Least Mean Square Algorithm with Application to Improved Adaptive Sparse Channel Estimation

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Abstract Least mean square (LMS) based adaptive algorithms have been attracted much attention since their low computational complexity and robust recovery capability. To exploit the channel sparsity, LMS-based adaptive sparse channel estimation methods, e.g., L_1-norm LMS or zero-attracting LMS (sparse LMS or ZA-LMS), reweighted zero attracting LMS (RZA-LMS) and L_p-norm LMS (LP-LMS), have been proposed based on L_p-norm constraint $(0 \leq p \leq 1)$. However, the above methods cannot exploit channel sparse structure information fully. To further improve estimation performance, in this paper, we introduce a L_0-norm LMS (L0-LMS) algorithm with application sparse channel estimation to full take advantage of the sparsity. In addition, due to LMS-based channel estimation methods have a common drawback which is sensitive to the scaling of random training signal. Therefore, it is very hard to choose a proper learning rate to achieve robust estimation performance. To solve this problem, we propose several improved adaptive sparse channel estimation methods by using normalized LMS algorithm (NLMS), which normalizes the power of input signal, with different sparse penalties, e.g., L_p-norm and L_0-norm. Computer simulation results demonstrate the advantage of the proposed channel estimation methods in estimation performance.

Keyword Least Mean Square (LMS); Adaptive Sparse Channel Estimation; Normalized LMS (NLMS); Sparse Penalty; Compressive Sensing (CS).

1. Introduction

The demand for high-speed data services is getting more insatiable due to the number of wireless subscribers roaring increase. Various portable wireless devices, e.g., smart phones and laptops, have generated rising massive data traffic [1]. It is well known that the broadband transmission is an indispensable technique in the next generation communication systems [2-7]. However, the broadband signal is susceptible to interference by frequency-selective fading. In the sequel, the broadband channel is described by a sparse channel model in which multipath taps are widely separated in time, thereby create a large delay spread [7-12]. In other words, unknown channel impulse response (CIR) in broadband wireless communication system is often described by sparse channel model, supporting by a few large coefficients. That is to say, most of channel coefficients are zero or close to zero while only a few channel coefficients are dominant (large value) to support the channel. A typical example of sparse channel is shown in Fig. 1, where the number of dominant channel taps is 4 while the length of channel is 16.

Traditional least mean square (LMS) algorithm is one of the most popular methods for adaptive system identification [13], e.g. channel estimation. Indeed, LMS-based adaptive channel estimation can be easily implemented by LMS-based filter due to its low computational complexity or fast convergence speed. However, the LMS-based method never takes advantage of channel sparse structure as prior information and then it may loss some estimation performance.

Recently, many algorithms have been proposed to take advantage of sparse nature of the channel. For example, based on the theory of compressive sensing (CS) [14], [15], various sparse channel estimation methods have been proposed in [16-21]. For one thing, these CS-based sparse channel estimation methods require that the training signal matrices satisfy the restricted isometry property (RIP) [22]. However, design these kinds of training matrices is non-deterministic polynomial-time (NP) hard problem [23]. For another thing, some of these methods achieve robust estimation at the cost of high computational complexity, e.g., sparse channel estimation using least-absolute shrinkage and selection operator (LASSO) [24]. To avoid the high computational complexity on sparse channel estimation, a variation of the LMS algorithm with L_1-norm penalty term in the LMS cost function has also been developed in [25]. The L_1-norm penalty was incorporated into the cost function of conventional LMS algorithm, which resulted in two sparse LMS algorithms, namely zero-attracting LMS (ZA-LMS) and reweighted-zero-attracting LMS (RZALMS) [25]. Following this idea, adaptive sparse channel estimation method using L_p-norm sparse penalty LMS (LP-LMS) was also proposed in order to further improve estimation performance [26].

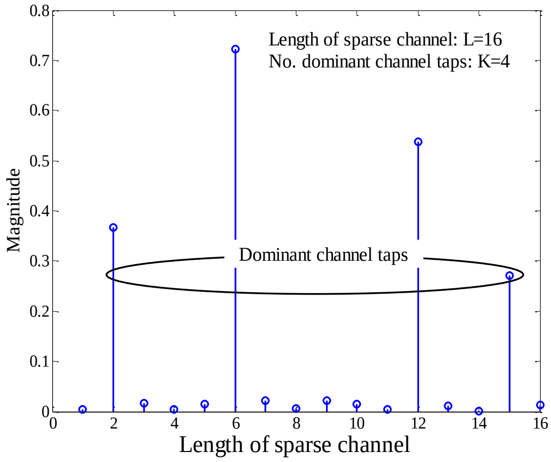

Fig. 1. A typical example of sparse multipath channel.

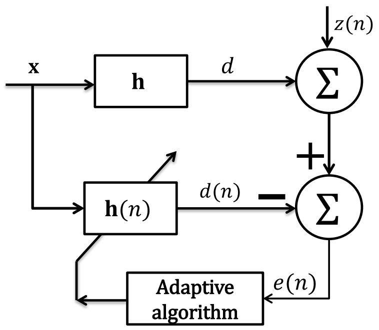

Fig.2. A sparse multipath communication system.

In this paper, we propose an improved sparse channel estimation method by introducing L_0-norm LMS algorithm (L0-LMS) which was proposed in [26]. In the following, based on above mentioned sparse LMS algorithms in [25-27], we propose two kinds of improved adaptive sparse channel estimation methods using L_p-norm normalized LMS (LP-NLMS) and L_0-norm normalized LMS (L0-NLMS), respectively. Effectiveness of propose approaches will be evaluated by computer simulations.

Section 2 introduces sparse system model and problem formulation. Section 3 discusses adaptive sparse channel estimation methods using different LMS-based algorithms. In section 4, computer simulation results are given and their performance comparisons are also discussed. Concluding remarks are resented in Section 5.

2. System Model

Consider a sparse multipath communication system, as shown in Fig. 2, the input signal $\mathbf{x}(n)$ and output signal $d(n)$ are related by

$$d(n) = \mathbf{h}^T\mathbf{x}(n) + z(n), \qquad (1)$$

where $\mathbf{h} = [h_0, h_1, \ldots, h_{N-1}]^T$ is a N-length unknown sparse channel vector which is supported only by K dominant channel taps, $\mathbf{x}(n) = [x(n), x(n-1), \ldots, x(n-N+1)]^T$ is N-length input signal vector and $z(n)$ is an additive noise variable at time n. The object of LMS adaptive filter is to estimate the unknown sparse channel coefficients $\mathbf{h}$ using the input signal $\mathbf{x}(n)$ and output signal $d(n)$. According to Eq. (1), channel estimation error $e(n)$ is written as

$$e(n) = d(n) - \mathbf{h}^T(n)\mathbf{x}(n), \qquad (2)$$

where $\mathbf{h}(n)$ is the LMS adaptive channel estimator. Based on Eq. (2), LMS cost function can be given by

$$L(n) = \frac{1}{2}e^2(n). \qquad (3)$$

Hence, the update equation of LMS adaptive channel estimation is derived by

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu e(n)\mathbf{x}(n), \qquad (4)$$

where $\mu \in (0, 2/\gamma_{\max})$ is a step size of gradient descend and $\gamma_{\max}$ is the maximum eigenvalue of the covariance matrix of $\mathbf{x}(n)$.

3. Adaptive Sparse Channel Estimation

From the above Eq. (4), we can find that the LMS-based channel estimation method never take advantage of sparse structure in $\mathbf{h}$. To a better understood, the standard LMS-based channel estimation can be concluded as

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \text{adaptive update}. \qquad (5)$$

Unlike the standard LMS method, we exploit the channel sparsity by introducing several L_p-norm ($0 \le p \le 1$) penalties to LMS-based cost function. Hence, the LMS-based adaptive sparse channel estimation can be written as

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \text{adaptive update} + \text{sparse penalty}. \qquad (6)$$

From above update Eq. (6), the objective of this paper is introducing different sparse penalties to take the advantage of sparse structure as for prior information.

3.1. ZA-LMS algorithm

To exploit the channel sparsity in CIR, the cost function of ZA-LMS [25] is given by

$$L_{ZA}(n) = \frac{1}{2}e^2(n) + \lambda_{ZA}\|\mathbf{h}(n)\|_1, \qquad (7)$$

where λ_{ZA} is a regularization parameter which balances the adaptive estimation error and sparse penalty of $\mathbf{h}(n)$. The corresponding update equation of ZA-LMS is

$$\mathbf{h}(n+1)=\mathbf{h}(n)-\mu\frac{\partial L_{ZA}(n)}{\partial\mathbf{h}(n)}$$
$$=\mathbf{h}(n)+\mu e(n)\mathbf{x}(n)-\rho_{ZA}\mathrm{sgn}\{\mathbf{h}(n)\},\qquad(8)$$

where $\rho_{ZA}=\mu\lambda_{ZA}$ and $\mathrm{sgn}\{\cdot\}$ is a component-wise function which is defined as

$$\mathrm{sgn}(h)=\begin{cases}h/|h|, & \text{when } h\neq 0\\ 0, & \text{when } h=0,\end{cases}\qquad(9)$$

where the h is one of taps of $\mathbf{h}$. From the update equation in Eq. (8), the second term attracts the small filter coefficients to zero, which speed up convergence when the most of the channel coefficients $\mathbf{h}$ are zeros.

3.2. RZA-LMS algorithm

The ZA-LMS cannot distinguish between zero taps and non-zero taps since all the taps are forced to zero uniformly; therefore, its performance will degrade in less sparse systems. Motivated by reweighted L_1-minimization sparse recovery algorithm [28], Chen et. al. proposed a heuristic approach to zero-attracting LMS (RZA-LMS) [25]. The cost function of RZA-LMS is given by

$$L_{RZA}(n)=\frac{1}{2}e^2(n)+\lambda_{RZA}\sum_{i=1}^{N}\log(1+\varepsilon_{RZA}|h_i|),\qquad(10)$$

where $\lambda_{RZA}>0$ is a regularization parameter which trades off the estimation error and channel sparsity. The corresponding update equation is

$$\mathbf{h}(n+1)=\mathbf{h}(n)-\mu\frac{\partial L_{RZA}(n)}{\partial\mathbf{h}(n)}$$
$$=\mathbf{h}(n)+\mu e(n)\mathbf{x}(n)-\mu\lambda_{RZA}\varepsilon_{RZA}\sum_{i=1}^{N}\frac{\mathrm{sgn}(|h_i(n)|)}{1+\varepsilon_{RZA}|h_i(n)|}$$
$$=\mathbf{h}(n)+\mu e(n)\mathbf{x}(n)-\rho_{RZA}\frac{\mathrm{sgn}(\mathbf{h}(n))}{1+\varepsilon_{RZA}|\mathbf{h}(n)|},\qquad(11)$$

where $\rho_{RZA}=\mu\lambda_{RZA}\varepsilon_{RZA}$ is a parameter which depends on step-size μ, regularization parameter λ_{RZA} and threshold ε_{RZA}, respectively. In the second term of Eq. (11), if magnitudes of $h_i(n), i=1,2,\dots,N$ are smaller than $1/\varepsilon_{RZA}$, then these channel coefficients will be replaced by zeros.

3.3. LP-LMS and LP-NLMS algorithms

Following the idea in Eq. (11), LP-LMS based adaptive sparse channel estimation method has been proposed in [26].The cost function of LP-LMS is given by

$$L_{LP}(n)=\tfrac{1}{2}e^2(n)+\lambda_{LP}\|\mathbf{h}(n)\|_p,\qquad(12)$$

where $\lambda_{LP}>0$ is a regularization parameter which balances the estimation error and channel sparsity. The corresponding update equation of LP-LMS is

$$\mathbf{h}(n+1)=\mathbf{h}(n)-\mu\frac{\partial L_{LP}(n)}{\partial\mathbf{h}(n)}$$
$$=\mathbf{h}(n)+\mu e(n)\mathbf{x}(n)-\rho_{LP}\frac{\|\mathbf{h}(n)\|_p^{1-p}\mathrm{sgn}(\mathbf{h}(n))}{\varepsilon_{LP}+|\mathbf{h}(n)|^{1-p}},\qquad(13)$$

where $\rho_{LP}=\mu\lambda_{LP}$ which is decided by step-size μ and regularization parameter λ_{LP}, and $\varepsilon_{LP}>0$. According to the updating equation of LP-LMS in Eq. (13), the update equation of LP-NLMS can be derived as

$$\mathbf{h}(n+1)=\mathbf{h}(n)+\mu_N\frac{e(n)\mathbf{x}(n)}{\mathbf{x}^H(n)\mathbf{x}(n)}-\rho_{LPN}\frac{\|\mathbf{h}(n)\|_p^{1-p}\mathrm{sgn}(\mathbf{h}(n))}{\varepsilon_{LPN}+|\mathbf{h}(n)|^{1-p}},\qquad(14)$$

where $\varepsilon_{LPN}>0$ and μ_N is a step size which controls the gradient descend speed and $\rho_{LPN}=\mu_N\lambda_{LPN}$ is a parameter which depends on step-size and regularization parameter.

3.4. L0-LMS and L0-NLMS algorithms

Consider the L_0-norm penalty on the cost function of LMS so that it can produce sparse channel estimator since this penalty term forces the channel taps values of $\mathbf{h}(n)$ to approach zero. Then, the cost function of L0-LMS is given by

$$L_{L0}(n)=\frac{1}{2}e^2(n)+\lambda_{L0}\|\mathbf{h}(n)\|_0,\qquad(15)$$

where $\lambda_{L0}>0$ is a regularization parameter. Since solving the L_0-norm minimization is a Non-Polynomial (NP) hard problem, we replace it with approximate continuous function

$$\|\mathbf{h}\|_0\approx\sum_{i=1}^{N}\left(1-e^{-\beta|h_i|}\right),\qquad(16)$$

According to the approximate function in Eq. (16), L0-LMS cost function can be changed as

$$L_{L0}(n)=\frac{1}{2}e^2(n)+\lambda_{L0}\sum_{i=1}^{N}\left(1-e^{-\beta|h_i|}\right),\qquad(17)$$

Then, the update equation of L0-LMS based adaptive sparse channel estimation can be derived as

$$\mathbf{h}(n+1)=\mathbf{h}(n)-\mu\frac{\partial L_{L0}(n)}{\partial\mathbf{h}(n)}$$
$$=\mathbf{h}(n)+\mu e(n)\mathbf{x}(n)-\rho_{L0}\beta\mathrm{sgn}(\mathbf{h}(n))e^{-\beta|\mathbf{h}(n)|},\qquad(18)$$

where $\rho_{L0}=\mu\lambda_{L0}$. It is worth mention that the exponential function in Eq. (18) causes high computational complexity. To reduce the computational complexity, the first-order Taylor series expansion of exponential functions is taken into consideration

$$e^{-\beta|h|}\approx\begin{cases}1-\beta|h|, & \text{when } |h|\leq 1/\beta\\ 0, & \text{others.}\end{cases}\qquad(19)$$

Then, the update equation of L0-LMS based adaptive sparse channel estimation can be derived as

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu e(n)\mathbf{x}(n) - \rho_{L0} J(\mathbf{h}(n)), \quad (20)$$

where $J(h)$ is defined as

$$J(h) \approx \begin{cases} 2\beta^2 h - 2\beta \mathrm{sgn}(h), & \text{when } |h| \leq 1/\beta \\ 0, & \text{others.} \end{cases} \quad (21)$$

Based on this algorithm in Eq. (20), we further propose an improved adaptive sparse channel estimation method by using L0-NLMS algorithm

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu_N \frac{e(n)\mathbf{x}(n)}{\mathbf{x}^H(n)\mathbf{x}(n)} - \rho_{L0} J(\mathbf{h}(n)), \quad (22)$$

where μ_N is the step size of gradient descend which is same as in Eq. (14).

4. Computer Simulation

In this section, we will compare the performance of the proposed channel estimators using 1000 independent Monte-Carlo runs for averaging. The length of sparse multipath channel $\mathbf{h}$ is set as $N = 16$ and its number of dominant taps is set as $K = 1$ and 4 respectively. The values of the dominant channel taps follow Gaussian distribution and the positions of dominant taps are randomly allocated within the length of $\mathbf{h}$ and is subjected to $\mathrm{E}[\|\mathbf{h}\|_2^2] = 1$. The received signal-to-noise ratio (SNR) is defined as $10\log(E_0/\sigma_n^2)$, where $E_0 = 1$ is received signal power and the noise power is $\sigma_n^2 = 10^{-\mathrm{SNR}/10}$. Here, we compare their performance with different SNR: 5dB~20dB. All of the step sizes and regularization parameters are listed in Tab. 1. It is worth noting that the (N)LMS-based algorithms can exploit more accurate sparse channel information at higher SNR environment. Hence, all of parameters are set direct ratio relation with noise power. For example, in the case of SNR=10dB, the parameters of LMS-based algorithm are matched with parameters which are given in [25]. Hence, the propose regulation parameter method can adaptive exploit channel sparsity in various SNR environment.

Tab. 1. Simulation parameters for (N)LMS-based adaptive sparse channel estimation.

Type of parameters	Value
μ	$5e-1$
μ_N	$5e-2$
λ_{ZA}	$0.02\sigma_n^2$
λ_{RZA}	$0.002\sigma_n^2$
λ_{LP}	$0.05\sigma_n^2$
λ_{LPN}	$0.005\sigma_n^2$
λ_{L0}	$0.02\sigma_n^2$
λ_{L0N}	$0.002\sigma_n^2$

The estimation performance between actual and estimated channel is evaluated by mean square error (MSE) which is defined as

$$\mathrm{MSE}\{\mathbf{h}(n)\} = \mathrm{E}\{\|\mathbf{h} - \mathbf{h}(n)\|_2^2\}, \quad (23)$$

where $\mathrm{E}\{\cdot\}$ denotes expectation operator, $\mathbf{h}$ and $\mathbf{h}(n)$ are the actual channel and its estimator, respectively.

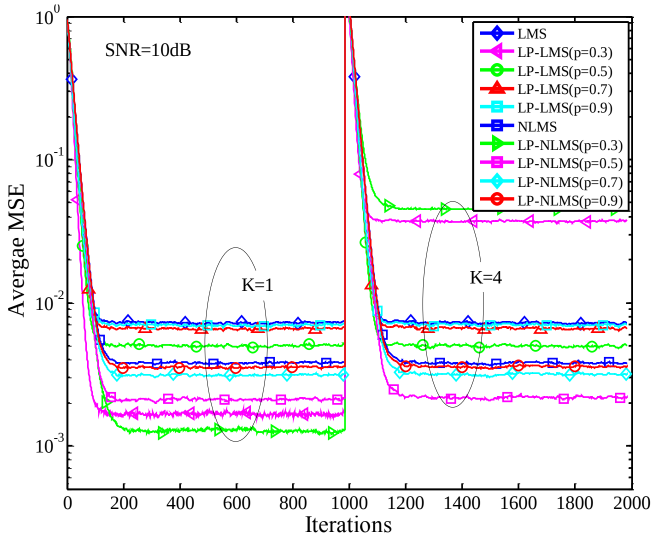

Fig. 3. Performance comparison of LP-(N)LMS with different p (SNR=10dB).

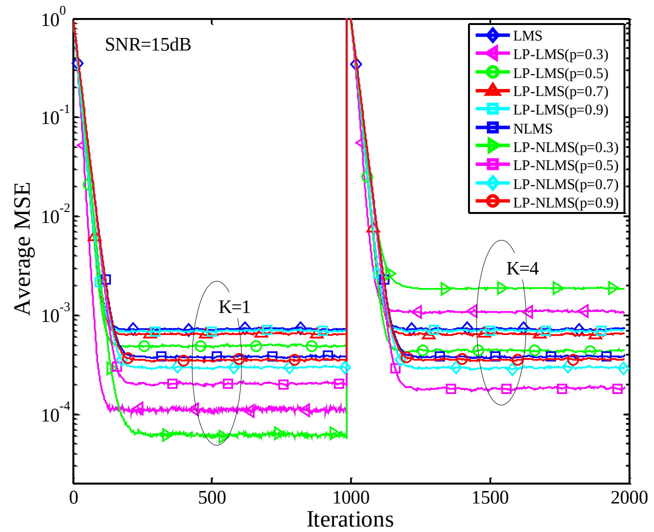

Fig. 4. Performance comparison of LP-(N)LMS with different p (SNR=20dB).

At the first experiment, we evaluate the estimation performance of LP-(N)LMS as a function of p which are shown in Figs. 3-4 in two SNR cases, i.e., $\mathrm{SNR} = 10\mathrm{dB}$ and 20dB. The parameter is set as $\varepsilon_{LP} = \varepsilon_{LPN} = 0.05$ which is suggested in [26]. The nonzero taps of two channels are assumed $K = 1$ and 4, respectively. From the two Figures, we can find that when the channel is very sparse (e.g.,$K = 1$), smaller p-sparse penalty (e.g., $p = 0.3$)

on (N)LMS algorithm can achieve better estimation performance. However, as the number of nonzero taps increase ($K = 4$), the small p-norm sparse penalty (e.g., $p = 0.3$) on (N)LMS algorithm is becoming unstable. In addition, simulation results in Figs. 3-4 are also shown that there is no direct relationship between the value of sparse penalty p and SNR. In order to tradeoff stability and estimation performance of LP-(N)LMS algorithm, it is better to set the value of the sparse penalty as $p = 0.5$. In the following simulation results, $p = 0.5$ is considered for LP-(N)LMS based adaptive sparse channel estimation.

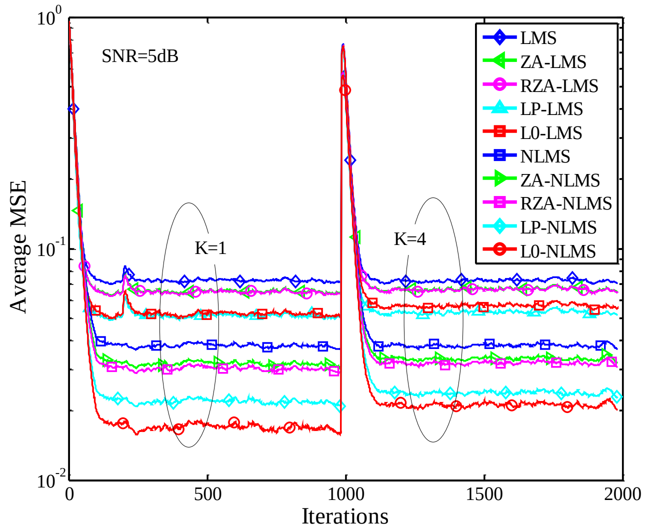

Fig. 5. MSE versus the number of iterations (SNR=5dB).

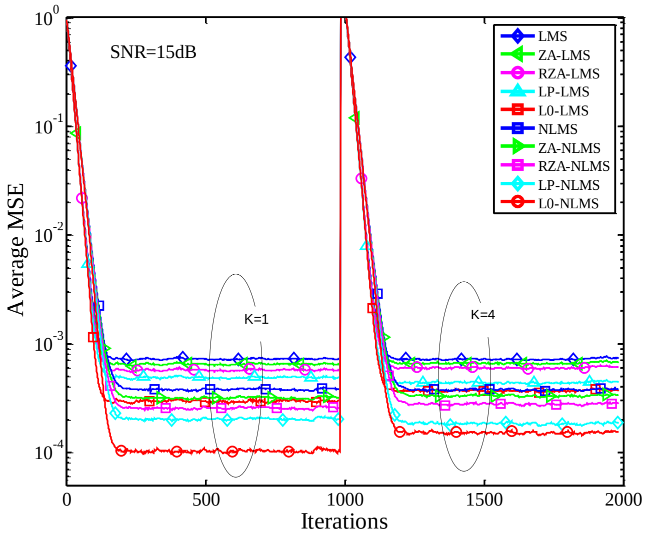

Fig. 6. MSE versus the number of iterations (SNR=15dB).

At the second experiment, we compare all the (N)LMS-based sparse adaptive channel estimation methods with different SNR: 5dB and 15dB as shown in Figs. 5-6, respectively. One can see Fig. 5 in SNR = 5dB environment (low SNR case), these MSE curves show that NLMS-based methods achieved better estimation performance than LMS-based one. For example, the curve of ZA-NLMS is much lower than ZA-LMS. That is to say, there exists a big performance gap between ZA-NLMS and ZA-LMS. However, as the SNR increasing, the obvious performance advantages of NLMS-based methods are reducing. Hence, comparing with LMS-based methods, we can conclude that NLMS-based methods not only work more reliable for unknown signal scaling, but also work more stable for noise interference, especially in low SNR environment. In addition, the simulation results are also shown that (N)LMS-based methods have inverse relationship with number of nonzero channel taps. In other words, for a sparser channel, (N)LMS-based methods can achieve better estimation performance and vice versa. In Figs. 5-6, when number of nonzero taps $K = 1$, channel performance gap bigger than the case where $K = 4$. Hence, these simulation results are also shown that adaptive sparse channel estimation performance is also decided by the number of nonzero channel taps. When the channel is no more sparse, the performance of these proposed methods will reduce to the performance of N(LMS)-based methods.

5. Conclusion

In this paper, we have investigated various (N)LMS-based adaptive sparse channel estimation methods by enforcing different sparse penalties, e.g. Lp-norm and L0-norm. First of all, we proposed an improved adaptive sparse channel estimation method using L0-norm sparse constraint LMS algorithm, when comparing with ZA-LMS, RZA-LMS and LP-LMS. In addition, to improve the robust performance and increase the convergence speed, we proposed NLMS-based adaptive sparse channel estimation methods by using different sparse penalties. Unlike LMS-based methods, the proposed NLMS-based methods have avoided the uncertain signal scaling and normalized the power of input signal with different sparse penalties. The proposed methods exhibit faster convergence and better performance which are confirmed by computer simulations under various SNR environments.

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