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Adaptive Sparse Channel Estimation Methods for Time-Variant MIMO Communication Systems

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Abstract Channel estimation problem is one of key technical issues in time-variant multiple-input multiple-output (MIMO) communication systems. To estimate the MIMO channel, least mean square (LMS) algorithm was applied to adaptive channel estimation (ACE). Since the MIMO channel is often described by sparse channel model, such sparsity could be exploited and then estimation performance could be improved by adaptive sparse channel estimation (ASCE) methods using sparse LMS algorithms. However, conventional ASCE methods have two main drawbacks: 1) sensitive to random scale of training signal and 2) unstable in low signal-to-noise ratio (SNR) region. To overcome the two harmful factors, in this paper, we propose a novel ASCE method using normalized LMS (NLMS) algorithm (ASCE-NLMS). In addition, we also proposed an improved ASCE method using normalized least mean fourth (NLMF) algorithm (ASCE-NLMF). Two proposed methods can exploit the channel sparsity effectively. Also, stability of the proposed methods is confirmed by mathematical derivation. Computer simulation results show that the proposed sparse channel estimation methods can achieve better estimation performance than conventional methods.

Keyword least mean square (LMS), least mean fourth (LMF), normalized LMF (NLMF), adaptive sparse channel estimation (ASCE), multiple-input multiple-output (MIMO).

1. Introduction

Signal transmission over multiple-input multiple-output (MIMO) channel is becoming one of mainstream techniques in the next generation communication systems. The major motivation is due to the fact that MIMO technology is a way of using multiple antennas to simultaneously transmit multiple streams of data in wireless communication systems. MIMO in cellular systems brings improvements on four fronts: improved data rate, improved reliability, improved energy efficiency, and reduced interference. However, coherent receive requires accurate channel state information (CSI) due to the fact that wireless signal propagates over frequency-selective fading channel. In these systems, the basic channel estimation problem is reduced to estimation multiple-input single-output (MISO) channel at each antenna at receiving side. One of typical examples is that use of very large number of antenna (so-called “massive MIMO”) at base station and only one antenna at mobile terminal (as shown in Fig. 1) makes high data communication possible with very low transmit power in a frequency-selective fading channel [1]. Besides, in the high mobility environment, the MIMO channel is subjected to time-variant fading (i.e., double-selective fading). The accurate estimation of channel impulse response (CIR) is a crucial and challenging issue in coherent modulation and its accuracy has a significant impact on the overall performance of communication systems.

During last decades, many channel estimation methods proposed for MIMO-OFDM systems [2–10]. However, all of the proposed methods can be categorized into two types. The first type is that linear channel estimation methods, e.g., least squares (LS) algorithm, based on the assumption of dense CIRs. By applied these approaches, the performance of linear methods depend only on size of MIMO channel. Note that narrowband MIMO channel may be modeled as dense channel model because of its very short time delay spread; however, broadband MIMO channel is often modeled as sparse channel model [11–13]. A typical example of sparse channel is shown in Fig. 2. It is well known that linear channel estimation methods are relatively simple to implement due to its low computation complexity [4–9]. But, the main drawback of linear channel estimations is unable to exploit the inherent channel sparsity. The second type is the sparse channel estimation methods using compressive sensing (CS) [14], [15]. Optimal sparse channel estimation often requires that its training signal satisfies restrictive isometry property (RIP) [16] in high probability. However, designing the RIP-satisfied training signal is a non-polynomial (NP) hard problem [17]. Although some proposed methods are stable while scarifying extra high computational burden, especially in time-variant MIMO-OFDM systems. For example, sparse channel estimation method using Dantzig selector was proposed for double-selective fading MIMO systems [9]. However, the proposed method needs to be solved by linear programming and then it incurs high computational complexity. To reduce complexity, sparse channel estimation methods using greedy iterative algorithms were also proposed in [8], [10]. However, their complexity depends on the number of nonzero taps of MIMO channel.

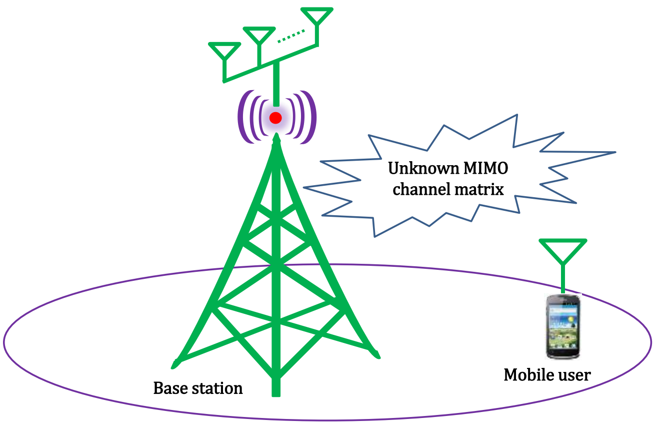

Fig.1. An example of time-variant MIMO system.

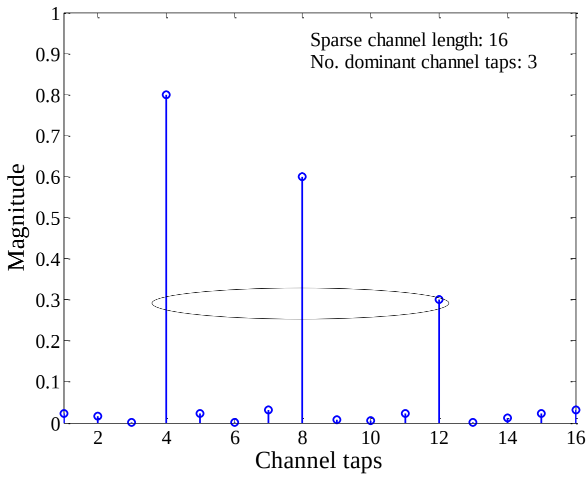

Fig. 2. A typical example of sparse multipath channel.

Unfortunately, above proposed methods cannot estimate channel adaptively. To estimate time-variant channel, adaptive sparse channel estimation (ASCE) methods using sparse least mean square algorithms (ASEC-LMS) were proposed in [21]. However, conventional ASCE-LMS methods have two main drawbacks: 1) sensitive to random scale of training signal and 2) unstable in low signal-to-noise ratio (SNR) region. To overcome the two harmful factors, in this paper, we propose a novel ASCE method using normalized LMS (NLMS) algorithm (ASCE-NLMS) for estimate MIMO channel. In addition, since normalized least mean fourth (NLMF) algorithm [18] outperforms the well-known normalized least mean square (NLMS) algorithm [19] in achieving a better balance between complexity and estimation performances. In our previous research in [20], stable sparse NLMF algorithm was also proposed to achieve better estimation than sparse NLMS algorithm [21]. Hence, for time-variant MIMO communication systems, we also propose improved ASCE methods using sparse NLMF algorithms (ASCE-NLMF). First of all, as shown in Fig. 3, MIMO-OFDM system model is formulated so that ASCE can estimate MIMO channel vector. Later, computer simulation results are presented to confirm the effectiveness of our proposed methods.

The remainder of this paper is organized as follows. A MIMO-OFDM system model is described and problem formulation is given in Section II. In section III, sparse NLMS and sparse NLMF algorithms are introduced and ASCE in time-variant MIMO-OFDM systems is highlighted. Computer simulation results are given in Section IV in order to evaluate and compare performances of the proposed ASCE methods. Finally, we conclude the paper in Section V.

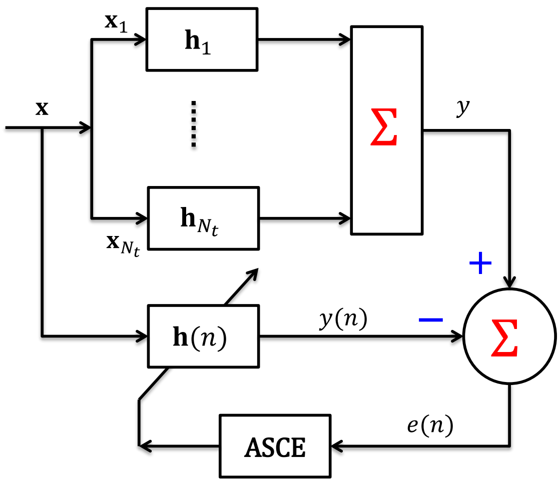

Fig. 3. ASCE for MIMO-OFDM communication systems.

2. System Model

Assume that the transmit power is $\{\|\bar{\mathbf{x}}_{n_t}(t)\|\} = KE_0$. The resultant vector $\mathbf{x}_{n_t}(t) \triangleq \mathbf{F}^{\mathrm{H}}\bar{\mathbf{x}}_{n_t}(t)$ is padded with cyclic prefix (CP) of length $L_{CP} \geq (K-1)$ to avoid inter-block interference (IBI), where $\mathbf{F}$ is a $K \times K$ DFT matrix with entries $[\mathbf{F}]_{kq} = 1/K\, e^{-j2\pi kq/K}$, $k,q = 0,1,\dots,K-1$. After CP removal, the received signal vector at the n_t-th antenna for time t is written as y. Then, the received signal y and input signal vector $\mathbf{x}$ are related by

$$y = \sum_{n_t=1}^{N_t} \mathbf{h}_{n_t}^{T}\mathbf{x}_{n_t} + z = \mathbf{h}^{T}\mathbf{x} + z, \qquad (1)$$

where $\mathbf{x} = [\mathbf{x}_1^T, \mathbf{x}_2^T, \dots, \mathbf{x}_{N_t}^T]^T$ combines all of the input signal vectors; additive noise variable z satisfies $\mathcal{CN}(0, \sigma_n^2)$ and the MIMO channel vector $\mathbf{h}$ can be written as

$$\mathbf{h} = \left[\mathbf{h}_1^T \quad \mathbf{h}_2^T \cdots \quad \mathbf{h}_{N_t}^T\right]^T \in \mathbb{C}^{NN_t \times 1}, \qquad (2)$$

where $\mathbf{h}_{n_t}$ $(n_t = 1,2,\ldots,N_t)$ is assumed equal N-length sparse channel vector from receiver to n_t-th antenna. In addition, we also assume that the each channel vector $\mathbf{h}_{n_t}$ is only supported by T dominant channel taps. A typical example of 16-paths sparse multipath channel, which is supported by 3 dominant channel taps, is depicted in Fig. 2. According to the system model in Eq. (1), the corresponding channel estimation error $e(n)$ at time t can be written as

$$e(n) = y - y(n) = y - \mathbf{h}^T(n)\mathbf{x}(n), \quad (3)$$

where $\mathbf{h}(n)$ denotes an adaptive MIMO channel estimator of $\mathbf{h}$ and $y(n)$ is the output signal. A diagram of ASCE method for MIMO-OFDM communication system was shown in Fig. 3. The goal of ASCE is to estimate MIMO channel $\mathbf{h}$ using error signal $e(n)$ and input training signal $\mathbf{x}(n)$. Traditional ASCE methods using sparse LMS algorithms were proposed to exploit channel sparsity. The cost function of ASCE method is concluded as

$$L_s(n) = \tfrac{1}{2}e^2(n) + \lambda_{slp}\|\mathbf{h}(n)\|_p. \quad (4)$$

where $0 \le p < 1$ and $\lambda_{slp} \ge 0$ denotes sparse regulation parameter which trades off the mean square error and sparsity of $\mathbf{h}$. Without loss of generality, corresponding update equation of ASCE methods can be written as

$$\mathbf{h}(n+1) = \mathbf{h}(n) - \mu_s \frac{\partial L_s(n)}{\partial \mathbf{h}(n)}$$

$$= \mathbf{h}(n) + \mu_s e(n)\mathbf{x}(n) - \rho_{slp}\frac{\|\mathbf{h}(n)\|_p^{1-p}\mathrm{sgn}(\mathbf{h}(n))}{\sigma + |\mathbf{h}(n)|^{1-p}}, \quad (5)$$

where $\rho_{slp} = \mu_s\lambda_{slp}$ and $\mu_s \in (0, \gamma_{\max}^{-1})$ is the step size of LMS gradient descend and $\gamma_{\max}$ is the maximum eigenvalue of the covariance matrix $\mathbf{R} = E\{\mathbf{x}(n)\mathbf{x}^T(n)\}$.

3. Proposed ASCE Methods

3.1. ASCE-NLMS

Consider L_p-norm sparse penalty on cost function of NLMS to produce sparse channel estimator since this penalty term forces the values for channel taps of $\mathbf{h}$ to approach zero. It is termed as LP-NLMS which was proposed for single-antenna systems in [21]. According to the (4) and (5), update equation of LP-NLMS based ASCE method can be derived as

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu_s \frac{e(n)\mathbf{x}(n)}{\|\mathbf{x}(n)\|_2^2} - \rho_{slp}\frac{\|\mathbf{h}(n)\|_p^{1-p}\mathrm{sgn}(\mathbf{h}(n))}{\sigma + |\mathbf{h}(n)|^{1-p}}, \quad (6)$$

where $\|\cdot\|_2$ is the Euclidean norm operator and $\|\mathbf{x}\|_2^2 = \sum_{i=1}^{N}|x_i|^2$. Following to this idea of the LP-NLMS algorithm on ASCE, if $p = 0$, then the zero-attracting forces the channel taps values of $\mathbf{h}$ to approach zero is L_0-norm penalty. It is termed as L_0-norm NLMS (L0-NLMS) [21] that the cost function is given by

$$L_{sl0}(n) = \tfrac{1}{2}e^2(n) + \lambda_{sl0}\|\mathbf{h}(n)\|_0, \quad (7)$$

where $\|\mathbf{h}\|_0$ is the L_0-norm operator that counts the number of nonzero taps in $\mathbf{h}$ and λ_{sl0} is a regularization parameter to balance the estimation error and sparse penalty. Since solve the L_0-norm minimization is a NP-hard problem [17], we replace it with approximate continuous function [22]

$$\|\mathbf{h}\|_0 \approx \sum_{l=0}^{N_tN-1}\left(1 - e^{-\beta|h_l|}\right). \quad (8)$$

According to the approximate function, L0-LMS cost function can be revised as

$$L_{sl0}(n) = \tfrac{1}{2}e^2(n) + \lambda_{sl0}\sum_{l=0}^{N_tN-1}\left(1 - e^{-\beta|h_l|}\right). \quad (9)$$

Then, the update equation of L0-LMS based ASCE can be derived as

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu_s e(n)\mathbf{x}(n) - \rho_{sl0}\beta\,\mathrm{sgn}(\mathbf{h}(n))e^{-\beta|\mathbf{h}(n)|}, \quad (10)$$

where $\rho_{sl0} = \mu_s\lambda_{sl0}$. It is worth mention that the exponential function in (10) will cause high computational complexity. To reduce the computational complexity, the first order Taylor series expansion of exponential functions is taken into consideration as [22]

$$e^{-\beta|h|} \approx \begin{cases} 1 - \beta|h|, & \text{when } |h| \le 1/\beta \\ 0, & \text{others.} \end{cases} \quad (11)$$

where h is an any element of channel vector $\mathbf{h}$. Then, the update equation of L0-NLMS based adaptive sparse channel estimation can be derived as

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu_s \frac{e(n)\mathbf{x}(n)}{\|\mathbf{x}(n)\|_2^2} - \rho_{l0}J(\mathbf{h}(n)), \quad (14)$$

where $J(\mathbf{h})$ is defined as

$$J(\mathbf{h}) = \begin{cases} 2\beta^2 h - 2\beta\,\mathrm{sgn}(h), & \text{when } |h| \le 1/\beta \\ 0, & \text{others.} \end{cases} \quad (15)$$

3.2. ASCE-NLMF

Unlike the proposed method in Section A, we propose a kind of improved ASCE methods using sparse NLMF algorithms for MIMO channel. At first, cost function $L_{nlmf}(n)$ of standard LMF can be constructed as

$$L_f(n) = \frac{1}{4}e^4(n). \quad (16)$$

The update equation of ASCE using LMF algorithm can be derived as

$$\mathbf{h}(n+1) = \mathbf{h}(n) - \mu_f \frac{\partial L_{nlmf}(n)}{\partial \mathbf{h}(n)} = \mathbf{h}(n) + \mu_f e^3(n)\mathbf{x}(n), \quad (17)$$

where $\mu_f \in (0,2)$ is a gradient descend step-size which controls convergence speed and steady-state performance; However, LMF algorithm only works stable in low SNR region [23]. Based on the our previous research in [21], ACE using normalized LMF (NLMF) algorithm is stable for different SNR region. Then, the update equation of NLMS based ACE is given by

$$\mathbf{h}(n+1)=\mathbf{h}(n)+\mu_f\frac{e^3(n)\mathbf{x}(n)}{\|\mathbf{x}(n)\|_2^2\left(\|\mathbf{x}(n)\|_2^2+e^2(n)\right)}$$

$$=\mathbf{h}(n)+\mu_f(n)\frac{e(n)\mathbf{x}(n)}{\|\mathbf{x}(n)\|_2^2}\ , \qquad (18)$$

where $\mu_f(n)=\mu_f e^2(n)/(\|\mathbf{x}(n)\|_2^2+e^2(n))$. Here, we can find that when $e^2(n)\gg\|\mathbf{x}(n)\|_2^2$, then $\mu_f(n)\to\mu_f$; when $e^2(n)\approx\|\mathbf{x}(n)\|_2^2$, then $\mu_f(n)\to\mu_f/2$; when $e^2(n)\ll\|\mathbf{x}(n)\|_2^2$, then $\mu_f(n)\to 0$. Hence, NLMF algorithm in Eq. (18) is stable which is equivalent to NLMS algorithm in Eq. (6). According to the previous research in [21] regarding single-antenna communication systems, if the standard NLMF algorithm is stable, then its corresponding ASCE method using sparse NLMF algorithm is also stable. Hence, stable ASCE using sparse NLMF algorithms are presented as follows.

For the MIMO channel vector $\mathbf{h}(n)$, its cost function of ASCE using LP-NLMF algorithm is given by

$$L_{flp}(n)=\frac{1}{4}e^4(n)+\lambda_{flp}\|\mathbf{h}(n)\|_p, \qquad (19)$$

where λ_{flp} is a regularization parameter which trades off the fourth-order mismatching estimation error and L_p-norm sparse penalty of $\mathbf{h}$. The update equation of ASCE method using LP-NLMF can be derived as

$$\mathbf{h}(n+1)=\mathbf{h}(n)+\mu_f(n)\frac{e(n)\mathbf{x}(n)}{\|\mathbf{x}(n)\|_2^2}-\rho_{flp}\frac{\|\mathbf{h}(n)\|_p^{1-p}\mathrm{sgn}(\mathbf{h}(n))}{\sigma+|\mathbf{h}(n)|^{1-p}} \qquad (20)$$

where $\rho_{flp}=\mu_f\lambda_{flp}$ depends on step-size μ_f and parameter λ_{flp}. Similarly, cost function of ASCE method using L0-NLMF algorithm can also be written as

$$L_{fl0}(n)=\frac{1}{4}e^4(n)+\lambda_{fl0}\|\mathbf{h}(n)\|_0, \qquad (21)$$

where $\lambda_{fl0}>0$ is a regularization parameter which trades off the fourth-order mismatching estimation error and sparseness of MIMO channel. Then, its updating equation algorithm can be written as

$$\mathbf{h}(n+1)=\mathbf{h}(n)+\mu_f(n)\frac{e(n)\mathbf{x}(n)}{\|\mathbf{x}(n)\|_2^2}-\beta_2 J(\mathbf{h}(n)), \qquad (22)$$

where $\beta_2=\mu_f\lambda_{fl0}$ and $J(\boldsymbol{h}(n))$ is a approximate sparse L_0-norm function which was defined in Eq. (15).

4. Computer Simulation

In this section, the proposed ASCE estimators using 1000 independent Monte-Carlo runs for averaging. The length of channel vector $\mathbf{h}_{n_t}$ between each antenna of transmit to receiver is set as $N=16$ and its number of dominant taps is set as $T=1$ and 3, respectively. Values of dominant channel taps follow Gaussian distribution and their positions are randomly allocated within the length of $\mathbf{h}_{n_t}$ which is subjected to $E\{\|\mathbf{h}_{n_t}\|_2^2=1\}$. The received signal-to-noise ratio (SNR) is defined as $20\log(E_0/\sigma_n^2)$, where $E_0=1$ is transmitted power at each antenna. Here, we set the SNR as 3dB, 6dB and 9dB in computer simulation. All of the step sizes and regularization parameters are listed in Tab. I. The estimation performance is evaluated by average mean square error (MSE) which is defined as

$$\text{Avergae MSE}\{\mathbf{h}(n)\}=\mathrm{E}\{\|\mathbf{h}-\mathbf{h}(n)\|_2^2\}, \qquad (23)$$

where $\mathrm{E}\{\cdot\}$ denotes expectation operator, $\mathbf{h}$ and $\mathbf{h}(n)$ are the actual MIMO channel vector and its n-th adaptive channel estimator, respectively.

TABLE I. SIMULATION PARAMETERS.

Parameters	Values
Gradient descend step-size: μ_s	0.5
Gradient descend step-size: μ_f	1.5
Regularization parameter: λ_{slp}	$(2\mathrm{e}-4)\sigma_n^2\log(N/T)$
Regularization parameter: λ_{flp}	$(2\mathrm{e}-6)\sigma_n^2\log(N/T)$
Regularization parameter: λ_{sl0}	$(2\mathrm{e}-3)\sigma_n^2\log(N/T)$
Regularization parameter: λ_{fl0}	$(2\mathrm{e}-5)\sigma_n^2\log(N/T)$

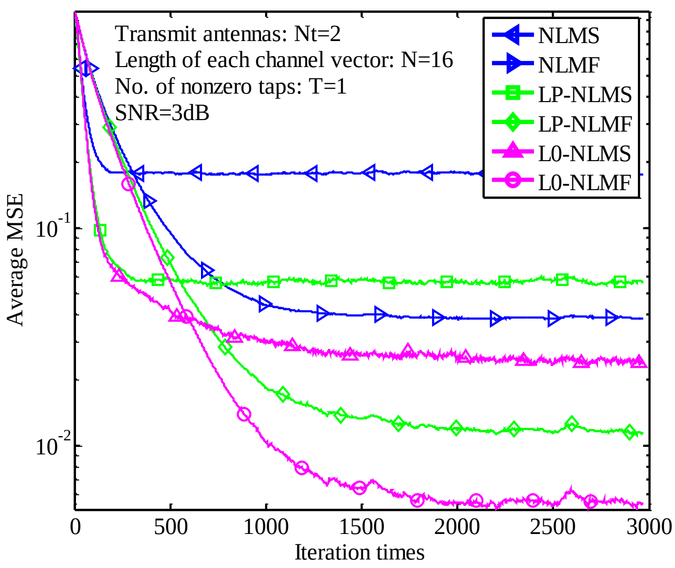

Fig. 4. Performance comparison at SNR = 3dB.

In the first example, the proposed methods are evaluated in Fig. 4 ($T=1$) and Fig. 5 ($T=3$) at SNR = 3dB. Since step-size of ASCE methods can balance the estimation performance and computation complexity. Hence, the step-size of sparse NLMS algorithms and sparse NLMF algorithms are set as $\mu_s=0.5$ and $\mu_f=1.5$, respectively. Note that the step-size $\mu_s=0.5$ was also recommended by the paper [21]. As the two figures shown, ASCE-NLMS methods achieved better estimation performance than ACE-NLMS. Similarly, ASCE-NLMF methods also achieved better estimation performance than ACE-NLMF methods. In addition, the two figures are also shown that ASCE-NLMF methods are much better than ASCE-NLMS methods but scarifying much computational complexity (iterative times). Relatively, the computational complexity of ASCE-NLMS is very low [21]. Hence, selecting a reasonable ASCE method may depend on the requirement of practical systems.

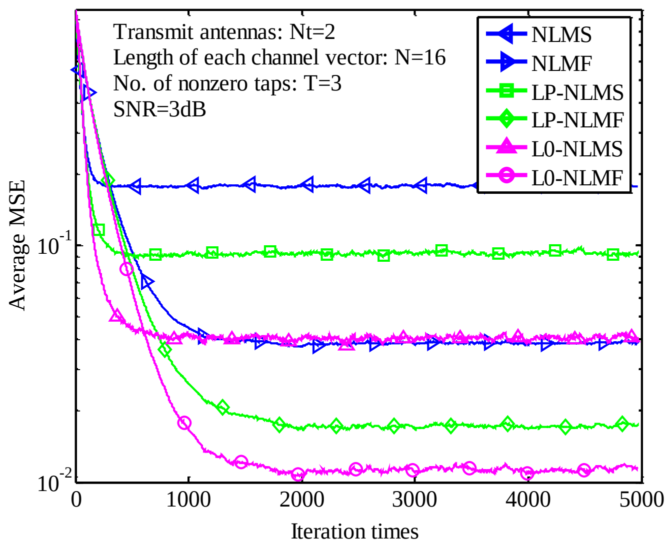

Fig. 5. Performance comparison at $\mathrm{SNR}=3\mathrm{dB}$.

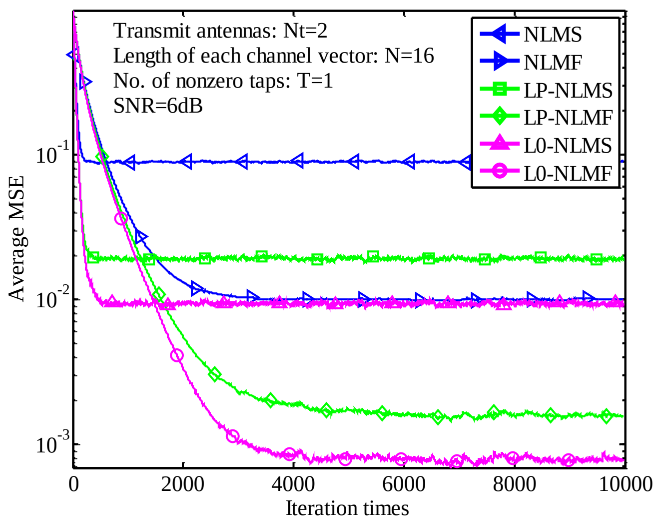

Fig. 6. Performance comparison at $\mathrm{SNR}=6\mathrm{dB}$.

In the second experiment, the proposed methods are evaluated at different SNR region as shown in Figs. 6-7. As SNR increasing from 6dB to 9dB, performance advantages of these proposed methods are shown obviously when comparing with conventional methods. Here, it was worth noting that computational complexity of ASCE-NLMF methods increase with SNR. Hence, how to reduce complexity of ASCE-NLMF is one of our future works.

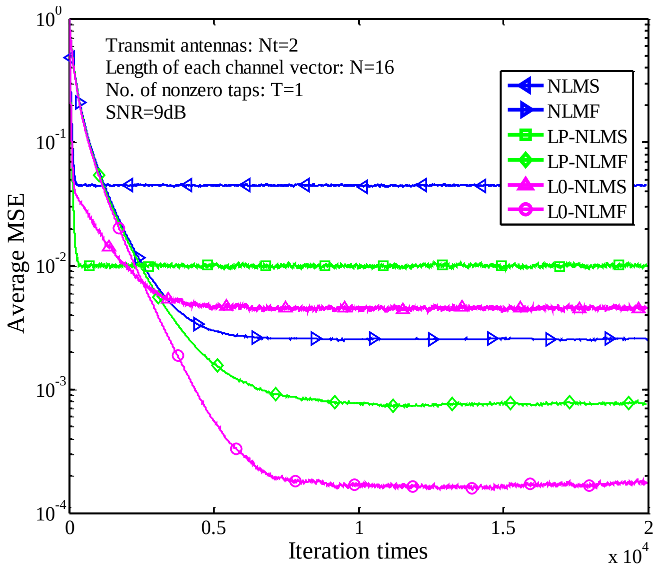

Fig. 7. Performance comparison at $\mathrm{SNR}=9\mathrm{dB}$.

5. Conclusion

In this paper, we proposed ASCE methods using sparse NLMS and sparse NLMF algorithms for time-variant MIMO-OFDM systems. First of all, system model was formulated to ensure each MIMO channel vector can be estimated. Secondly, cost function of the two kinds of proposed methods were constructed using sparse penalties, i.e., L_p-norm and L_0-norm. Later, MIMO channel vector was estimated using ASCE methods. Simulation results were shown that proposed ASCE-NLMS methods achieved better performance than standard ACE-NLMS method without scarifying much computational complexity. The simulation results are also shown that proposed ASCE-NLMF methods are even better than ASCE-NLMS methods while scarifying amount of computation complexity.

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