mathbf{W}_t\mathbf{W}_t^H] = M$. We can see that Tx-FDE weight in (16) is similar to one in [7], but the power constraint, which actually affects κ, is different. Note that there exists frequency-domain channel response $H(k)$ in $W_t(k)$, which implies that channel state information should be presence in both transmitting side and receiving side.

To briefly conclude the proposed transmission using excess-bandwidth joint Tx/Rx MMSE-FDE, we firstly copy the frequency-domain transmit signal to entire DFT-point. Next, as a replacement of conventional filter, Tx-FDE weight is used for $J$ = (1+α)$M$ points. At the receiver, Rx-FDE and spectrum combining are applied after DFT. We expect that using excess-bandwidth joint Tx/Rx MMSE-FDE outperform conventional transmission without bandwidth expansion.

## 4. Simulation Results

Simulation parameters are summarized follow. We assume block transmission consisting of $M$ = 256 QPSK modulated symbols. The number of available fast Fourier transform (FFT) point is $N_c$ = 2$M$ = 512. The transmission is conducted under 16-path block Rayleigh fading with uniform power delay profile, where the delay-time difference between each path equals the symbol length. Perfect channel estimation and zero timing offset are also assumed in this simulation model. Guard interval length is assumed as cyclic prefix where $N_g$ = 32. Such particular values of filter roll-off factor, i.e., α = 0, 0.25, 0.5, 0.75, and 1 are evaluated for proposed joint Tx/Rx MMSE-FDE. System performance is evaluated in aspects of PAPR, BER, and system throughput. In this paper, we select the joint Tx/Rx MMSE-FDE without roll-off factor (α = 0) [7] and transmission using square-root raised cosine filter [11] as a conventional scheme [7].

### 4.1 PAPR Performance

PAPR over a block of transmission is defined as

$$PAPR = \frac{\max\left\{|s(n)|^2\right\}}{E[|s(n)|^2]}, n = 0,...,N_c-1. \tag{17}$$

We use the complementary cumulative distribution function (CCDF) as the indicator of PAPR performance. Figure 3 shows the CCDF of PAPR of proposed Tx-FDE with particular values of filter roll-off factor. It is seen that PAPR performance changes when roll-off factor changes. However, PAPR of transmission using Tx-FDE is better than one without roll-off factor. We can also observe that the best PAPR performance of proposed transmission scheme is achievable when the roll-off factor is 1, and proposed scheme also provides lower PAPR at every roll-off factor.

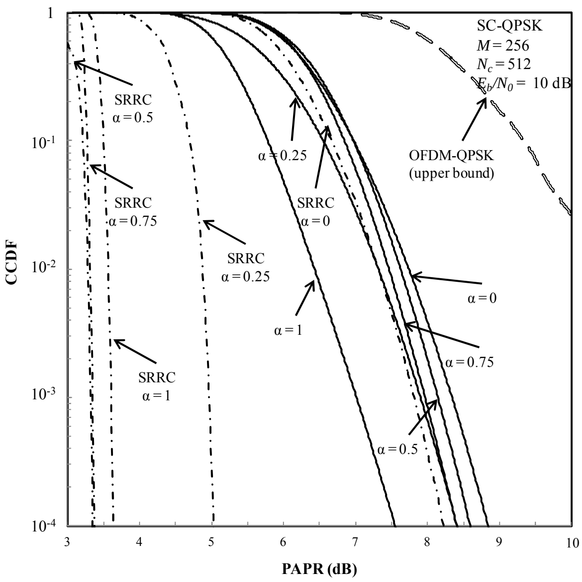



**Fig.3** CCDF of PAPR.

### 4.2 BER Performance 10

BER performances of Tx-FDE with and without roll-off factor are shown in Figure 4 as a function of average received bit energy-to-noise power spectrum density ratio $E_b/N_0 = (E_s/N_0)(1+N_g/N_c)/2$. It is obviously seen that the Tx-FDE with roll-off factor outperforms conventional Tx-FDE without roll-off factor. It is also noticed each roll-off factor provides different BER improvement. For example, a big improvement can be observed from the region that roll-off factor is more than 0.5, where the others provide only small improvement. This may be explained as the small amount of frequency diversity gain does not give a big merit since Tx-FDE itself is strong enough to deal with fading problem.

We also evaluate the BER performance based on energy efficiency by showing the BER performance of a function of peak $E_s/N_0$. Peak $E_s/N_0$ is simply defined as $E_s/N_{0,peak} = E_s/N_{0,average} + PAPR_{0.1\%}$ where we can see that PAPR is also included.

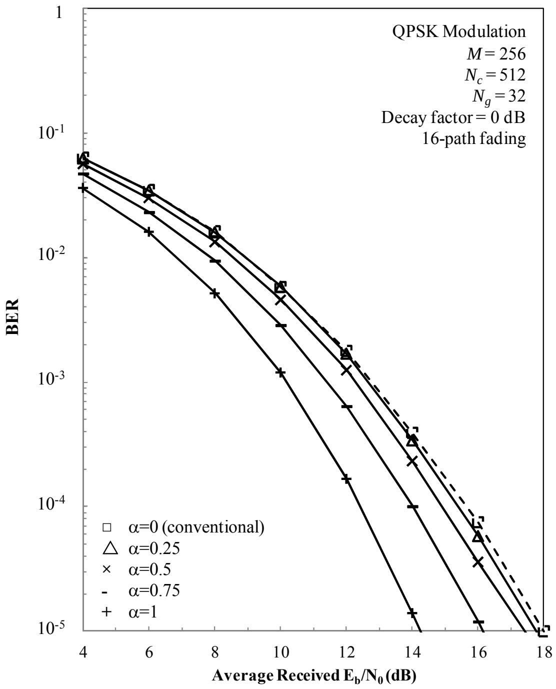



**Fig.4** BER performance of Tx-FDE with roll-off factor.

As shown in Fig.5, we can get the best BER performance when we use roll-off factor equals 1. However, we can also observe that the performance gap at alpha is 1 becomes larger. This is because PAPR performance at $\alpha = 1$ is very good. In addition, we can also observe that the performance of $\alpha = 0.25$ and $\alpha = 0.5$ are very close.

We also compare the BER performance of transmission using joint Tx/Rx MMSE-FDE with the one using square-root raised cosine filter [11]. Fig.6 and Fig.7 show the BER performance comparison in aspects of average received $E_b/N_0$ and peak $E_s/N_0$, respectively. In Fig.6, we can observe that transmission using Tx-FDE outperforms the transmission using Nyquist filter in every roll-off factor due to the lower residual ISI. However, when we consider the BER performance in aspects of energy efficiency, as shown as a function of peak $E_s/N_0$, transmission using Tx-FDE provides worse BER performance because of its high PAPR. In addition, using Tx-FDE without roll-off factor still provides the better BER performance, as shown in Fig.7.

### 4.3 Throughput Performance

The throughput $\eta$ (in bps/Hz) is expressed by [13]

$$\eta = N \times (1 - PER) \times \frac{1}{1+\alpha} \times \frac{N_c}{N_c + N_g}, \tag{18}$$

where $N$ represents number of bits per symbol and PER is packet error rate. A packet length is set to be 512 bits. Simulation result of throughput performance as a function of peak $E_s/N_0$. As shown in Fig.8, at the place where peak $E_s/N_0$ is low, excess-bandwidth Tx-FDE weight outperforms one without roll-off factor due to additional frequency diversity gain. However, at high peak $E_s/N_0$, increasing roll-off factor reduces maximum throughput by a factor of (1+α), and hence leads to decreasing of maximum spectrum efficiency.

## 5. Conclusion

In this paper, we proposed an excess-bandwidth joint Tx/Rx MMSE-FDE using spectrum combining to obtain an additional frequency diversity gain while mitigating the residual ISI. Simulation results confirmed that proposed scheme outperforms conventional Tx-FDE without roll-off factor in terms of both BER and PAPR performance. We also evaluated the energy efficiency by considering the BER performance as a function of peak $E_s/N_0$.

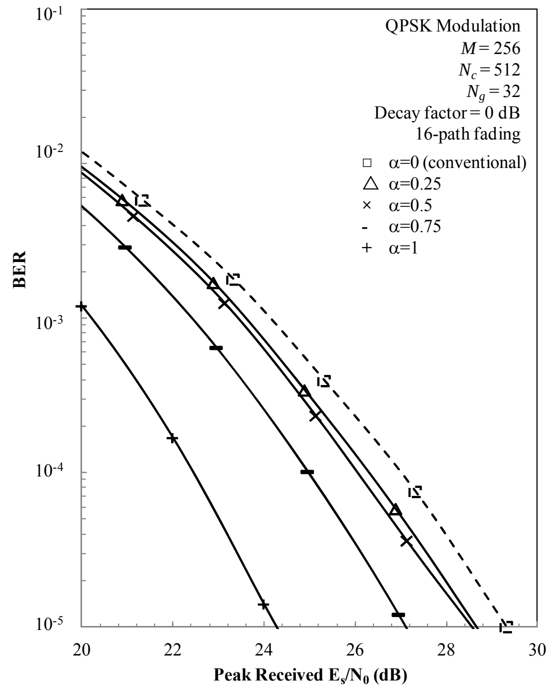



**Fig.5** BER performance Tx-FDE in aspects of energy efficiency as a function of peak $E_s/N_0$.

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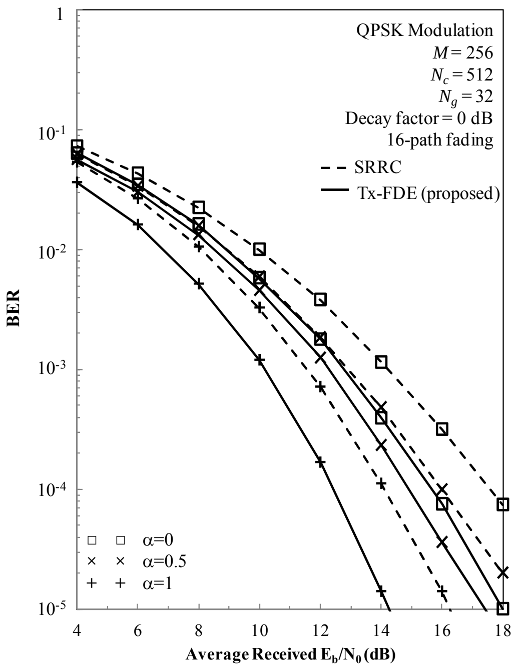


**Fig.6** BER performance comparison of Tx-FDE and square-root raised cosine filter.

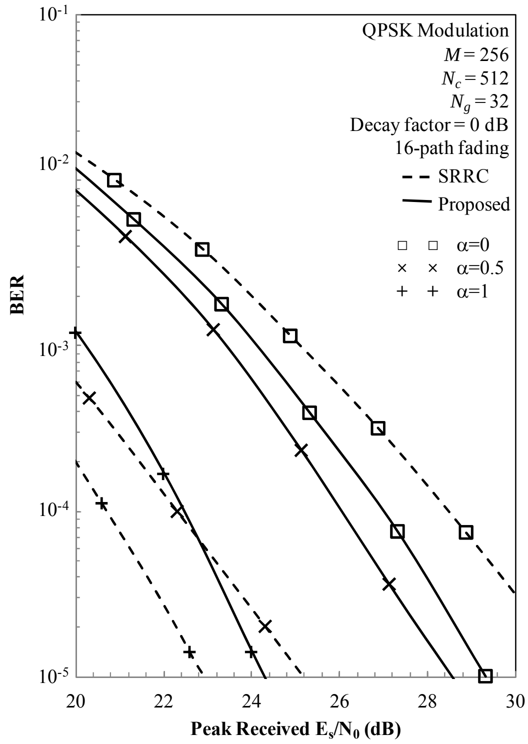


**Fig.7** BER performance comparison in aspects of energy efficiency as a function of peak $E_s/N_0$.

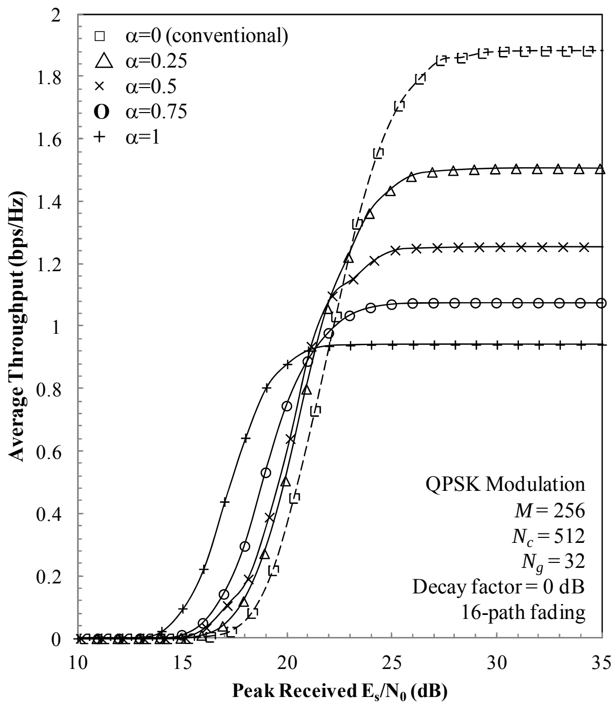


**Fig.8** Throughput performance Tx-FDE in aspects of energy efficiency as a function of peak Es/N0.