

Dantzig Selector Algorithm for Sparse Multipath Channel Estimation

Guan Gui^[1,2], Qun Wan^[1] and Fumiyuki Adachi^[2]

[1] Dept. of Electrical Eng., University of electrical Science and Technology of China, Chengdu, 611731, China

[2] Dept. of Electrical and Commu. Eng., Graduate School of Eng., Tohoku University, Sendai, 980-8579, Japan

E-mail: gui@mobile.ecei.tohoku.ac.jp, wanqun@uestc.edu.cn, adachi@ecei.tohoku.ac.jp

1. Introduction

Sparse multi-path channels (MPC) are often encountered in high data rate wireless communication applications. Conventional linear MPC methods, such as the least squares (LS), do not exploit the sparsity [1] of MPC. Based on the compressed sensing (CS) theory[2], accurate sparse MPC estimator can be obtained by solving a Lasso problem even in the presence of noise. In this paper, a novel CS-based sparse MPC method by using Dantzig selector (DS) [3] is proposed. This method exploits a channel's sparsity to reduce the length of training sequence and, hence, increase spectral efficiency. Simulation results demonstrate a significant reduction of the number of training sequence compared to the LS and Lasso algorithms.

2. Channel Model and DS Algorithm

In a typical sparse multipath channel estimation scenario, we try to estimate a channel vector $\mathbf{h} \in \mathbb{R}^L$ based on the standard underdetermined linear model

$$\mathbf{y} = \mathbf{X}\mathbf{h} + \mathbf{z}, \quad (1)$$

where $\mathbf{y} \in \mathbb{R}^n$ is the observation vector at the receiver, $\mathbf{X} \in \mathbb{R}^{n \times L}$ is a known training matrix with columns normalized to unit ℓ_2 norm at the transmitter, and $\mathbf{z} \sim N(0, \sigma^2 \mathbf{I})$ is an independent and identically distributed (i.i.d.) measurement noise vector, where σ^2 is the noise power. Based on the compressed sensing theory [2], we assume that MPC vector \mathbf{h} is S -sparsity [1], where S ($1 \leq S \ll L$) is the number of nonzero taps in \mathbf{h} . If we want to acquire accurate channel state information, designing a training matrix of appropriate structure is a crucial procedure on sparse channel estimation [4]. In the context of CS, accurate sparse MPC estimation is guaranteed if the training signal obey a restricted isometry property (RIP) [5]. The S -restricted isometry constant of $n \times L$ training matrix \mathbf{X} , denoted by δ_S , is defined as the smallest value such that

$$(1 - \delta_S) \|\mathbf{h}\|_2^2 \leq \|\mathbf{X}\mathbf{h}\|_2^2 \leq (1 + \delta_S) \|\mathbf{h}\|_2^2, \quad (2)$$

hold for any S -sparsity vector \mathbf{h} . Then the training signal matrix \mathbf{X} is said to satisfy RIP of order of S if $\delta_S \in (0, 1)$. Given the channel model in (1), the proposed algorithm for sparse MPC estimation can be described as follows:

$$\min_{\mathbf{h} \in \mathbb{R}^L} \|\mathbf{h}\|_1 \quad \text{subject to} \quad \|\mathbf{X}^T(\mathbf{y} - \mathbf{X}\mathbf{h})\|_\infty \leq \lambda_{DS} \cdot \sigma, \quad (3)$$

where $\lambda_{DS} = \sqrt{2 \log L}$ is a regularized parameter [3], and here, $\|\cdot\|_1$ and $\|\cdot\|_\infty$ denote ℓ_1 -norm and ℓ_∞ -norm of a vector, respectively.

3. Simulation and Conclusion

The parameters used in the simulation are listed in Tab. 1. To illustrate the performance of proposed algorithm, Figure.2 shows the nonzero taps estimation error by employing LS, Lasso, DS

and ideal (known positions of nonzero taps). The estimation error using root mean square error (RMSE) evaluation criterion can be defined as:

$$\text{RMSE} = \sqrt{1/M \sum_{m=1}^M \|\mathbf{h} - \hat{\mathbf{h}}_m\|_2^2}.$$

Simulation results clearly demonstrate that proposed CS-based sparse MPC estimation method using DS algorithm can reduce the length of training sequence compared to LS and Lasso. Thus the proposed DS algorithm can increase the spectral efficiency.

Tab. 1 Simulation Parameters

Estimation methods	Linear	Convex	
	LS	Lasso	DS
Channel model	Frequency-selective Rayleigh fading		
Channel length L	120		
Taps amplitude	[-0.8, -0.2] \cup [0.2, 0.8]		
Num. of nonzero taps	6 (distributed uniformly over \mathbf{h})		
SNR	10dB		
Training signal \mathbf{X}	Random Toeplitz structure sequence		
Training signal length	50~150		
Monte Carlo	$M=1000$ trails		

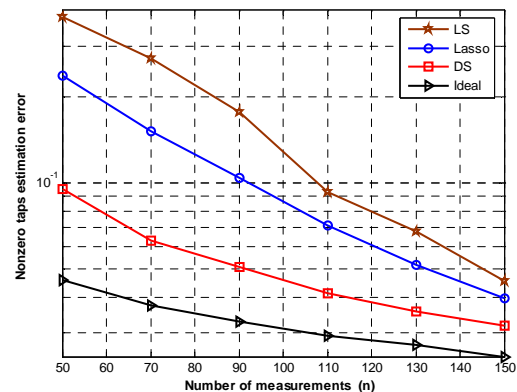


Fig. 2 RMSE of LS, Lasso, DS and ideal changing with number n of measurements of training signal.

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