SB-8-1

Equivalence of Time- and Frequency-domain Channel Estimation for Multicarrier Signal Reception

Shinsuke Takaoka and Fumiyuki Adachi
Department of Electrical and Communication Engineering
Graduate School of Engineering, Tohoku University

1. Introduction

Ultra high-speed data transmission capability will be required in the next generation wireless communication systems in a severe frequency-selective fading channel [1]. In a mobile radio channel, the transmitted signal is reflected and diffracted by many obstacles and is received as a multipath signal with different time delays at a receiver, thus creating the frequency-selective fading. Recently, the multicarrier technique, e.g., orthogonal frequency division multiplexing (OFDM) and multi-carrier code division multiple access (MC-CDMA), has been attracting much attention [2], [3]. For achieving high-quality data transmissions, the use of coherent modulation rather than differential modulation is essential. So far, many channel estimation schemes in time- and frequency-domain have been proposed for coherent detection of multicarrier signals in a severe frequency-selective fading channel [4]-[6].

In this paper, time-domain representation of the frequency-domain channel estimation is presented and equivalence of time- and frequency-domain channel estimation is pointed out.

2. Time domain representation of frequency-domain channel estimation

Frequency-domain channel estimation using 2M+1-tap FIR filter is considered for OFDM and MC-CDMA with N_c subcarriers. The filter structure is illustrated in Fig. 1. Channel estimation is carried out using the noisy instantaneous channel gains $\{\hat{H}_k; k=0\sim N_c\text{-}1\}$, which are obtained from received pilot symbols or reverse modulation to remove data-modulation from the received subcarriers, where k represents the subcarrier index. Using 2M+1 tap weights $\{W_k;\ k=-M\sim+M\}$, the channel gain $\tilde{H}_k$ at the k-th subcarrier is estimated as

$$\begin{aligned}\tilde{H}_k &= \sum_{m=-M}^{M}\hat{H}_{k-m}W_m \\ &= N_c\sum_{\tau=0}^{N_c-1}\left\{\hat{h}(\tau)w(\tau)\right\}\exp(-j2\pi k\frac{\tau}{N_c})\end{aligned}, (1)$$

where $\hat{h}(\tau)$ and $w(\tau)$ represent the channel impulse response corrupted by noise and the delay-time window, respectively. In the above, edge effect in the frequency-domain filtering is neglected. Eq. (1) implies that the frequency-domain filtering is equivalent to multiplying the noisy impulse response estimate by delay-time window. Therefore, the problem of finding the frequency-domain filter tap weights can be replaced by finding the good delay-time window. Once the delay-time window is found, $\hat{h}(\tau)w(\tau)$ can be computed and then, using N_c-point FFT, the channel gain at the k-th subcarrier is obtained.

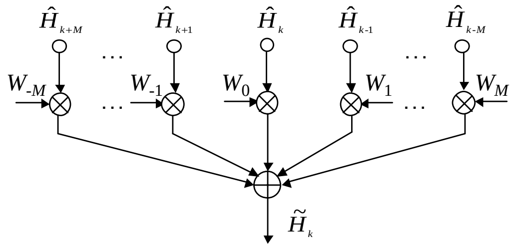

Fig. 1 Frequency-domain FIR filter structure.

3. Numerical results

The multipath fading channel is assumed to have an L=8-path exponential power delay profile with decay factor α and the time delay separation of τ samples. Multicarrier modulation using N_c=256 subcarriers and QPSK data modulation is assumed.

A 2M+1-tap frequency-domain filter based on minimum mean square error (MMSE) criterion [7] is considered. The optimum tap weight vector $\mathbf{W}_{opt}=\{W_k;\ k=-M\sim+M\}$ is given by

$$\mathbf{W}_{opt}=\left(\mathbf{R}_f+\frac{1}{\mathrm{SNR}}\mathbf{I}\right)^{-1}\mathbf{r}_f\ , (2)$$

where $\mathbf{I}$ is the identity matrix, $\mathbf{R}_f$ is 2M+1-by-2M+1 frequency correlation matrix having the element $\rho_{j,k}=E[H^*_{m+j}H_{m+k}]$ for $|j|\le M$, $|k|\le M$, and $\mathbf{r}_f$ is 2M+1-by-1 frequency correlation vector having the element $\sigma_k=E[H^*_m H_{m+k}]$ for $|k|\le M$. Applying N_c-point FFT to $\mathbf{W}_{opt}$ gives the delay-time window $\mathbf{w}=\{w(t);\ t=0\sim N_c\text{-}1\}$. In the following, how the delay-time window is impacted by various parameters is discussed, e.g., the number of frequency-domain filter taps, the power delay profile shape, the time delay separation between adjacent path and the average received signal energy per bit-to-noise power spectrum density ratio E_b/N_0.

First, we will show the impact of the number of taps on the delay-time window in Fig. 2. Since we are assuming L=8 and delay time separation of τ=8 samples, the real channel impulse response becomes zero after 56 samples. As seen from Fig. 2, however, if too small value of M is used, e.g., M=6, the delay-time window width becomes wider than the maximum time-delay difference.

This result can be used to determine the frequency-domain filter size. Another implication of Fig. 2 is that for the time-domain channel estimation, first we estimate the noisy impulse response and then multiply the impulse response estimate by the delay-time window only inside the guard interval as proposed in [4], [5]. Proper choice of the delay-time window shape could result in a better channel estimation than frequency-domain channel estimation.

As the decay factor α increases for the given number *L* of paths, the effective number of paths becomes less. This is clearly indicated in Fig. 3. The smaller width of delay-time window can be used as α increases. The same is true when the time-delay separation between the adjacent paths becomes shorter for the given number *L* of paths. This is seen in Fig. 4. Assuming an exponential power delay profile, for a low average E_b/N_0 region, the delay-time window width can be made narrower since the contribution of weaker paths becomes less (or the effective number of paths becomes less as the average E_b/N_0 decreases). This is clearly indicated in Fig. 5.

4. Conclusion

In this paper, time-domain representation of the frequency-domain channel estimation was presented and equivalence of time- and frequency-domain channel estimation was pointed out. The frequency-domain channel estimation is equivalent to multiplying the noisy impulse response estimate by delay-time window. Therefore, the problem of finding the frequency-domain filter tap weights can be replaced by finding the delay-time window. In this paper, the equivalent delay-time widow was obtained for the frequency-domain MMSE FIR filtering. The study on time-domain channel estimation is ongoing to find the optimum delay-time widow. The result will be reported in a succeeding paper.

Acknowledgement

This work is supported by Grant-in-Aid for JSPS Fellows.

Reference: [1] F. Adachi, IEICE Trans. Fund., Vol. E84-A, pp.55-60, Jan. 2001. [2] L. J. Cimini, Jr., IEEE Trans. Commun., pp. 665-675, July 1985. [3] S. Hara et al., IEEE Commun. Mag., pp. 126-144, Dec. 1997. [4] J.-J. van de Beek et al., in Proc. VTC'95, pp. 715-719, July 1995. [5] T. Fukuhara et al., in Proc. VTC'03, pp. 2343-2347, April 2003. [6] S. Takaoka et al. in Proc. VTC'03, pp. 1576-1679, April 2003. [7] S. Haykin, *Adaptive filter theory,* Prentice Hall, 1996.

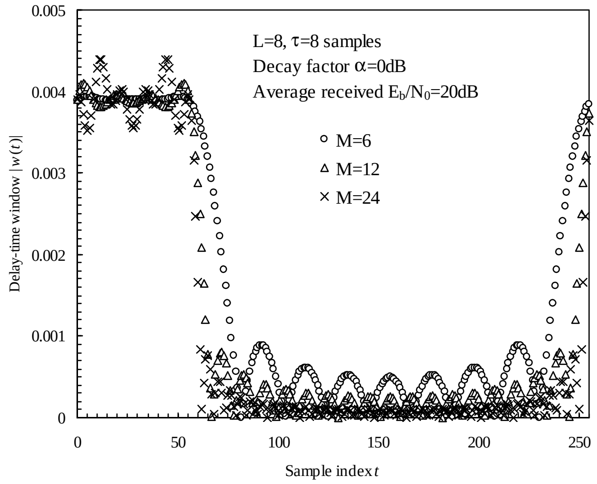

Fig. 2 Impact of number 2*M*+1 of taps.

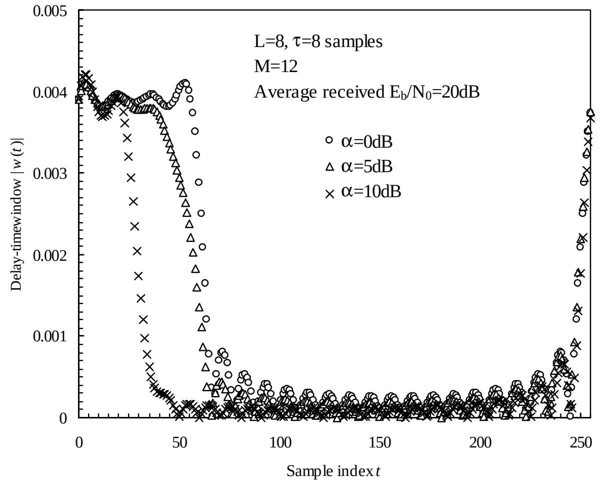

Fig. 3 Impact of the decay factor *α*.

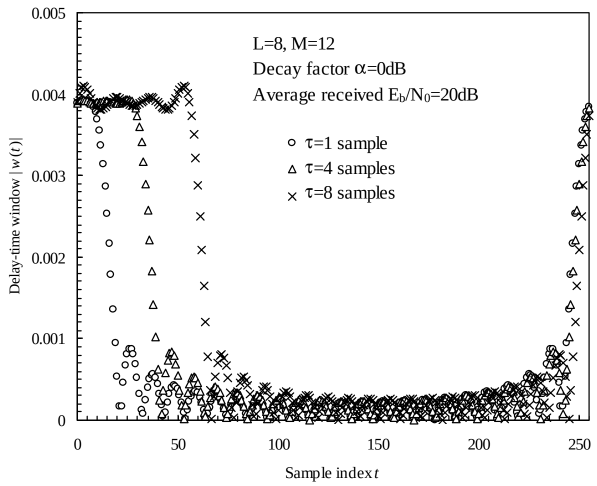

Fig. 4 Impact of the time delay separation *τ*.

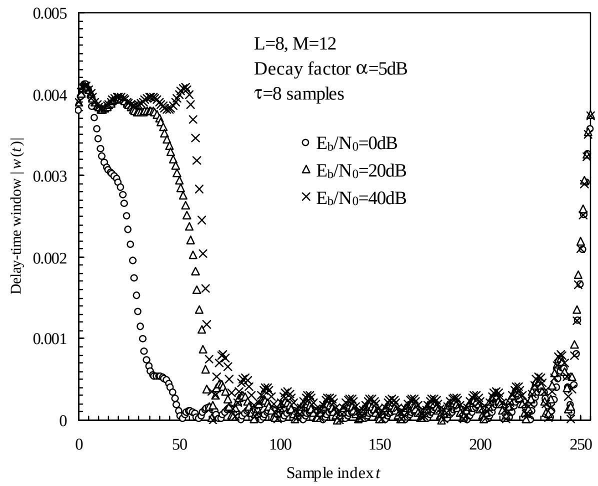

Fig. 5 Impact of average received E_b/N_0.